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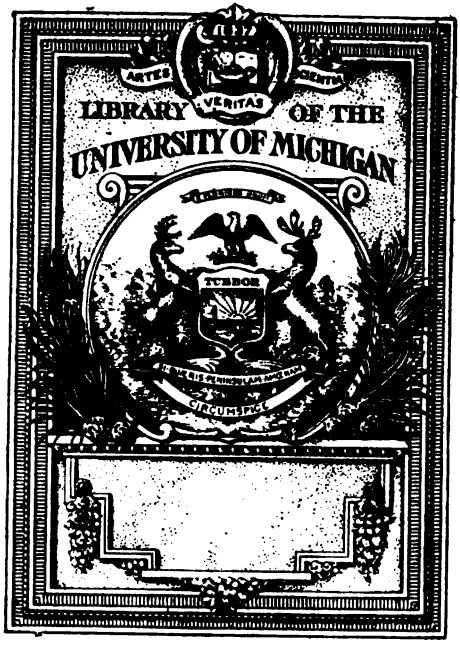
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A

TREATISE

OF

ARITHMETIC,

IN

Whole Numbers & Fractions.

Wherein all the

Neceſſary Practical RULES are laid down in the *Plaineſt* and *moſt Familiar Terms*, and the *Truth* of each RULE demonſtrated.

To which is Annex'd,

A Large *APPENDIX,*

Containing many *neceſſary* TABLES, and a great Variety of *curious* and *uſeful* QUESTIONS, intermix'd with *inſtructive* DIRECTIONS and *proper* REMARKS.

The Whole being Deſign'd and Compos'd, not only for the Benefit of *Young Students*, but of *Thoſe* who are deſirous of being *rationally* skill'd in the *Science* of ARITHMETIC.

By THOMAS WESTON, late Maſter of the *Academy* at *Greenwich*: And Publiſh'd by JOHN WESTON, *now* Maſter thereof.

LONDON:

Printed for J. HOOKE, at the *Flower-de-Luce*, over-againſt St. *Dunſtan's* Church in *Fleet-ſtreet*.

M.DCC.XXIX.

TO THE

YOUNG GENTLEMEN

OF THE

ACADEMY

IN

GREENWICH,

For *whoſe* USE it was *Originally* COMPOS'D,

THIS

TREATISE

OF

ARITHMETIC

Is now *reſpectfully* and *publickly* Dedicated,

By their moſt Faithful,

And moſt Humble Servant,

J. WESTON.

THE

PREFACE.

THE *Excellency and Univerſal Uſefulneſs of my* Subject *is allow'd by all; ſo far, therefore, I am ſecure.*

All Mankind ought, in ſome meaſure or other, to underſtand Arithmetic; *and (as Taſtes are different) I may, poſſibly, pleaſe and help ſome.*

If haply then any ſhall like my Manner of Handling this important Subject, and allow my Rules *to be plainer, and my Demonſtrations of their Truth to be more intelligible, than they can find elſewhere, I have my End.*

What

PREFACE.

What Rules *I have thought neceſſary to meddle with here, may be ſeen in the Table of the Contents.* Logarithms, Compound Intereſt, *and the Buſineſs of* Arithmetical *and* Geometrical Progreſſionals *I ſhall treat of elſewhere.*

I have, indeed, omitted ſome Things commonly met with in Books of Arithmetic, *concerning Operations in* Addition, Subtraction, Multiplication, *and* Diviſion *of* Whole Numbers, *as alſo ſome Compendious Methods of managing ſome of them, it being more my preſent Deſign to give the* Foundations *of theſe Operations, to demonſtrate the Reaſon of every Step taken in them.*

The young Arithmetician, taught in this Way, will not only be able to perform theſe Operations with much more Profit and Delight, than he can poſſibly do, who has only learn'd them by Rote; but will alſo be able longer to remember what he has learn'd.

For the Complaints often heard, viz. *That the* Mathematicks, *and* (*particularly*) Arithmetic,

metic, *are ſoon* loſt, *unleſs they are daily applied to, can ariſe from nothing ſo much as this,* viz.

That moſt *are taught the* Practice *of Arithmetic, or the Methods of Working by* heart *only, and of courſe are wholly ignorant of the Reaſons of thoſe Operations.*

Whilſt he who does not with an implicit Belief follow Rules *got only by* heart, *but on the contrary, is led by* Judgment *more than* Memory, *will hardly* ever *loſe what he has once gain'd in ſo* rational *a Way.*

Now how far this Treatiſe (wherein nothing is left unexplain'd) is capable of contributing to the End propos'd, I do (without vainly hoping to eſcape the Cenſorious) humbly leave to the Judgment of the Candid and Impartial.

T. W.

The PUBLISHER

TO THE

READER.

I Shall ſay little or nothing of this Performance particularly, but leave that to ſpeak for itſelf; and no more of the Author than many, to whom he was known, can juſtify, (and ſo much, in Juſtice to him, I ought to ſay); which is, That he was Eloquent and Judicious; he had a juſt and happy Way of Expreſſing himſelf; and in his Manner of Communicating his Sentiments, and Imprinting his Inſtructions upon the Minds of his Pupils, he excell'd moſt Men.

The Reader therefore, I think, need not doubt of finding in this Treatiſe *(which comes from one thus qualified) ſomething ſerviceable to him, and well worth his Acceptance.*

J. W.

Hon. OF Lucky ARITHMETIC.

CHAP. I.

ARITHMETIC takes its Name from its * *Object*, which is *Number*, and is the Art of rightly accompting or reckoning by *Numbers*.

* ἀπὸ τῆ ἀριθμῆ.

Numbers are either *Whole* or *Broken*.

A *Whole* Number or *Integer* consists of one or more Units; and

A *Broken* Number, or *Fraction*, of Part, or Parts of an Unit.

The common *Notes* or *Characters* by which we express Numbers, whether *Whole* or *Broken*, are these following, *viz.*

1	2	3	4	5	6
One,	Two,	Three,	Four,	Five,	Six,
7	8	9	0		
Seven,	Eight,	Nine,	Nothing.		

The laſt of which, tho' by itſelf it ſignifies *Nothing*, yet when joined to the right Hand of any one or more of the aforeſaid *Notes*, *Characters*, or *Figures*, it increaſeth its Value Ten Times.

ARITHMETIC hath Two Parts, *Notation* (commonly called *Numeration*) and *Numeration* (properly ſo called).

Notation teaches how to *deſcribe*, or *write down* any *Number* by the aforeſaid Characters; or to *read* the *Value* of any *Number* already *deſcribed* or *wrote down*.

Numeration teaches us to *account* by Numbers; it conſiſts of Four Parts, *viz. Addition*, *Subtraction*, *Multiplication*, and *Diviſion*.

But before I treat of theſe, I ſhall premiſe, That ſince there is no Number ſo great, but one or more Units may be added to it, the Variety of Numbers muſt be *infinite*; and therefore the Pains of contriving diſtinct Names to each Number, or to keep them in Memory, if found, and to apply them to Uſe, muſt alſo be *infinite*.

But now, to ſave this Labour of finding *new Words* for each Acceſſion of Unity, by which the Numbers produced might be named, the Sagacity of the Antients hath invented (or rather, the divine Indulgence hath ſhewn) a Method, never to be enough admired, by which they might deſign this infinite Variety of Numbers by Terms or Names *Finite* in Number; and, indeed, by a *very few*, as ſhall be ſhewn in the next Place.

To every one, then, of the Ten firſt and leaſt Numbers they gave a Name, calling them *One*, *Two*, *Three*, *Four*, *Five*, *Six*, *Seven*, *Eight*, *Nine*, *Ten*.

After

After this, they rather numbered *Tens* than *Units*, *viz.* One *Ten*, two *Tens*, three *Tens*, *&c.* to ten *Tens*; or (as they ſhorter ſpeak) *Ten*, *Twenty*, *Thirty*, *Forty*, *Fifty*, *Sixty*, *Seventy*, *Eighty*, *Ninety*, One *Hundred*; for *Twenty* is the ſame as two *Tens*, *Thirty* as three *Tens*, *&c.* and an *Hundred* is of the ſame Value with ten *Tens*, or *One Hundred.*

After they came to an *Hundred*, they rather number'd *Hundreds*, than *Tens* or *Units*, *viz.* One *Hundred*, two *Hundreds*, three *Hundreds*, 4, 5, 6, 7, 8, 9, 10 *Hundreds*, or ſhorter, an *Hundred*, two *Hundred*, three *Hundred*, *&c.* to a *Thouſand*; for it's the ſame thing to ſay two *Hundred*, as two *Hundreds*, three *Hundred*, as three *Hundreds*, *&c.* and a *Thouſand*, as *Ten Hundreds.*

But when they came to a *Thouſand*, they rather numbered *Thouſands*, than *Hundreds*, *Tens*, or *Units*, *viz.* A *Thouſand*, two *Thouſand*, three *Thouſand*; or, one *Thouſand*, two *Thouſands*, three *Thouſands*, *&c.* to *Ten Thouſands*, *Hundreds* of *Thouſands*, *Thouſands* of *Thouſands*, *Tens* of *Thouſands* of *Thouſands*, &c. *ad Infinitum.*

Thus 'tis plain, that a very few Words are ſufficient to expreſs Numbers in a manner *infinite*; for theſe *Twelve* Words, *One*, *Two*, *Three*, *Four*, *Five*, *Six*, *Seven*, *Eight*, *Nine*, *Ten*, a *Hundred*, a *Thouſand*, with their *Conjugates*, are ſufficient to expreſs All *Numbers whatſoever.*

Theſe being premiſed, we ſhall, in the next Chapter, fully treat of the *Notation* of Numbers.

CHAP. II.

NOTATION (as has been already ſaid) teaches how to *deſcribe* or *write down* any *Number* by the foremention'd *Characters*; and to *read* the *Value* of any *Number* already *deſcribed* or *wrote down*.

But now the *Notation* of Numbers is not the ſame in ALL Nations; for altho' all agree in this, *viz.* To deſcribe Numbers, for Brevity's ſake, by *ſome* Characters or *other*, yet different Nations have uſed different Methods of doing this: Thus for Inſtance, The * *Hebrews*, *Greeks*, and *Romans*, denoted Numbers by the Letters in their Alphabets, inſtead of our Characters: But having juſt mentioned this, I ſhall come at once to the Method which is of greateſt Moment; and herein are uſed the common numeral Figures before mentioned, which are, as has been ſhewn, *Ten* in Number, *viz.* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, whoſe Values we ſhall now at large explain.

* See my Writing-Book.

The firſt (0) ſignifies *Nought*, or *Nothing*, and is commonly called a *Cypher*; the other Nine ſignify ſo many *Numbers*, *viz.* 1, *One*; 2, *Two*; 3, *Three*, *&c.* as in the laſt Chapter; and theſe are the *firſt* or *primary* Values of them, *viz.* which they by *themſelves* ſignify; for they have *ſecondary* ones, which they aſſume, not when ſet by themſelves, but according to the Place where they are found; for as it is one thing to ſay two *Angels*, and another, two *Men*; ſo alſo, it is one thing to ſay two *Units*, and another, two *Tens*, or two *Hundreds*;

dreds; for altho' in all theſe, the *numbering* Number, or *Numerator* be the ſame (*viz. Two*) yet the Thing *numbered*, or *Denominator*, is not the ſame; and this *ſecondary* Value of the Figure is *occaſionally* changed, according to the *Place* it *poſſeſſes*, or is found in.

Thus, that Figure which, to the *Right Hand* is *Firſt*, numbers ſo many *Units* as its Value ſhews; that in the *Second*, ſo many *Tens*; that in the *Third*, ſo many *Hundreds*; that in the *Fourth*, ſo many *Thouſands*; that in the *Fifth*, ſo many *Tens* of *Thouſands*; that in the *Sixth*, ſo many *Hundreds* of *Thouſands*; that in the *Seventh*, ſo many *Thouſands* of *Thouſands*, or, if you will, ſo many *Millions*; the *Eighth*, ſo many *Tens* of *Millions*; the *Ninth*, ſo many *Hundreds* of *Millions*; the *Tenth*, ſo many *Thouſands* of *Millions*, or *Thouſands* of *Thouſands* of *Thouſands*, *&c.* every Place always increaſing the Value, by a continual *decuple* Proportion.

Thus, then, the Number exhibited by theſe Characters or Figures, 2684, is Two Thouſand, Six Hundred, Eighty-four, or Two *Thouſands*, Six *Hundreds*, Eight *Tens*, and Four *Units*; This, I ſay, is called the *ſecondary* Value of Figures, becauſe it ariſes from the *Seat* or *Place* where the Figure is found.

But now, theſe *very* Places (that they may be the better underſtood, eſpecially where there be *many* of them) are uſually diſtinguiſhed into *Periods*, each *Period* conſiſting of *Six* Figures; and *each Period* again into *half Periods*, or *leſſer Periods*, each conſiſting of *Three* Figures; and theſe Periods (eſpecially where there are many of them, as in great Numbers) moſt Authors uſually diſtinguiſh, by placing *Points* over them, or by ſome other Way at Pleaſure, by which the Numbers may be more eaſily read.

Now the *Firſt* greater Period denotes *Units*; the *Second*, *Millions*; the *Third*, *Millions* of *Millions*; the *Fourth*,

Fourth, *Millions* of *Millions* of *Millions*, &c. ſo that if the Learner rightly underſtand how to read a Number of Six Figures, he may eaſily read any other Number whatſoever. Again,

The *Firſt leſſer* Period denotes alſo *Units*; the *Second*, *Thouſands*; the *Third*, *Thouſands* of *Thouſands*, or *Millions*, &c.; for, in this Caſe, as one Place exceeds another in a *decuple* Proportion, ſo one *Period* exceeds another in a *millecuple* one; thus, then, if the Learner can read a Number of *Three* Figures, he has, in effect, learn'd to read any other Number whatſoever.

Thus, for Example, this Number, 5438912769, is thus read, Five Thouſand, Four Hundred, and Thirty-eight *Millions*, Nine Hundred and Twelve *Thouſand*, Seven Hundred and Sixty-nine; and after this Manner muſt we judge of all other Numbers, as well where a given or propoſed Number is to be *wrote down*, as where a Number, already wrote down, is *to be read*; for in both theſe, the Reaſon is the ſame.

Here Note, That the *Cypher*, altho' by itſelf, *whereſoever* it be placed, ſignifies *Nothing*; for it is all one whether we ſay, *no Units*, or *no Tens*, or *no Hundreds*, or *no Thouſands*, &c. yet this *Cypher* is often of Neceſſity made uſe of to fill a Place, ſo that all the Figures in Places above (or to the left Hand of) it, may obtain their *proper* Value: Thus for Inſtance, If in this Number, 5438012769, the *Cypher* placed in the *Sixth* Place (from the right Hand) be wanting, the next Figure 8, would not be in the *Seventh*, but in the *Sixth* Place, and therefore would not ſignify 8 *Thouſands* of *Thouſands*, but 8 *Hundred Thouſands*; The ſame muſt be judged of the remaining ones, *viz.* The Figure 3 would not be, as now, in the *Eighth*, but in the *Seventh* Place, and 4 in the *Eighth*, and 5 in the *Ninth*; and thus their Values would be *Ten times leſs*.

Thus

Thus again, 1 placed by itſelf, ſignifies *One* (becauſe it's in the *Firſt* Place); 10, *Ten* (becauſe 1 is in the *Second* Place; for altho' the adjoined Cypher [0] by itſelf ſignifies *Nothing*, yet it cauſes that the Figure 1 is now in the *Second* Place, which, without it, would have been in the *Firſt*) thus 100 ſignifies *One Hundred*, &c. but if Cyphers are placed in the laſt Place (*i. e.* to the left Hand) they are of no uſe at all in whole Numbers, at leaſt, there's no Neceſſity for them; for 0001, 001, 01, and 1, all ſignify the ſame Thing, *viz.* One *Unit*; ſometimes, indeed, for Neatneſs ſake, or that Numbers, perhaps, may be more readily added (eſpecially when you are employed with Numbers of *different* Denominations) *Cyphers may be* placed to the *left* Hand, as in the following Example.

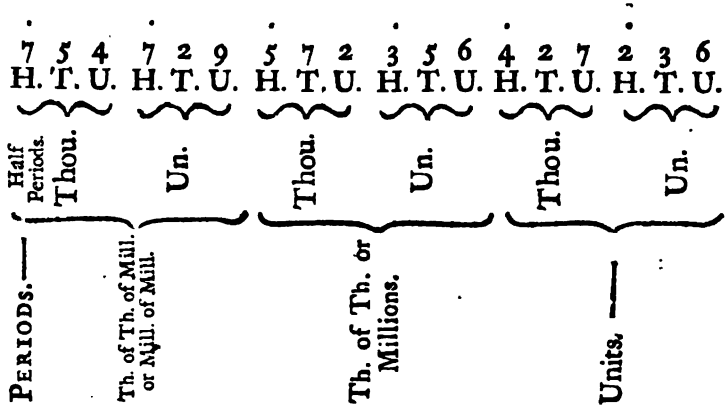
l.	*s.*	*d.*
19	16	04
52	08	10
08	06	05

But all the Cyphers *ſo placed*, in this, and all other Examples, ſignify no more, than if they were wanting.

This may be ſufficient for explaining the *ordinary* or *common Notation*, or, as ſome call it, *Numeration*.

But before I quit this Subject, I ſhall inſert the following *Table* (grounded on what has been ſaid in this Chapter) for the Uſe of young Beginners.

The NOTATION *Table.*



Tho' in the foregoing *Table* I have noted but 18 Figures, yet from it, and what has been ſaid before, we may learn to value any other Number, how great ſoever (tho' larger Numbers are very ſeldom met with in Practice) thus one Figure *more* to the left Hand in the Table, would have ſignified ſo many *Tens* of *Millions* of *Millions*, as there are *Units* in that Figure ſingly taken: Thus again, one Figure *more* would have been ſo many *Hundreds* of *Millions* of *Millions*, *&c.* Thus the Value of the following Number, conſiſting of *Thirty* Figures, *viz.*

859590638476754729572356427236

Will be Eight Hundred Fifty-nine thouſand, Five Hundred and Ninety *Millions* of *Millions* of *Millions* of *Millions*; Six Hundred Thirty-eight Thouſand, Four Hundred Seventy-ſix *Millions* of *Millions* of *Millions*; Seven Hundred Fifty-four Thouſand, Seven Hundred and Twenty-nine *Millions* of *Millions*; Five hundred Seventy-two Thouſand, Three Hundred Fifty-ſix *Millions*; Four

Four Hundred Twenty-ſeven *Thouſand*, Two Hundred Thirty-ſix.

Here indeed it muſt be owned, that by the foregoing ordinary Way of reading this Number, we can hardly have any clear Notion of it; but by giving new and orderly Denominations, theſe, and many more Figures, if needful, may, perhaps, be more eaſily and diſtinctly *read*, and *Ideas* of them got more eaſily to ourſelves, and more plainly ſignified to others; as the following Example will evince.

897643,	291678,	546789,	987654,	954321,
Nonilions,	Octilions,	Septilions,	Sextilions,	Quinquillions,
659863,	960543,	953267,	854986,	736598,
Quadrillions,	Trillions,	Billions,	Millions,	Units.

But of this enough, and, may be, more than was needful.

Before I enter upon the 3d Chapter, I will inſert ſome few *Characters* or *Marks*, which hereafter may more conveniently be uſed, than the *Words* they are made to ſignify; and they are theſe that follow.

This Character	+	*Stands for*	Added to.
	−		Subtracted from.
	×		Multiplied by.
	÷		Divided by.
	=		Equal to.

The next Chapter treats of NUMERATION, *i.e.* Of ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION *in General.*

CHAP. III.

THESE Four Operations, *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*, are in Reality but Two, *viz.* *Addition* and *Subtraction*; The Sign of the former being, as above noted, + ; and that of the latter —. Thus 3 + 4 = 7. (*i. e.* 3 added to 4, is equal to 7) And 8—2=6. (*i. e.* 8 leſs 2, is equal to 6) but now if you would have the Number 4 not to be added only *once* to the Number 3, as in the foregoing Example of *Addition*, but ſuppoſe it to denote the *Number* of times, the Figure 3 is to be added to itſelf (*viz.* 4 times) then ſuch ſort of *Addition* is called by the peculiar Name of *Multiplication*, whoſe Sign is, as before ſhewn, ×: Thus 3 × 4 (*i. e.* 3 + 3 + 3 + 3) = 12. In like manner, if you would have the Number 2, not to be taken only *once* from 8 (as in the foregoing Example of *Subtraction*) but ſuppoſe it to denote how often it can be taken from it, then this Sort of *Subtraction* is called by the peculiar Name of *Diviſion*, whoſe Sign is, as before ſhewn, ÷: Thus 8 ÷ 2 = 4, or thus, 2) 8 (4, or $\frac{8}{2} = 4$; *i. e.* it may be taken 4 times out of 8, or 8 — 2 — 2 — 2 — 2 = 0.

Here follow ſome uſeful *Notes*, and *General Rules*, to be carefully obſerved in *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*.

NOTE I.

NOTE I.

EVERY Number may be ſaid to be compoſed of two Parts, one whereof may be called the *Numerator*, the other the *Denominator*; *This* teaches WHAT is numbered; *The former*, HOW OFTEN.

Thus in the Number 1717 Years; *Years* are the Things *Numbered*, or the *Denominator*, and 1717 the Thing *Numbering*, or the *Numerator*.

NOTE II.

BECAUSE *this* Denominator *(viz. Years)* does not *eſſentially* belong to the Numerator 1717, (for 1717 will always ſignify 1717, let the Things *numbered* be *Years*, *Hours*, *Minutes*, or what elſe you pleaſe) I call it the *Secondary* Denominator.

NOTE III.

BUT now the Denominators, or Denominations of *each* Figure of the Numerator 1717, *viz. Units*, *Tens*, *Hundreds*, *Thouſands*: *Theſe* I call *Primary* Denominators, becauſe theſe *eſſentially* belong to the Numerator; for if they were changed, the Numerator *itſelf* muſt inevitably be changed alſo.

GENERAL RULE I.

PLACE every Figure of the ſame Denomination (whether *Primary* or *Secondary*) directly under each other: Thus, in *Primary* Denominations, place *Units* under *Units*, *Tens* under *Tens*, *&c.* Again, in *Secondary* ones, place, for Inſtance, *Years* under *Years*, *Hours* under *Hours*, *&c.*

Pounds under *Pounds*, *Shillings* under *Shillings*, *&c.* not that there is an abſolute Neceſſity for this, but 'tis done for the greater Regularity and Eaſe in working.

GENERAL RULE II.

RESOLVE the particular Sum of *every* Denomination (as well *Secondary* as *Primary*) into as many of the next ſuperior Denominations, as it can be reſolved into: Thus, 24 *Units* may be reſolved into 2 *Tens* and 4 *Units*; 30 Pence into 2 Shillings and 6 Pence, *&c. each* to be put down ſeparately in their proper Places, according to the *Firſt Rule*: This not out of an abſolute Neceſſity, but becauſe 'tis the more regular and eaſy Way.

Since it is abſolutely neceſſary to the right obſerving the Second General Rule foregoing, that the Learner know exactly the *Ratio* or *Proportion*, according to which the various Denominations of Things (at leaſt of thoſe for our preſent Purpoſe) proceed: I have therefore, in the APPENDIX, placed ſeveral *Tables* very uſeful to that End, which may be applied to on Occaſion.

Before I end this Chapter, the young Arithmetician may be told, in order to his better underſtanding the *Theory* of *Addition*, and its following Rules, that in *Arithmetick*, as well as *Geometry*, and other *Diſciplines*, certain ſelf-evident *Principles*, muſt be of Neceſſity granted, by Help of which, other Things may be proved; which Principles, are ſuch as no-body can deny or doubt their Truth, if only the bare Words be underſtood: Of this kind are thoſe AXIOMS or *common Notes*, which are premiſed in the Elements of EUCLID, moſt of thoſe being as applicable to *Arithmetick*, as they are to *Geometry*: Nor, indeed, are they propoſed by EUCLID as applicable to *Corporeal Magnitudes* only, but ſuch as may be applied, as Occaſion ſerves, to any *other Quantities*:

From

From them, then, I'll take what may ſerve my preſent Purpoſe, and they are theſe that follow.

AXIOMS.

I. THINGS *equal* to the *ſame* Thing, or to *equal* Things, are *equal* to one another.

II. If *equal* Things be added to *equal* Things, or to *one* and the *ſame* Thing, the *Wholes* will be *equal* to one another.

III. If from *equal* Things be taken away *equal* Things, or *one* and the *ſame* Thing, the *Remainders* will be *equal* to one another.

IV. If to *unequal* Things *equal* Things or *one* and the *ſame* Thing, be added, the *Wholes* will be *unequal.*

V. If from *unequal* Things *equal* Things, or *one* and the *ſame* Thing, be taken away, the *Remainders* will be *unequal.*

VI. Things which are *double* to the *ſame* Thing, or to *equal* Things, are *equal* one to another: Underſtand the ſame of *Triple*, *Quadruple*, *&c.*

VII. Things which are the *Halfs* of one and the ſame Thing, or of *equal* Things, are *equal* the one to the other: Conceive the ſame of *Subtriple*, *Subquadruple*, *&c.*

VIII. Every *Whole* is greater than its *Part.*

IX. Every *Whole* is *equal* to *all its Parts* taken together.

Theſe *Axioms* or *Maxims* being granted, I may proceed directly to the Explanation of ADDITION.

CHAP.

CHAP. IV.

ADDITION collects Numbers (Numerators) of the *same Denomination*, into one *Sum* or *Aggregate*.

First, *Place* the Numbers to be added, according to the 1st General Rule, after which, you may proceed to their *Addition*; remembering, in Numbers of the *same secondary* Denomination, for every *Ten* in the *Units* Row, to carry *One* to the Row of *Tens*; and for every *Ten* (in Value really 100) in the Row of *Tens*, to carry *One* to the Row of *Hundreds*; and for every *Ten* (in Value really 1000) in the Row of *Hundreds*, to carry *One* to the Row of *Thousands*, &c.

And in Numbers of different *secondary* Denominations; for Instance, *Pounds*, *Shillings*, *Pence*, and *Farthings*; remembering, for every *Four* in the Row of *Farthings*, to carry *One* to the *Pence* Row; and for every *Twelve Pence* in the *Pence* Row, to carry *One* to the *Shillings*; and for every *Twenty Shillings* in the *Shillings* Row, to carry *One* to the *Pounds*; according to the 2d General Rule.

EXAMPLES.

To	4986	To	590867659875
Add	979	Add	900986983989
Total	5965	Total	1491854643864

Add

Add theſe Numbers together, *viz.* 26, 987, 5678, 98765.

			2	6	
			9	8	7
		5	6	7	8
	9	8	7	6	5
Total -	1	0	5	4	5	6

Add *l.* 6 18 9, *l.* 5 14 6¾, *l.* 4 14 9¼, and *l.* 3 0 9½, into one Sum.

	l.	*s.*	*d.*
	6	18	09
	5	14	06¾
	4	14	09¼
	3	00	09½
Total - -	20	08	10½

After the ſame Manner muſt we proceed in other Additions, let the Quantities, or Things to be added, be what they will; the Difference being only *this*, *viz.* That in ſome Caſes, *more* Units, and in others, *fewer*, are = *one* Unit of the *next ſuperior* Denomination; but this, the forementioned Tables in the APPENDIX ſhew at large; there can therefore be no need of any farther Directions as to this Matter.

	Years	M.	W.	D.	H.	M.	S.
	1717	12	3	6	23	59	59
	598	09	2	5	19	46	19
	364	06	0	3	08	38	26
	59	07	3	4	22	49	56
Total	2740	10	3	0	03	14	40

Leag.

Leag.	Mil.	Yards.	Feet	Inch.
563	2	1759	2	11
496	1	986	1	10
99	0	897	0	9
67	2	789	2	8
1227	1	913	2	2

Sig.	°	′	″
1	29	59	59
2	19	36	46
4	18	09	57
2	13	18	09
11	21	4	51

Troy Weight.

Lib.	Oz.	dwt.	Gr.
12	11	19	23
126	09	18	22
19	10	17	19
14	07	19	23
174	4	16	15

Averdup. Weight.

Tuns	Hun.	Q.	P.	Oz.
0	19	3	27	15
0	17	3	19	14
0	14	2	22	15
0	9	1	19	09
3	2	0	6	05

Thus much for the Addition of Numbers of the *ſame* Denomination.

But now, Numbers of *different* Denominations are not *collected* into one Sum, *i. e.* are not added *properly*, but rather *connected* by the intermediate Sign + either expreſſed or underſtood: Thus,

12 Shillings	23 Minutes	6 Angels
10 Pence	15 Seconds	5 Men, *&c.*
s. *d.*	′ ″	A M.
12+10	23+15	6+5

Except the *different* Denominations be firſt reduced into *one* and the *ſame* Denomination, thus 1*s.* + 2*d.* (reducing the Shilling into 12 Pence) the Sum is 14 Pence; but how theſe Reductions are to be made, ſhall be ſhewn in the 8th Chapter of this Treatiſe.

The *Reaſon* of this Rule of *Addition*, is ſufficiently evident from the 9th AXIOM.

The

The moſt *certain* Proof of ADDITION is by SUBTRACTION, as ſhall be ſhewn hereafter, though the Methods commonly taught, are by *Addition*; ſuch are theſe following, Repeat the Operation of *Addition* in a *retrograde* or *inverted* Order, and if the Sum Total ſo found, agree with the former, you have good Reaſon to believe it true; or part the given Numbers into *Two* Parcels (or *more* if you pleaſe) then add up each Parcel by itſelf, and if the particular Sums, ſo found, added together, be = the Sum Total firſt found, then you have good Reaſon to believe you are true; but, if not, Care muſt be taken to diſcover and correct the Error.

EXAMPLES.

l.
985
463
987
296
543
789
649
578
907
546
398
979
8120
7135
8120

l.
985
463
987
296
543
789
649
578
907
546
398
979
8120

l.
2731
2559
2830
8120

To entertain the young Arithmetician, the foregoing *Precepts* may be applied to common Uſes in the following, and ſuch-like Queſtions.

QUESTIONS.

I. A Merchant has 5 Debtors, *A*, B, C, D, E; A owes him *l.* 199 15 04½; B, *l.* 68 13 09½; C, *l.*420 00 10½; D, *l.*48 09 11¾; and E, *l.*419 00 00; What is due in all to this Merchant?

*Anſwer. l.*1156.

II. What Sum is that from which, if you take *l.*468 11 4½, there ſhall remain *l.*40 19 11¼?

*Anſwer. l.*509 11 04.

III. The Moon runs thro' the *Zodiack* in 354 Days, 8 Hours, 48 Minutes (which is the *Lunar* Year) and is 10 Days, 21 Hours, 1 Minute, ſooner than the Sun; what then is the *Length* of the *Solar* Year? or, In what Time does the Sun run through the Zodiack?

Anſwer. 365 Days, 5 Hours, 49 Minutes.

IV. What's the Number of Days between the 1ſt of *Auguſt* 1714, and the 9th of *December* 1715, firſt and laſt Days included?

Anſwer. 496 Days.

To anſwer this, and ſuch-like Queſtions, *See* 2d *Note*, under *Tab.* 2. in the APPENDIX.

See alſo more *Queſtions* in the APPENDIX.

I now proceed to the Explanation of *Subtraction.*

CHAP.

CHAP. V.

SUBTRACTION takes Numbers (*Numerators*) of the *ſame Denomination* from each other; the Number *remaining* after *Subtraction*, ſhewing the *Difference* of the given Quantities, or the *Exceſs* of the greater Quantity above the leſs.

Here, as in *Addition*, Care muſt be taken truly to place the Numbers to be ſubtracted from each other, according to our 1ſt General *Rule*; this being done, begin with the Figure next your right Hand (not out of Neceſſity, but Conveniency) and ſubtract the under Sum from the upper, and ſet down the Remainder.

EXAMPLES.

	l.			*l.*	*s.*	*d.*
From	5987		From	56798	15	09 ½
Take	3643		Take	35686	14	09 ½
Rem.	2344		Rem.	21112	01	00

But if it happens, that any Figure of the Number to be ſubtracted from the other, be *greater* than the Figure (placed over it) from which it ſhould be taken, then the Figure ſo ſet over, muſt be *increaſed*, by borrowing *One* from the next *ſuperior* Denomination.

Thus, in the following Example, I can't take 9 from 6, therefore I borrow 1 (in Value 10) from the next

Row, and add it to 6, which makes it 16, by which Means I am enabled to take 9 from it, and the Remainder is 7; but now we muſt take care that the next Figure in the Number I am to ſubtract from, muſt be ſuppoſed to be *leſs* by *One* than it was before I borrow'd from it (or, which is the ſame thing in effect, tho' not ſo eaſy to the Learner) the Figure next to be taken from it, muſt be ſuppoſed to be encreaſed by an Unit, that Figure you are to take it from, remaining unaltered: Thus then, in the ſame Example, I go on, and either ſay, 7 from 7 and there remains 0; or, 1 and 7 is 8 from 8, and there remains 0.

	4986
	3679
Remainder . -	1307

Thus again, in the following Example, I can't take 2 Farthings from 1 Farthing, I therefore borrow 1 Penny (in Value 4 Farthings) from the next, *viz.* the Pence Row, and add them to 1, which makes it 5, then I can take 2 from 5, the Remainder being 3; then I go on and ſay, 9 Pence from 6 Pence I can't, but, borrowing 1 Shilling (or 12 Pence, from the Row of Shillings) 9 from 18 and there remains 9; or, which comes to the ſame thing, 10 from 7 I can't, but 10 from 19 (borrowing a Shilling) and there remains 9: Then going on, I ſay, 15 from 13 I can't, but, borrowing 1 Pound (or 20 Shillings) 15 from 33, and there remains 18; or, which is the ſame thing, 16 from 14 I can't, but 16 from 34, and there remains alſo 18; then 6 from 8, or 7 from 9, and there remains 2; and 1 from 4, there remains 3; and laſtly, 3 from 5 and there remains 2; ſo that after *l.* 316 15 09 $\frac{1}{2}$ is ſubtracted from *l.* 549 14 07 $\frac{3}{4}$, the Remainder is *l.* 232 18 09 $\frac{1}{4}$.

From

	l.	*s.*	*d.*
From	549	14	07 ¼
Take	316	15	09 ½
Rem.	232	18	09 ¾

After the ſame Manner muſt we proceed in other Subtractions, let the Quantities or Things to be ſubtracted be what they will, the Difference being only *this*, *viz.* That in *ſome* Caſes *more* Units, and in *others*, *fewer*, are = *One* Unit of the *next ſuperior* Denomination: Here can therefore be no need of any farther Directions.

Thus much for the Subtraction of Numbers of the *ſame* Denomination.

But now Subtraction does not ſo much take Numbers of *different* Denominations from each other, as it ſhews, by the intermediate Sign —, that they are *to be* ſo taken: Thus,

12 Shillings	23 Minutes
10 Pence	15 Seconds
s. *d.*	′ ″
12 — 10	23 — 15

Except the *different* Denominations be firſt reduced into *one* and the *ſame* Denomination; Thus, 12 *s.* reduced into Pence, are 144 *d.* then 10 *d.* taken from 144 *d.* leaves 134 *d.* or, 11 *s.* 2 *d.* But how theſe Reductions are to be made, ſhall be ſhewn in the VIIIth Chapter of this Treatiſe.

But now, in *every* Subtraction *actually* to be performed, we muſt note, that the Quantity *to be* ſubtracted, ought to be *leſs* (at leaſt *not bigger*) than the Quantity it is to be taken *from*: for it is impoſſible to take a *greater* Quantity from a *leſs* (ſince the *Part* cannor

not be *greater*, nor *ſo great*, as the *Whole*): Thus, for Example, it is impoſſible to ſubtract 8 from 5.

But now, tho' this, in itſelf, be *impoſſible*, yet it is often required to be done amongſt Arithmeticians, eſpecially amongſt Algebraiſts, as a thing *poſſible*; for they ſuppoſe (beſides *real* ones) certain *imaginary* Quantities, leſs than nothing, which they call *Negative* Quantities: And as (for Diſtinction's ſake, where it is needful) to *true* Quantities (which they call *poſitive* or *affirmative* ones) they prefix this Character, +, the Sign of ADDITION or AFFIRMATION; ſo, to *imaginary* or *negative* Quantities, they prefix this Character, —, the Sign of SUBTRACTION or NEGATION.

Thus, for Example, if 8 be to be taken from 5, they ſay, there remains — 3, or a Quantity *leſs than nothing*, by the Number *Three*: Which may be farther explained, thus,

He that *hath* 5 (ſuppoſe Shillings) and *owes* 8 *s.* is ſaid to *have* — 3 *s.* *i. e.* 3 *s.* leſs than nothing; or (what is the ſame thing) to *owe* 3 *s.*

Nor are theſe *negative* Quantities introduced without very great Advantage by Arithmeticians (alſo by Geometricians, and other Mathematicians) as might be ſhewn, was this a proper Place. But, to go on,

The *Reaſon* of this Rule of SUBTRACTION (whether we begin at the *left* Hand, or the *right*) is ſufficiently evident from the 9th AXIOM, *viz.* *The Whole is = all its Parts taken together*: Thus, the Subtraction of all the *Particulars*, is equivalent to the Subtraction of the *Whole* SUBDUCEND, or Number *to be* ſubtracted.

In the laſt Chapter it was ſaid, That the moſt *certain* Proof of ADDITION is by SUBTRACTION; the Method may now be taught, and is as follows.

Having

Having added, ſuppoſe, *two* Numbers together, if *one* of them be ſubtracted from the Sum, and the Remainder be = the *other*, the *Addition is right*. Again,

If you have added *divers* Numbers together, if *any one* of them be ſubtracted out of the Sum total, and the Remainder be = *all* the reſt (added together); or if the Sum of all *but one* be ſubtracted out of the Total, and the Remainder be = the Number *left*, then your *Addition is right*.

Here follow ſome Examples, which, at the ſame time as they will explain to you *this* Method of *proving* what you have before done in ADDITION, may alſo ſerve for your *farther* Practice in SUBTRACTION.

From	5965	1491854643864
Take	976	590867659875
Rem.	4989	900986983989

		l.	*s.*	*d.*
From	105456	20	08	10 ½
Take	98765	13	10	01 ½
Rem.	6691	6	18	09

	Years	M.	W.	D.	H.	M.	S.
From	2740	10	3	0	03	14	40
Take	1022	10	3	0	03	14	41
Rem.	1717	12	3	6	23	59	59

	Leag.	M.	Y.	F.	I.	Sig.	°	′	″
From	1227	1	913	2	2	11	21	04	51
Take	663	1	913	2	3	9	22	05	52
Rem.	563	2	1759	2	11	1	28	58	59

Troy

	Troy Weight.				*Averdup. Weight.*				
	Lib.	Oz.	dwt.	Gr.	Tuns	H.	Q.	P.	Oz.
From	174	04	16	15	3	02	0	06	05
Take	161	04	16	16	2	02	0	06	06
Rem.	12	11	19	23		19	3	27	15

The *Demonſtration* of this Proceſs, and all others of the like Nature, depends (as will eaſily be ſeen) on the IXth AXIOM.

But now, though *this* Proof be the moſt *certain*, yet ſince it may *poſſibly* be more *tedious* than the *firſt* Operation *itſelf*; perhaps it may be better for the young Arithmetician (to whom *chiefly*, if not *only*, theſe, or any other Proofs, are *needful*) in its Stead, to *repeat* the Operation of *Addition*, eſpecially if he do it in a *retrograde* or *inverted* Order, as was ſhewn in the laſt Chapter.

From the forementioned IXth Axiom, is alſo naturally deduced a Method of *proving* SUBTRACTION: 'Tis as follows,

Add the leſſer Number and the Remainder together, and if the Sum of theſe two (here called *Parts*) be = the bigger, or that from which the leſſer was taken (which is here called the *Whole*) then the Work is right; if not, Care muſt be taken to diſcover and correct the Error.

EXAMPLES.

		l.	*s.*	*d.*
From	4986	549	14	07 $\frac{1}{4}$
Take	3679	316	15	09 $\frac{1}{2}$
Rem.	1307	232	18	09 $\frac{3}{4}$
Proof	4986	549	14	07 $\frac{1}{4}$

Now,

Now, by theſe *Reciprocal* Proofs, the young Arithmetician may eaſily, and uſefully enough, ſee that ADDITION is, as it were, the * *Geneſis* of *that*, whereof SUBTRACTION is the † *Analyſis*: Or, That what ADDITION *creates* or *does*, SUBTRACTION *reſolves* or *undoes*.

* Γένεσις, a Γίνομαι, fio.

† Ἀνάλυσις, ab ἀνὰ, rè, & λύω, ſolvo.

QUESTIONS.

I. A Merchant had 5 Debtors, A, B, C, D, and E; which together owe him *l.* 1156; B, C, D, and E, together owe him *l.* 737: What is A's Debt?

Anſwer. *l.* 419.

II. What Sum is that to which, if I add *l.* 40 19 11 $\frac{1}{4}$, the Sum will be *l.* 509 11 4?

Anſwer. *l.* 468 19 4 $\frac{3}{4}$.

III. The Sun runs through the *Zodiack* in 365 Days, 5 Hours, 49 Minutes; The Moon in 354 Days 8 Hours 48 Minutes: What is the Difference of each Period?

Anſwer. 10 Days, 21 Hours, 1 Minute.

IV. The Sum of 2 Numbers is 1964, and the leſſer of them is 856: What is the greater?

Anſwer. 1108.

V. Two Perſons are of different Ages; the elder of them, A, is 95 Years and 5 Months, and the Difference of their Ages is 32 Years and 6 Months: What is B's Age?

Anſwer. 62 Years 11 Months.

See more *Queſtions* in the APPENDIX.

I now proceed to the Explanation of *Multiplication*.

CHAP. VI.

TO MULTIPLY, is *to find a Number which may contain a* GIVEN *Number as often as 'tis required.*

As, if it be ſought *what* Number will contain a *given* Number, *twice, thrice, four times, twenty times, an hundred times,* or *a thouſand times,* &c. MULTIPLICATION teaches *to find this Number.*

That Number, whoſe *Double, Triple, Quadruple,* &c. is ſought, is called the *Multiplicand.*

That Number which ſhews *how often* the Number ſought muſt contain the *Multiplicand,* is called the *Multiplier.*

Both of theſe, *viz. Multiplicand* and *Multiplier,* are called *Factors.*

Laſtly, The Number *ſought* is called the *Fact* or *Product.*

Thus, if 10 × by 2 be = 20, 10 will be the *Multiplicand,* 2 the *Multiplier* (both called *Factors*) and 20 the *Fact* or *Product.*

Though it is all one which of the two *given* Numbers we make the *Multiplicand,* and which the *Multiplier,* the *Product,* either Way, being the *ſame*; for three times four, is the ſame as four times three; twelve being

produced either Way, *i.* $4 \times 3 = 3 \times 4 = 12$: The ſame in others.

The forementioned Terms being underſtood, the following *Definition* of *Multiplication* may, without much Difficulty, be underſtood.

MULTIPLICATION finds a Number which contains the *Multiplicand*, as often as the *Multiplier* contains an *Unit*.

Hence is manifeſt what was ſaid at the Beginning of Chap. III. *viz.* That MULTIPLICATION (at leaſt *ſtrictly taken*) is ſcarcely any thing elſe but a *manifold* ADDITION, or a *Compendium* of ADDITION; for it is the ſame thing to *triple*, or to *multiply* by 3, as to ſet down the Quantity *three times*, and *add* them; the *Product* of that *Multiplication* being the *ſame* as the *Aggregate* of this *Addition*:

Thus, $2 \times 2 = 2 + 2$. $2 \times 3 = 2 + 2 + 2$. $2 \times 4 = 2 + 2 + 2 + 2$, *&c.*

Where, by the Way, you may note, That the Number 2, whether it be *added to*, or *multiplied by* itſelf, is increaſed alike, *i. e.* $2 + 2 = 2 \times 2$, which happens only in this Number 2.

You may alſo, by the Way, note, That the *Product*, or Number *produced* by *Multiplication*, is often called a *Rectangle*, or *Plane*, or *Plain Number*, a METAPHOR borrowed from *Geometricians*, where, from multiplying a *Length* into a *Breadth*, is gained a *Plane*, or the *Area*, or *Content* of a *Rectangle*, or right-angled *Parallelogram*. But, to return;

Having now, may be, more than enough explained the *Nature* of Multiplication, it now remains to ſhew *how to multiply*.

And here, firſt, let us note, That MULTIPLICATION is exerciſed or imployed either in *ſimple* Numbers (I mean in Numbers expreſſed by *one* Figure) or in *compound* Numbers, *i. e.* ſuch as are expreſſed by *more* Figures *than one.*

As to what relates to the Multiplication of *ſimple* Numbers, we muſt be obliged to our *Memory*, or to the *Principles* of ADDITION, until Uſe and Exerciſe ſhall have made it familiar: Thus, if any one knows not what 3 times 3 is, or 5 times 7, he may add 3 *threes*, or 5 *ſevens* together; the ſame in others: For, as I juſt before have ſhewn, $3+3+3=3\times3$; and $7+7+7+7+7=7\times5$, *i. e.* to gain the deſired *Product*, the *Multiplicand* muſt be ſo often *added to* itſelf, as an *Unit* is contained in the *Multiplier.*

But, leſt theſe *Additions* ſhould appear tireſome to the young Arithmetician, a TABLE may be at once, by the ſame Method, made; which Table is uſually applied to on theſe Accounts, and which, *at Sight*, gives the *Products* of any two *ſimple* Numbers.

The TABLE.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
	3	9	12	15	18	21	24	27
		4	16	20	24	28	32	36
			5	25	30	35	40	45
				6	36	42	48	54
					7	49	56	63
						8	64	72
							9	81

I now

I now proceed to ſhew how to multiply *compound* Numbers, when the *Multiplier* is a SIMPLE Number.

EXAMPLE.

Suppoſe the Number 3421 be to be *doubled*, or multiplied by 2.

Under the *Multiplicand* 3421 (which, not out of *Neceſſity*, as has been already ſaid, but out of *Convenience*, is generally choſe, as here, the *biggeſt* of the two given Numbers) place the *Multiplier* 2, and (drawing a Line) under each of the Figures of the *Multiplicand*, place its *Double* (becauſe the *Multiplier* is 2).

Thus the Double of one *Unit*, are two *Units*; the Double of two *Tens*, are four *Tens*, or *Forty*; the Double of four *Hundreds*, are eight *Hundreds*; the Double of three *Thouſands*, are ſix *Thouſands*.

```
3421
   2
------
   2
  40
 800
6000
------
6842
```

Which Numbers collected into one Sum, *viz.* 6842, are double the whole Multiplicand 3421; for, ſince all the Parts taken together, are = the *Whole*, therefore all the Parts, *twice taken*, will be = the Whole *twice taken*, i. e. the *Double* of all the *Parts*, will be = the *Double* of the *Whole*.

Which

Which *Demonſtration* (*mutatis mutandis*) is of the ſame Force, if applied to the *Triple*, *Quadruple*, *Quintuple*, or *any other Multiple*.

Again, If the ſame Number, 3421, be to be multiplied by 8, all the Figures of the Multiplicand muſt be *octupled*, or taken 8 times; and each of their *Octuples* muſt be ſubſcribed in their Places, and then collected into one Sum, Regard being had to their Places: Thus,

1 Unit *octupled*, gives 8 Units; 2 Tens *octupled*, give 16 Tens; 4 Hundreds, 32 Hundreds; and 3 Thouſands, 24 Thouſands; which being collected into one Sum by ADDITION, you gain the *Octuple* of the *whole Multiplicand*, viz. $27368 = 3421 \times 8$, as is manifeſt;

```
 3421
    8
 ----
    8
  160
 3200
24000
-----
27368
```

Thus you ſee that the particular *Multiples* of each Figure, may be placed in ſo many *different* Lines; or rather, in *two* Lines; thus,

```
 3421
    8
 ----
24268
  31
 ----
27368
```

For,

†

For, ſo long as *this* is in *both* taken care of, *viz.* To place each Figure in its proper *Place* or Degree, it matters not whether you make *two* Lines or *more*; the ſame Sum being either way gained.

But now, for the greater Advantage and Expedition in working, the SAME *Product* or *Sum* of *Addition* may be found by a *running* Work of *Multiplication* (though, for *Clearneſs* ſake, the foregoing Method was *firſt* inſerted).

Thus in the foregoing Example,

After 8 (*viz.* the Octuple of the Figure 1) is placed in the Unit's Place, the next is 16 *Tens* (*viz.* the Octuple of *two* Tens) 6 of which being put in the Place of *Tens*, for the other 10 Tens reſerve 1 *Hundred*; then 8 times 4 Hundred is 32 Hundred, to which the 1 Hundred (reſerved) muſt be added, which makes 33 Hundred, 3 of which being put in the *Hundreds* Place; for the other 30 Hundred, reſerve 3 *Thouſand*; but if to theſe 3 Thouſand 24 *Thouſand* be added (*viz.* 8 times 3 Thouſand) you will have 27 Thouſand to be ſet down in their proper Places, as in the *annex'd* Example.

3421
8

27368

Thus has been enough (I believe) taught how to proceed in multiplying *compound* Numbers, when the Multiplier is a *ſimple* or *ſingle* Number.

But, if not only the *Multiplicand*, but alſo the *Multiplier* be a *compound* Number, the *ſame* Operation as before, muſt be repeated *as often* as there are *Figures* of the Multiplier.

The

The Numbers ſo found (having due Regard to their Places) muſt be collected into one Sum by *Addition*.

EXAMPLE.

If the *ſame* Number 3421 be to be multiplied by 28, after you have ſubſcribed 8 *times* the *whole Multiplicand* (found by the Method juſt before taught) you muſt alſo ſubſcribe 20 *times* the Multiplicand (*viz.* 68420) or (for the greater Eaſe in working) the *Double* of the Multiplicand (found as taught above) ſo that its *firſt* Figure to the Right Hand be put in the Place of *Tens*; its *next* in the Place of *Hundreds*, &c. or by adding a Cypher to the Right Hand of this Double 6842, you will have *ten* times its *Double*, that is, *twenty* times the *whole* Multiplicand; which added to 8 times the ſame Multiplicand before found, you will have 28 times the *whole* Multiplicand; as is manifeſt, *viz.* 95788 = 3421 × 28.

```
  3421
    28
 -------
 27368
 6842
 -------
 95788
```

In like Manner muſt you do in others, be the Number of Figures in the *Multiplier* what they will.

More Examples may be as follow,

```
  56783
    398
 -------
 454264
 511047
 170349
 -------
 22599634
```

```
  59832
    907
 -------
 418824
 5384880
 -------
 54267624
```

870-

8706543	759837
54086	90705
52239258	3799185
69652344	53188590
348261720	68385330
43532715	737421808 5
470902084698	

But if there be *Cyphers* at the End of *either* Multiplicand or Multiplier, or *both*, you may multiply *the remaining* Figures (as has been already ſhewn) and to the right Hand of *that* Product, put *ſo many* Cyphers *as* there are Cyphers at the right Hand of *either* Multiplier or Multiplicand, or *both*; as in the following *Examples*,

5968000	5683
6	9000
35808000	51147000

97638000
4670000
683466
585828
390552
455969460000000

Again, If you are required to multiply any Number by 10, 100, 1000, 10000, *&c.* you need only annex the Cyphers of the *Multiplier*, to the right Hand the Figures of the *Multiplicand*, and the Thing is done.

Thus, $869 \times 10 = 8690$, $636 \times 1000 = 636000$, $786 \times 100 = 78600$, $895 \times 10000 = 8950000$, *&c.*

But now, Numbers whoſe Denominations are *different* (of which Sort are to be eſteemed Numbers only of *ſecondary* Denominations) can't be *otherwiſe* multiplied *than* by the intermediate Sign ×, either expreſſed or underſtood.

Thus, if 4 Feet be to be multiplied by 2 Inches, the Product will be 4 Feet × 2 Inches.

Though, if theſe be firſt reduced into the *ſame* Name, they *alſo* may be multiplied by each other.

Thus (4 Feet *i. e.*) 48 Inches × 2 Inches = 96 Inches, *i. e.* 48 Inches × 2 = 96 Inches.

Again, altho' the *primary* Denominations of Numbers *are* different amongſt themſelves (*v. g. Tens* DIFFERENT FROM *Hundreds*, whence, in ADDITION, *Tens* and *Hundreds* can't be ADDED together) yet *All of them*, as they are Denominators of *Units* (for *v. g. Tens* are Tens of *Units*; *Hundreds*, Hundreds of *Units*, &c.) are to be accounted *alike*; and therefore *may*, all of them, *be* MULTIPLY'D by each other; for Inſtance, Not only *Tens* by *Tens*, *Hundreds* by *Hundreds*, but alſo *Tens* by *Hundreds*, &c.

And for the *like* Reaſon in *Decimal* FRACTIONS, and alſo in *Vulgar* ones, not only *Tenths* by *Tenths*, *Hundredths* by *Hundredths* (&c.) *Thirds* by *Thirds*, *Fourths* by *Fourths* (&c.) are multiply'd, but alſo *Tenths* by *Hundredths* (&c.) *Thirds* by *Fourths* (&c.) *viz.* As they are *All* ſuppoſed to be *Parts of* the *ſame* UNIT or THING, and therefore *Homonimous*.

On

On the contrary, if (*v. g.*) 3 *fourths* be accounted *Parts* of a SHILLING, and 2 *thirds*, *Parts* of a PENNY, they can't be multiplied together; for as 3 *s.* × 2 *d.* makes 6, not EITHER *Shillings* or *Pence*; ſo, for the *like* Reaſon, 3 *fourths* of a SHILLING × 2 *thirds* of a PENNY, make not 6 *twelfths* EITHER of a *Shilling* or a *Penny*.

Before I proceed to the PROOF of *Multiplication*, the Learner may here uſefully enough obſerve, that EVERY *Multiplier* is really to be eſteemed an *abſolute* Number, *i. e.* of no *ſecondary* Denomination; for it is manifeſt, that the *only* Buſineſs of the *Multiplier*, is *to number* the *Times* the *Multiplicand* is to be *added to* itſelf; but to *Numeration*, or *Numbering*, no *ſecondary* Denomination has the leaſt Regard.

Hence, he that multiplies 4 Feet by 2 Feet, does no more than multiply 4 Feet by the Number 2; and thus the Product, *viz.* 8 Feet, is only the *Aggregate* of the *Multiplicand* 4 Feet + 4 Feet; thus again, 4 *s.* × 2 *s.* is no more than 4 *s.* × the Number 2, *&c.*

Thus is the Reaſon plain, why * *Homogeneal* Numbers *may* be multiplied together, but † *Heterogeneal* ones *not*; for Inſtance, 4 Feet × 2 Feet = 8 Feet; but 4 Feet × 2 Inches, are = 8 neither Feet nor Inches, *&c.*

* Ὅμοιος, ſimilis, & Γένος, genus.

† Ἕτερος, alius, & Γένος, genus.

Thus, if to *both* Factors be put the *ſame ſecondary* Denomination, then Multiplication is converſant about Things of the *ſame ſecondary* Denomination, *i. e.* (becauſe the *Multiplier* is really to be eſteemed an *abſolute* Number, or of no *ſecondary* Denomination; therefore) the *Multiplicand* is to be eſteemed as an *Aggregate* of Things of *one ſecondary* Denomination; but if *both* Factors have *different ſecondary* Denominations, then Multiplication is employed about Things of *different ſecondary* Denominations, *i. e.* for the Reaſon before given, the *Multiplicand* is to be eſteemed as an *Aggregate* of *different ſecondary*

condary Denominations; but ſince ARITHMETIC allows of *no ſuch* Aggregate, therefore *neither* does it admit of the *Repetition* or *Multiplication* of *ſuch* Aggregate.

I now proceed to the PROOF of *Multiplication.*

As ADDITION is *proved* by SUBTRACTION (its Contrary) ſo MULTIPLICATION is beſt *proved* by DIVISION (its Contrary) for what is *done* by MULTIPLICATION, is *diſſolv'd* or *undone* by DIVISION: For,

When a third Number is produced by the Multiplication of two other Numbers, if that third Number be divided by either of the other two, the remaining one will be, by that Means, produced: Thus,

If by multiplying 6 into 8, or (which is the ſame thing) 8 into 6, I get 48; then, to try whether my Work be right, I divide this Product by *either* of the Factors, and if the *other* be produced, it is an Argument that it *is* right; but if otherwiſe, it is certain that there is a Fault committed ſomewhere.

Now, after this Manner may *every* Operation in this Rule be prov'd; yet, becauſe DIVISION has not been *here* yet taught, I ſhall inſert a Way of Proof, which may ſerve the Learner at preſent: 'Tis as follows,

Let that which before was your MULTIPLICAND, be now made the MULTIPLIER, and the contrary, and if the *Product* be the SAME it was before, there is *great Reaſon* to believe it is a true one; but if not, that it is erroneous. Here follow ſome

QUESTIONS.

I. What Number divided by 4, will give (the Quotient) 3?

Anſwer. 12.

II. A Third of what Number is 235?

Anſwer. 705.

III. Admit the Earth's Circumference is 360 Degrees, and that one Degree is 60 Miles, how many Miles is it round the Earth?

Anſwer. 21600.

IV. If in a Mile there be 1760 Yards, how many Yards are there in 1760 Miles?

Anſwer. 3097600.

V. A certain Sum of Money is divided betwixt 110 Men, their Shares come to 15 *l.* each, how much Money was diſtributed in all?

Anſwer. 1650 *l.*

VI. Since a Year contains 365 Days (for we will not here meddle with the odd Hours, Minutes, *&c.* which, every 4th Year, make one Day, and is the Cauſe of LEAP-YEAR) how many Hours are there in a Year?

Anſwer. 8760.

I proceed now to explain DIVISION.

CHAP.

CHAP. VII.

TO DIVIDE, is *to Part any given Number into any Number of* EQUAL *Parts*; or, *to find a Number which may ſhew* HOW OFTEN *any given Number is contained in any given Number.*

Now, in the Diviſion of *Numbers*, there are *two* Numbers GIVEN, *viz.* The DIVIDEND, which is to be parted, and the DIVISOR, which denotes the *Number* of Parts into which it is ſuppoſed to be divided; *which* being GIVEN, a *third* is SOUGHT, which is called THE QUOTIENT; for it ſhews (QUOTIES) HOW OFTEN the DIVISOR is contained in the DIVIDEND, or (which is the *ſame* thing in effect) HOW OFTEN the *former* can be taken from the *latter*.

Thus, if the Number 12 be to be divided into 3 equal Parts, the *Quotient* (ſuppoſe 4) ſhews the Value of one 3d of it, or ſhews How OFTEN the DIVISOR 3, is contained in the DIVIDEND 12.

Thus again, if the Number 20 be to be divided by (the DIVISOR) 4, or to be parted into 4 *equal* Parts, or if 4 divides 20; or *laſtly*, if it be ſought How OFTEN 4 is contained in 20, or How OFTEN it can be taken *from* it; the Number ariſing, or QUOTIENT (ſuppoſe 5) ſhews what a 4th Part of that Number is, *i. e.* If the Number 20 be ſuppoſed to be divided into 4 equal Parts, *what* each of thoſe Parts *is*, *viz.* 5; or How

How OFTEN 4 is contained in 20, or How OFTEN it can be taken from it, *viz.* FIVE times.

What has here been ſaid, being well obſerved, the following *Definition* of DIVISION will be eaſily underſtood.

DIVISION *finds a Number which may contain an* UNIT, *as often as the* DIVIDEND *contains the* DIVISOR; *or, which is contained in the* DIVIDEND, AS OFTEN *as an* UNIT *is contained in the* DIVISOR.

Hence is manifeſt what was ſaid at the Beginning of Chap. III. *viz.* That DIVISION (at leaſt when the *Diviſor* is an INTEGER) is ſcarcely any thing elſe but a *manifold* SUBTRACTION, or a *Compendium* of SUBTRACTION; for he that asks How OFTEN the Number (for Inſtance) 12 contains 3, does the *ſame* as ask How OFTEN 3 can be taken from 12; and if by a *continual* Subtraction he finds How OFTEN 3 can be taken from 12, without a Remainder, he finds, at the ſame time, How OFTEN, *preciſely*, 3 is contained in 12; which is the very *ſame* thing as TO DIVIDE 12 by 3.

Whoſo now well underſtands what has been ſaid, muſt ſee thro' the *genuine* Foundation of DIVISION.

But ſince the Labour would be almoſt *infinite*, in large Diviſions, *ſeparately* to perform *ſo many* Subtractions as are ſufficient, the Maſters of *practical* Arithmetic, have contrived wonderfully to *ſhorten* this great Labour. Thus, inſtead of an OFTEN SUBTRACTION, they ſubtract SOME MULTIPLE of the DIVISOR, which comes to the *ſame* thing: Thus, in the foregoing Example, where the Number 12 is propoſed to be divided by 3; it is the *ſame* thing whether we ſubtract 3 *four times*, or 4 *times* 3 *once* (becauſe $3 + 3 + 3 + 3 = 3 \times 4$), 0 remaining either Way.

But

But now, by *how much* the greater the DIVIDEND and QUOTIENT ſhall be, *ſo much* the more the Advantage of this *Compendium* appears.

If 20 be to be divided by 2, in the Room of *ten* Subtractions of the Number 2, I ſubſtitute the Subtraction of 2 taken 10 times; for $2 \times 10 = 20$.

If 360 be to be divided by 6, inſtead of *Sixty* Subtractions of the Number 6, I *once* ſubtract 60 times 6: The like in others.

Having now, as I believe, ſufficiently explained the NATURE of *Diviſion*, and COMPAR'D it with *Multiplication*, the next thing will be to explain its *Practcie*, or to teach *how to divide*.

And here firſt let us note, That in the Practice of Diviſion, the *Diviſor* will be either a SIMPLE Number, to be denoted by ONE Figure, or a COMPOUND, to be expreſs'd by MORE Figures than ONE.

If the *Diviſor* be a SIMPLE Number, and the *Dividend* alſo be LESS than 10 times the *Diviſor*, we muſt be obliged to our *Memory* for a Solution, or elſe make Uſe of the TABLE in the laſt Chapter.

EXAMPLE.

If it be ſought How OFTEN 3 is contained in 24, firſt ſet down the *Diviſor* and *Dividend* thus,

Diviſor.	Dividend.	Quotient.
3)	24 (	8
	24	
	—	
	0	

Having

Having thus placed the Numbers, begin to *divide*, asking how many times 3 (the *Diviſor*) can be had in 24 (the *Dividend*); now, your *Memory*, or the foregoing *Table*, will anſwer us, 8 times; which 8 place in the *Quotient*, and multiply the *Diviſor* thereby, and ſubtract the *Product* (24) from the *Dividend*, and there remains 0: Thus the Work is finiſhed.

But if the Number to be divided, be *not* leſs than 10 times the *Diviſor*, we muſt proceed by Degrees, and ſubtract DIVERSE *Multiples* of the *Diviſor* ſucceſſively.

EXAMPLE.

Suppoſe I am to *divide* 4544 by 8.

Firſt I conſider How OFTEN I can have the *Diviſor* in the firſt Figure, or Figures, to the left Hand the *Dividend* that is or are either *equal to* or next *greater than* itſelf: As here,

I conſider How MANY times 8 can be had (not in 4, for that would be 0 times, and a Cypher at the left Hand of any *whole* Number, is of no Value, but) in 45, and find it 5 times; then ſetting 5 in the *Quotient*, I multiply it by 8 (the *Diviſor*) which makes 40; this I take from 45, and ſet down the *Remainder* (5) then, to this Remainder I bring down 4, the next Figure in the *Dividend*, making a Point under it (as a *Mark* that no Figure may be either omitted, or twice brought down); then I conſider How OFTEN 8 can be had in 54, and find it 6 times; then ſetting 6 after the former Figure in the Quotient, I multiply *it* alſo by (the *Diviſor*) 8, it makes 48, which I take from 54, and ſet down the *Remainder* 6, to which I bring down the next Figure 4 (making alſo a *Point* under *it*); laſtly, I conſider How OFTEN 8 can be had in 64, and find it *exactly* 8 times, putting 8 in the Quotient, and multiplying (the

Diviſor) 8 by it, the *Product* is 64, which I take from 64, and there remains 0; and now all the Figures of the *Dividend* being already brought down and divided, the Work is done, and the *Quotient* is 568.

Diviſor. Dividend. Quotient.

```
8 ) 4544 ( 568
    40
    --
     54
     48
     --
      64
      64
      --
       0
```

Here the Learner may obſerve,

1. That in *this* Way of *Diviſion*, the *Number* of Figures in the *Quotient*, will be always *one more* than the Number of *Points* in the *Dividend*.

2. That, thro' the *whole* Operation, *every* Remainder muſt be *leſs* than the *Diviſor*, otherwiſe the *Quotient* is too *little*, or the Operation *wrong*.

But to go on,

There is one *Objection* which may poſſibly be ſtarted by the Learner, concerning the Method of *Dividing* before taught; but what follows will eaſily remove it.

Suppoſe, for Inſtance, 486 was to be divided by 3, proceeding by the Rules before given, the Work will ſtand as follows,

```
3 ) 486 ( 162
    3 ..
    ---
    18
    18
    --
     6
     6
     -
     0
```

Here the Learner may *object*, that the Figure 4 in the Place of *Hundreds*, and 8 in the Place of *Tens*, do not ſignify *ſimply* ſo many *Units*; but that the *former* ſignifies 4 *Hundreds*, and the *latter* 8 *Tens*; and therefore, that 3 is not contained barely *twice* in 400, but *two hundred* times; and in 80, not *four*, but *forty* times.

This, indeed, is *true*, but it is in like manner *true*, that of the Figures 1 and 6 placed in the Quotient (altho' when they were *firſt* wrote down, *they* might ſeem to ſignify *ſimply* ſo many *Units*; *yet*, when the *whole* Diviſion is perform'd, it is manifeſt that) the *former* ſignifies *one Hundred*, and the *latter*, *Sixty*; and, therefore, we do not ſay, the *Diviſor* 3 is contained *once*, and *ſix* times, and *twice* in the *Dividend* 486, but *One Hundred Sixty* and *Two* times: And, indeed, in the very Beginning of the Work, if we have regard to the *Places* of the Figures, we do not ſay, *ſimply*, that *once Three* is to be taken from 4, but one *Hundred* times *Three* from 400, or *three* times *one Hundred* from *four Hundred*.

Thus again, in the next Work, we do not ſo much take *Six* times *Three* from 18, as *Sixty* times *Three* from 180; ſo that the Work ſhould really ſtand as follows,

```
3 ) 486 ( 100
    300
   ----
    186 ( 60
    180
   ----
      6 ( 2
      6
     ---
      0
```

But, for Brevity's Sake, in *Practice*, the *ſetting down*, and *ſubtracting* thoſe Cyphers, is omitted.

For the Learner's farther Practice in *dividing* by a *ſingle* Figure, ſome of the firſt Examples in MULTIPLICATION may be *proved* thus,

Divide the *Products* $\left\{\begin{matrix}6842\\27368\\35808000\end{matrix}\right\}$ by $\left\{\begin{matrix}2\\8\\6\end{matrix}\right\}$ the *leſſer*

of the *Factors* (not out of Neceſſity, but to ſave Trouble) and if the *Quotients* be = the *other* Factors, the Work is right, if not, there's a Fault committed ſomewhere.

```
2)6842(3421     8)27368(3421     6)35808000(5968000
  6...            24...            30.....
  --               --               --
  8                33               58
  8                32               54
  --               --               --
  4                16               40
  4                16               36
  --               --               --
  2                 8               48
  2                 8               48
  --               --               --
  0                 0                 000
```

With

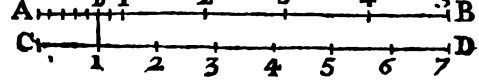
With a little Experience, the Learner will be able to divide *any* Number by a ſingle Figure, without *actually* ſetting down the ſeveral *Products* and *Remainders*, as is done in the foregoing Examples.

But now, if any *Part* of the *Dividend* remains after the *Diviſor* is taken out of it *as often* as poſſible, *ſuch* Remainder will be the *Numerator* of a FRACTION, whoſe *Denominator* will be the *Diviſor*; and *this* Fraction, or at leaſt, *another* of the *ſame* Value, in *leſs* Terms, muſt be placed after the INTEGERS of the *Quotient*; thus will the *exact* Quotient be gained. Thus,

If the Number 560873 be to be divided by 7, proceeding as before taught, you'll at laſt find the Number 5 remaining; therefore, after the *whole* Numbers 80124 of the *Quotient*, you muſt place the *Fraction* $\frac{5}{7}$; Thus, 80124 $\frac{5}{7}$ is the *exact* Quotient ſought, or the *ſeventh* Part of the Number 560873; or by this we find, that the Number 560873 contains the Number 7, 80124 times and $\frac{5}{7}$ (five ſeventhſ) of another time; and laſtly, that *this* Number 80124 $\frac{5}{7}$ (the *Quotient*) is contained *exactly* 7 times in 560873 (the *Dividend*); The like in others.

For tho' the *Remainder* 5 be actually 5 UNITS of the *Dividend*, yet we muſt have but a 7th Part of it to add to the *whole* Numbers of the Quotient, and a 7th Part of 5 is $= \frac{5}{7}$, as may be illuſtrated enough for the preſent, *thus*,

Let the Line AB repreſent the 5 *remaining* UNITS, *one* of which, as A 1, let be divided into 7 *equal* Parts: Let the Line CD (= AB) be divided into 7 equal Parts, then will C 1 be *one ſeventh* of AB $= \frac{1}{7}$ (*five ſeventhſ*) of A 1, or of 1 UNIT.



7)

```
7 ) 560873 ( 80124 1/7
    56....
    ------
      08
       7
      --
       17
       14
      --
        33
        28
        --
         5
```

Before I leave *Diviſion* by a SINGLE Figure, the Learner may know, that BIPARTITION, or Halving, or Dividing by 2, is *beſt* done as follows:

Suppoſe 546378 be required to be divided by 2, or halv'd, firſt I ſay, the Half of 5 is 2, but 5 being odd, I carry the 1 that's left to the next Figure 4, and ſay, the Half of 14 is 7, then the Half of 6 is 3, the Half of 3 is 1, the Half of 17 is 8, and the Half of 18 is 9: Thus, 546378 being divided by 2, the *Quotient* is 273189.

And now, SIMPLE *Diviſion*, or *Diviſion* by a SINGLE Figure, being well underſtood, COMPOUND *Diviſion*, or *Diviſion* by TWO, THREE, or MORE Figures, will not be very difficult, *this* being built upon the very *ſame* Principles with the *other*: However, for the *Learner's* Sake, the following Helps are given.

EXAMPLE.

EXAMPLE.

Divide 6763082 by 379; First consider how many Figures at the *left* Hand the *Dividend*, make a Number either *equal to*, or *next greater than* the DIVISOR (here they are *three*, for 6, or 67, are *each* LESS than the DIVISOR, and 676 the NEXT GREATER); then consider How OFTEN 379 can be had in 676, or, if you will, How OFTEN 3 in 6, having *Regard* to the *following* Figures in the DIVISOR (particularly to the *next* Figure THERE, *viz.* 7, for tho' the Number 3 is found TWICE in 6, yet TWICE 37, *viz.* 74, can't be had in 67, much less TWICE 379 in 676, therefore) finding it *but* ONCE, set 1 in the *Quotient*, and multiply the *Divisor* 379 by *it*, and subtract the *Product* 379 from 676, and set down the *Remainder* 297; to the *right* Hand whereof, bring down the *next* Figure of the *Dividend* (*viz.* 3.) having first made a *Point* at it, and then proceed, as already taught, 'till the *Division* is finish'd, and you'll find the *true* QUOTIENT to be $17844\frac{206}{379}$. See the whole WORK.

```
379) 6763082 ( 17844 206/379
     379....
     ------
      2973
      2653
      ----
       3200
       3032
       ----
        1688
        1516
        ----
         1722
         1516
         ----
          206
```

The

The following Examples are PROOFS to ſome in the *laſt* CHAPTER.

```
28 ) 95788 ( 3421        398 ) 22599634 ( 56783
     84                        1990
     ---                       ----
     117                       2699
     112                       2388
     ---                       ----
      58                       3116
      56                       2786
      --                       ----
      28                       3303
      28                       3184
      --                       ----
       0                       1194
                               1194
                               ----
                                  0
```

But theſe, and ſuch like Operations, may alſo be performed without *actually* ſetting down the ſeveral PRODUCTS, though 'twill be neceſſary to ſet down the REMAINDERS: Thus,

```
8706543 ) 4709020846 98 ( 54086
          35574934
           74876269
            52239258
                0
```

The following Obſervations may be of good Uſe to the Learner.

1. When any Number is given to be divided, if a Number offer itſelf that will *divide* both the *Dividend* and *Diviſor*, without a *Remainder*; *theſe Quotients* will be

be in the *ſame* Proportion *thoſe Numbers* were in before *ſuch Diviſion.*

As for Example;

If 48 was to be divided by 12, the *Quotient* would be 4; and if the *Half* of *theſe* (24 and 6) or again, the *Half* of *theſe* (12 and 3) be taken, the *Quotient* will alſo be 4; for,

12)48(4 6)24(4 3)12(4
0 0 0

2. If any Number hath 5 or 0 in the UNIT's Place, 5 will divide *ſuch* Number without a *Remainder.*

3. But, when any thing *does* remain after Diviſion is ended, although it be a *Part* of the *Dividend*, yet, as it has Relation to the *Diviſor*, it muſt be of the *ſame* Denomination with the *Quotient*, as it is a *Fraction* or *Part* of an INTEGER: Thus, if 10 *s.* were to be divided equally amongſt 4 Men, the *Quotient* will be 2 *s.* and the *Remainder* 2 *s.* but now, as this *Remainder* (2 *s.*) has Relation to 4 Men, it is in Value but 6 *d.* or *two fourths* of a SHILLING, the Denomination of the *Quotient*, and muſt be annex'd unto it, as before taught.

4. If the *Diviſor* has CYPHERS on the *right* Hand, they may be omitted, the *like* Number of Figures (whether they happen to be CYPHERS or not) being likewiſe cut off from the *right* Hand of the *Dividend*, and the Diviſion may be made with the *reſt* of the Figures, obſerving only, that whatever Figures were cut off in the *Dividend*, muſt be brought down at laſt, to compleat the *Remainder.* See the following *Examples.*

23|00)567|83(24 $\frac{1583}{2300}$ 56|000)2360|000(42 $\frac{8000}{56000}$ or $\frac{1}{7}$
107 120
1583 8000

5. Hence 'tis evident, that to divide any Number by 10, 100, 1000, *&c.* we need only cut off *ſo many* Figures from the *right* Hand the *Dividend*, *as* there are CYPHERS in the *Diviſor*; for thus will the *remaining* Figures of the *Dividend* be the TRUE *Quotient*, if to them be annexed thoſe Figures *ſo* cut off, which are a DECIMAL Fraction.

EXAMPLE.

If 658396 be divided by 10000, the *Quotient* will be 65 and $\frac{8396}{10000}$ or 65,8396.

Thus, 10000)65,8396

Laſtly, The Learner may note, that if at any time in Diviſion, the *Quotient* be = the *Diviſor*, then is the *Dividend* a SQUARE Number, and the *Diviſor* is its ROOT: But more of this in its proper Place.

Before I proceed to the PROOF of *Diviſion*, I ſhall (according to my Method in the laſt CHAPTER) acquaint the Learner, that Numbers whoſe Denominations are *different* (of which Sort are to be eſteemed Numbers *only* of *ſecondary* Denominations) can't be *otherwiſe* divided, *than* by ſome SIGN of *Diviſion*, as ÷ inter-plac'd.

Thus, If 4 Feet be divided by 2 Inches, the Quotient will be 4 Feet ÷ 2 Inches.

Except theſe be firſt reduced into the *ſame* Name; for then they *may* be divided by each other: Thus, (4 Feet, *i. e.*) 48 Inches ÷ 2 Inches = 24 Inches.

After the foregoing Manner, is expreſſed the Diviſion of a *leſſer* Number by a *greater*; for Example, 2 divided by 3 is thus denoted, 2 ÷ 3, or $\frac{2}{3}$.

I now

I now proceed to the PROOF of *Diviſion.*

The Learner firſt NOTING, that this PROOF of *Diviſion*, as alſo THOSE of *Addition*, *Subtraction*, and *Multiplication*, he muſt not take to be applied as DEMONSTRATIONS of the TRUTH of the RULES by which theſe Operations are perform'd (for the TRUTH of THESE is ſufficiently manifeſt from the PRINCIPLES on which they are built); but they are made uſe of *only* to find whether any Fault has, by Accident, crept into the Operation, *i. e.* whether the Operation performed be conformable to thoſe Rules. But now, tho' this PROOF may be made by *repeating* the *Work*, yet it is much better done by *changing* the *Form* of the Operation; for thus there is the *leſs* Danger of falling again into the ſame Error.

Now, as SUBTRACTION is beſt *proved* by ADDITION (its Contrary) ſo DIVISION (or a manifold Subtraction) is beſt *proved* by MULTIPLICATION (or a manifold Addition) thus,

If the QUOTIENT *be multiply'd into the* DIVISOR *(or* THIS *into* THAT*) the* DIVIDEND *will be produc'd:* For,

What by *dividing* was DISSOLV'D, by *multiplying* (by the *ſame* Number) is again RESTOR'D.

Thus, If 12 ÷ 3 be = 4, then 3 × 4 (*i. e.* the Diviſor by the Quotient) = 12.

Underſtand the ſame in greater Numbers: Thus,

If by dividing 154518 by 546, the *Quotient* is 283; or, If by dividing the ſame Number 154518 by 283, the *Quotient* be 546, then, on the contrary, by multiplying 546 into 283, or *this* into *that*, you will again have the *Dividend*, as you ſee.

546) 154518 (283	546	283
	283	546
4531		
1638	1638	1698
0	4368	1132
	1092	1415
	154518	154518

But if, after Diviſion is perform'd, there be any *Remainder*, we muſt take care to add it, for otherwiſe there will be produced *only* a *Part* of the Dividend: Thus for Example,

If the Number 387 ÷ 24, gives the Quotient 16, and 3 *remain*, this *Remainder* ſhews, that in *that* Dividend the *Diviſor* is contained 16 times (or the *Quotient* contained 24 times) and moreover the Number 3; and therefore to the Product made by the Diviſor × the Quotient, or this by that, muſt be added the Number 3.

The *Reaſon* of this whole Proceſs (if, by what has been already ſaid, it is not ſufficiently underſtood) may be farther ſeen thus,

Since the *Multiplicand* is SO OFTEN contain'd in the *Product*, As an *Unit* is contained in the *Multiplier* (for it is *ſo often* taken *as* the *Multiplier* bids); if by *Diviſion* be ſought *how often* the *Multiplicand* is contained in the *Product*, there ought to come forth (if the Operation be rightly performed) for the *Quotient* of the Diviſion, the *ſame* Number as *before* was *Multiplier*.

And on the contrary, ſince the *Quotient* ſhews *how often* the *Diviſor* is contained in the *Dividend* (either preciſely, or with *what* Remainder); if that *Diviſor* (by multiplying) be taken *ſo often* (and the *Remainder*, if there be any, *added*) the *Dividend* ought to be produc'd, *viz.*

viz. That Number in which the *Diviſor* is *ſo often* contained as was ſaid.

Now, beſides this PROOF of *Diviſion* by a CONTRARY Operation, it *may* be proved by the SAME: For ſince the *Diviſor* no leſs ſhews *how often* the *Quotient* is contained in the *Dividend*, than *how often* the *Diviſor* is contained in the *ſame* Dividend (for they are the *ſame* in reſpect to the *Dividend*, as the *Factors* in *Multiplication*, to the *Fact* or *Product*); if, after Diviſion is performed, there ariſe any Doubt of the Truth of the Operation.

Let that which before was the QUOTIENT, *be now made the* DIVISOR; and if for a *Quotient* THAT Number comes forth WHICH *before* was the *Diviſor*, 'tis an Argument the Work is rightly done; but if otherwiſe, that there's ſome Miſtake.

EXAMPLE.

If 154518 be ÷ 546, the *Quotient* is 283, of which, if any Doubt be made, let the *ſame* Number 154518 be ÷ the *Quotient* 283, and there'll come forth the Number 546, which before was the *Diviſor*, whence it is manifeſt, the Work is right.

Work	546)154518(283	Proof	283)154518(546
	4531		1301
	1638		1698
	0		0

But now, the *Remainder*, if there be any, will be the *ſame* both Ways.

Thus, Since if 387 be ÷ 24, the *Quotient* is 16, and the *Remainder* 3; if 387 be ÷ 16, the *Quotient* will be 24, and the *Remainder* alſo 3, as you may ſee;

Work

Work	24)387(16	Proof	16)387(24
	147		67
	3		3

Here may be noted, That theſe Proofs of *Diviſion*, as alſo thoſe of *Multiplication*, in the laſt CHAPTER, take Place in *every* Multiplication or Diviſion *whatſoever*; whether in WHOLE Numbers, *Vulgar* or *Decimal* FRACTIONS, or in ALGEBRAIC Operations.

Before I end this CHAPTER, I ſhall, for the Learner's farther Help, ſay ſomething concerning *Multiplication* and *Diviſion* of Quantities of MANY Denominations.

1. When a Quantity of *many* Denominations is to be × another of only *one* Denomination, begin the Work at the *loweſt* or *leaſt* Denomination, and ſo go on to the *higher* or *greater*, always carrying to the *higher* what is neceſſary.

EXAMPLE.

If any one ſpends each Day 14 Pounds 10 Shillings and 8 Pence of *Engliſh* Money, how much does he ſpend in a Week?

It will immediately be anſwered, That his Week's Expence is ſeven times the Expence of each Day (ſince each Week contains ſeven Days) and therefore the daily Expence muſt be multiply'd by 7. Thus, 8 Pence × 7 = 56 *d.* But now, ſince 12 *d.* = 1 *s.* 56 *d.* are = 4 *s.* + 8 *d.* therefore ſetting down 8 *d.* I reſerve 4 *s.* to be added by-and-by to the other Shillings: Then 10 *s.* × 7 = 70 *s.* to which the reſerved 4 *s.* being added, they become 74 *s.* But 20 *s.* = 1 *l.* therefore 74 *s.* = 3 *l.* + 14 *s.* therefore ſetting down 14 *s.* I reſerve 3 *l.* to be added by-and-by to the reſt: Laſtly, 14 *l.* × 7 = 98 *l.* to which adding 3 *l.* I get 101 *l.* which being ſet down, it's manifeſt the Week's Expence is *l.* 101 14 8.

The

The Work ſtands thus,

l.	*s.*	*d.*
14	10	8
		7
101	14	8

But if the *Multiplier* conſiſts of *different*, either *primary* or *ſecondary* Denominations (the *Multiplicand* ſtill conſiſting, as in the laſt Example, of *different ſecondary* ones) it is, for the moſt Part, the better Way (and, indeed, ſometimes it almoſt *muſt* be done) to reduce thoſe Quantities of *different* Denominations, to Quantities of *one* Denomination, and then to multiply them; and after *Multiplication* (if it be needful) thoſe *former* Denominations may be reſtored.

EXAMPLE I.

If to each Man be owing *l.* 28 13 9, How MUCH is owing to 283 Men?

Before you begin to multiply, it will be proper to reduce the whole Sum of Money propos'd, to *one* Denomination of *Pence*, which is done by Multiplication: Thus,

Becauſe every *Pound* (of *Engliſh* Money) contains 20 *Shillings*, multiply the propos'd Number of *Pounds* by 20, to get the Number of *Shillings* contain'd in thoſe 28 *Pounds*, to which the 13 *s.* (mentioned in the Queſtion) muſt be added, to get the Number of *all* the Shillings, *viz.* 573; then, becauſe each *Shilling* contains 12 *Pence*, multiply that Number of Shillings (573) by 12, to get the Number of *Pence* contained in them, to which ſtill muſt be added 9 *d.* (mention'd in the Queſtion) to get the Number of *all* the Pence, *viz.* 6885 *d.* = *l.* 28 13 9. Laſtly,

Laſtly, multiply 6885 (the Number of *Pence*) by the propos'd *Multiplier* 283, and there's produc'd 1948455 Pence. See the Work.

l.	*s.*	*d.*	
28	13	9	
20			
560			
13			
573			Shill.
12			
6876			
9			
6885			Pence
283			
20655			
55080			
13770			
1948455			Pence owing to 283 Men.

But now, to know how many *Pounds* and *Shillings* are contained in this Number of *Pence*, *divide* them as follows,

Becauſe 12*d.* = 1*s.* divide 1948455, the Number of *Pence*, by 12, thus are gotten 162371 *Shillings*, and moreover 3 *Pence* remaining after this Diviſion; then, becauſe 20*s.* = 1*l.* divide the Number of *Shillings* by 20, thus are gained 8118 *Pounds*, and moreover 11 *Shillings* remaining; and thus, 1948455*d.* = *l.*8118 11 3. the Sum ſought.

12)

12)1948455(162371

74 8118 11 3
28
44
85
15
3

EXAMPLE II.

If from a Source, or Spring, each Hour runs 34 Pounds and 7 Ounces of Water, how much will run out in 26 Days and 15 Hours?

Firſt reduce the aſſign'd Quantity of Water to Ounces by multiplying 34 (the Number of Pounds Weight) by 12 (becauſe 1 Pound is = 12 Ounces) and add the 7 Ounces; Reduce alſo the aſſign'd Time into Hours, by multiplying 26 (the Number of Days) by 24 (becauſe 1 Day is = 24 Hours) and add the 15 Hours: Then multiply 415, the Number of Ounces running out every Hour, by 639, the Number of Hours in the propos'd Time, and there is produced 265185, the Number of Ounces running out in that whole Time: Which Number of Ounces divide, laſtly, by 12 (for ſo many Ounces are contain'd in one Pound) and there come forth 22098 Pounds 9 Ounces, the ſought Quantity of Water running out in that whole Time.

See the WORK.

P.	O.			D.	H.
34	7	415	Ounces	26	15
12		639	Hours	24	
415	Ounces	3735		109	
		1245		53	
		2490			
			P. O.	639	Hours
		12)265185(22098	9		
		25			
		118			
		105			
		9			

But now, what has been ſaid of the *Multiplication* of Quantities conſiſting of *many* Denominations, the ſame, almoſt, muſt be underſtood of their *Diviſion*; only *here* we rather begin with the *higheſt* Denomination, when, *there* we began with the *loweſt*.

EXAMPLE I.

If *l.* 183 10 9 are to be divided equally among 4 Perſons, what is each Perſon's Share?

1. Divide 183 *l.* by 4, and you'll have the Quotient 45, and 3 Pounds remaining; then (becauſe 1 Pound is = 20 Shillings, and therefore 3 *l.* = 60 *s.*) inſtead of 3 Pounds, ſubſtitute 60 Shillings, adding the 10 Shillings (mentioned in the Queſtion) thus have you 70 Shillings, which ÷ 4, gives the Quotient 17, and 2 remain: Laſtly, ſince 2 Shillings are in Value 24 Pence, adding to 24 Pence the 9 Pence (mention'd in the Queſtion)

ſtion) they become 33 Pence; theſe ÷ 4, give the Quotient 8 Pence; to which, becauſe 1 remains (ſince you are come to the *loweſt* Denomination) join the FRACTION $\frac{1}{4}$; and thus the *whole* Quotient is *l.*45 17 8$\frac{1}{4}$, and therefore each Perſon's Share is 45 Pounds 17 Shillings and 8 Pence Farthing.

See the WORK.

l.	*s.*	*d.*	*l.*	*s.*	*d.*
4) 183	10	9	(45	17	8$\frac{1}{4}$

23
3 Pounds
× 20
—
60
+ 10
—
4)70(17
30
2
× 12
—
24
+ 9
—
4)33(8
1

EXAMPLE II.

In 24 JULIAN Years, 34 Days, 2 Hours, and 32 Minutes, how many SYNODICAL Months or LUNATIONS?

Becauſe a JULIAN Year conſiſts of 365 Days, and 6 Hours, *i. e.* (multiplying 365 by 24, the Hours in

one Day, and adding 6) of 8766 Hours; 24 Years will consist of 210384 (as will be manifest by multiplying); to which if 34 Days, 2 Hours, and 32 Minutes be added, *i. e.* 818 Hours and 32 Minutes, you'll have 211202 Hours and 32 Minutes, *i. e.* (by multiplying 211202 by 60, the Minutes in one Hour, and adding 32) 12672152 Minutes, which, at length, are of the same Value with 24 JULIAN Years, 34 Days, 2 Hours, and 32 Minutes.

Now a SYNODICAL Month (suppose from NEW-Moon to NEW-Moon) is stated 29 Days, 12 Hours, and 44 Minutes, *i. e.* (as is manifest from the following Calculation) 42524 Minutes: what is sought therefore is this, HOW OFTEN 42524 Minutes (or *one* SYNODICAL Month) are contain'd in 12672152 Minutes (the Time propos'd) THIS Number then divided by THAT, gives the Quotient precisely 298: So many SYNODICAL Months therefore are contain'd in 24 JULIAN Years, 34 Days, 2 Hours, and 32 Minutes.

And thus must be managed Operations in MULTIPLICATION and DIVISION, when the Quantities consist of DIFFERENT Denominations.

See the WORK.

D.	H.
365	6
24	
1460	
730	
6	
8766	Hours
× 24	
35064	
17532	
210384	Hours in 24 Years
+ 818	32
211202	32
× 60	
12672120	
+ 32	
12672152	Minutes

D.	H.	
34	2	32
24		
136		
68		
2		
818	32	
D.	H.	
29	12	44
24		
116		
58		
12		
708	Hours	
60		
42480		
+ 44		
42524	Minutes	

42524)12672152(298

416735

340192

0

Here, indeed, there's need of ſome Judgment to manage the preparatory REDUCTIONS moſt fitly for the preſent Purpoſe, that the *whole* Buſineſs may be perform'd

form'd with as little Labour as poſſible; but THIS, by frequent Exerciſe and Uſe, will become familiar.

Here follow ſome Queſtions in *Diviſion*, which may ſerve as Proofs of thoſe in *Multiplication*.

QUESTIONS.

I. What Number is that, which, if multiply'd by 4, makes 12?

Anſwer. 3.

II. The Triple of what Number is 705?

Anſwer. 235.

III. In 21600 Miles (the Circumference of the Earth, ſuppoſe) how many Degrees, allowing 60 Miles to a Degree?

Anſwer. 360.

IV. If in 1760 Miles there be 3097600 Yards; how many Yards are there in one Mile?

Anſwer. 1760.

V. If 1650 Pounds be diſtributed equally among 110 Men, what is each Man's Share?

Anſwer. 15 Pounds.

VI. If in a Year there be 8760 Hours, how many Days are there in the ſame Time?

Anſwer. 365.

See the APPENDIX for more Queſtions in *Multiplication* and *Diviſion*.

Before I end this Chapter, the Learner may note, that ſuch of the foregoing Operations in *Multiplication* and *Diviſion*, as require the turning any Quantity of *one* Denomination into an *equivalent* Quantity of *another* Denomination, belong to the *Rule* commonly call'd REDUCTION: A farther Explanation of which, is the Buſineſs of the next Chapter.

CHAP.

CHAP. VIII.

REDUCTION, tho' taught as a RULE by itſelf, by moſt Authors, is not properly any RULE of *Arithmetic* DIFFERENT from MULTIPLICATION and DIVISION already taught, but rather an APPLICATION of thoſe two Rules, by which is ſhewn, how to REDUCE or turn any Quantity of *one* Kind or Denomination, into an *equivalent* Quantity of *another* Denomination; and this (for our preſent Purpoſe) is *twofold.*

1. The *firſt* is that which turns an INTEGER of a *greater ſecondary* Denomination, into an *equivalent* INTEGER of a *leſſer* one; and is called *Reduction* DESCENDING.

2. The *other* is that which turns an INTEGER of a *leſſer ſecondary* Denomination, into an *equivalent* INTEGER of a *greater* one; and is called *Reduction* ASCENDING.

The { *firſt* / *latter* } is perform'd by { *multiplying* / *dividing* } the

Number to be *reduc'd* by *as many* UNITS of a LESSER Denomination, *as* are *equivalent* to ONE *Unit* of the GREATER.

Thus,

Thus, *l.* *s.* *d.* 55 16 4 ¾ are *reduc'd* into 53587 Farthings,
× 20 = the Sh. in ONE Pound, and add the 16 *s.*

it makes 1116 the Shillings in *l.* 55 16.
× 12 = the Pence in ONE Sh. and add the 4 *d.* it

makes 13396 the Pence in *l.* 55 16 4.
× 4 = the Far. in ONE Penny, and add the ¾, it

makes 53587, the Farthings in *l.* 55 16 4 ¾.

Thus again 53587 Far. are *reduc'd* into *l.* 55 16 4 ¾.

÷ 4, the Far. in 1 *d.*) 53587 (

÷ 12, the Pen. in 1 *s.*) 13396 = the Pen. in 53587 *Far.* & 3 *Far.* rem.

÷ 20, the Sh. in 1 *l.*) 1116 = the Sh. in 53587 *Far.* & 4 ¾ rem.

55 = the Poun. in 53587 *Far.* & 16 *s.* 4 ¾ *d.* remaining.

Thus are the 53587 Farthings turn'd into *l.* 55 16 4 ¾.

No more need be ſaid concerning theſe Sorts of *Reduction*, if what has been already ſaid be well underſtood; nor *any thing* of the REASON of *this Rule*, it being manifeſt from its PRACTICE.

I ſhall, therefore, only inſert a few Queſtions, with their Anſwers, for the farther Exerciſe of the Learner, who, if he knows not readily, when he comes to work them, *how many* UNITS of a LESSER Denomination, are *equivalent to*, or do make *one* of a GREATER, will find TABLES in the APPENDIX to inform him.

QUESTIONS.

I. In *l.*4963 15 5 $\frac{1}{2}$, how many Shillings, how many Pence, and how many Farthings?

Anſwer. 99275 *s.* + 5 *d.* $\frac{1}{2}$. 1191305 *d.* + 2 *Farth.* and 4765222 *Farthings.*

II. In 4765222 Farthings, how many Pence, how many Shillings, and how many Pounds?

Anſwer. 1191305 *d.* + 2 *Farth.* 99275 *s.* + 5 *d.* $\frac{1}{2}$, and *l.*4963 15 5 $\frac{1}{2}$.

III. In 350 *Engliſh* Miles, how many Yards?

Anſwer. 616000 Yards.

IV. In 616000 Yards, how many *Engliſh* Miles?

Anſwer. 350 Miles.

V. In 11 Signs, 28 Degrees, 49 Minutes, 46 Seconds, how many Seconds?

Anſwer. 1291786 Seconds.

VI. In 1291786 Seconds, how many Signs, Degrees, Minutes, and Seconds?

Anſwer. 11 Signs, 28 Degrees, 49 Minutes, and 46 Seconds.

VII. In 264 Acres, 3 Roods, 36 Perches, how many Perches?

Anſwer. 42396 Perches.

VIII. In

VIII. In 42396 Perches, how many Acres?

Anſwer. 264 Acres + 3 Roods + 36 Perches.

IX. In 7 Pounds 17 Shillings and 6 Pence, how many Piſtoles at 17 Shillings and 6 Pence?

Anſwer. 9.

X. In 12 Pounds 10 Shillings, how many Rix-Dollars, at 4 Shillings and 2 Pence?

Anſwer. 60.

See more Queſtions in the APPENDIX.

CHAP. IX.

IN the foregoing Chapter I have explain'd the CHIEF PARTS OF ARITHMETIC, *viz.* ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; by Help of which, the following PRACTICAL Rules of PROPORTION, as they come in Order, may be perform'd; but, that THEIR THEORY, as well as PRACTICE, may ALSO be rightly underſtood, this Chapter is firſt inſerted.

EUCLID defines the Word PROPORTION, or RATIO thus (ſee Def. 3. of Book 5. of Elem.)

Λογος ἐστί δυό μεγεθῶν ὁμογενῶν ἡ κατὰ πηλικότητα ποσὸς ἄλληλα ποια σχέσις.

PROPORTION, *or* RATIO, *is a certain mutual Relation of* HOMOGENEAL *Quantities, according to Quantity*; or it is a determinate MODE, or MANNER, by which any one Quantity contains another, or is contained by it; which *Mode* or *Manner*, we may eaſily comprehend by Help of the following

EXAMPLES.

The *Mode* or *Manner* by which a POUND, *Troy-Weight*, contains an OUNCE, ſignifies no more, but that the *former* contains the *latter* 12 times, or that the *former* is to the *latter*, as 12 to 1. Again,

The

The *Mode* or *Manner* by which a FOOT is contain'd in a PACE, is nothing elſe but this, *viz. That* is the fifth Part of *This*, or is to *This* as 1 to 5.

EUCLID defines the Word PROPORTIONALITY, or ANALOGY, thus,

Ἀναλογία ἐστὶν ἡ τῶν λόγων ὁμοιότης.

ANALOGY *is a Similitude or Likeneſs of* RATIOS *or* PROPORTIONS; thus,

There is an *Analogy* or *Proportionality* between 9 to 3, and 21 to 7, the *Ratios* of 9 to 3, and of 21 to 7, being alike, *viz.* both TRIPLE.

Theſe DEFINITIONS being underſtood, and what has been ſaid in the foregoing Pages conſider'd, 'tis eaſy to ſee there are TWO Ways of comparing Numbers, or whatever elſe is capable of *Quantity* or *Menſuration*, with others *homogeneal* to *themſelves*; the ONE according to their DIFFERENCE, the OTHER according to their RATIO or PROPORTION.

N. B. Becauſe Things are to be compar'd to each other, by means of ſome COMMON Thing which is IN, or ſuppoſed to be IN, BOTH the Things *to be compar'd*, and not by any thing which belongs only to ONE of them, or to NEITHER of them; therefore only HOMOGENEAL Quantities CAN be compar'd together, whether the Compariſon be made as to their DIFFERENCE, or to their RATIO.

Thus a TIME with a TIME, a LINE with a LINE, may be compar'd both as to DIFFERENCE and to RATIO; but, for any one to compare a TIME with a LINE, ſeeking either their DIFFERENCE, or their RATIO, is abſurd.

Theſe

Theſe Things being premiſed, I come now to explain what was before mention'd, *viz.* That

HOMOGENEAL or LIKE Quantities (*i. e.* ſuch as are capable of being compar'd to each other) may (according to *that Meaſure* by which they are *homogeneal)* be TWO Ways compar'd; for,

I. It may be either ſimply enquir'd about their DIFFERENCE or EXCESS, *viz.* whether they DO differ, or How MUCH they differ, *i. e.* by How MUCH the ONE is GREATER or LESS than the OTHER: Or,

2. It may be enquir'd about their RATIO or PROPORTION; as ſuppoſe, How OFTEN *this* contains *that*, or How MUCH *this* is OF *that*; whether, for Inſtance, *this* contains *that*, ONCE, TWICE, THRICE, *&c.* or whether *that* contains *this* ONCE and $\frac{1}{2}$, TWICE and $\frac{1}{3}$, THRICE and $\frac{1}{4}$ *this*.

In either Method of Compariſon, the ONE is call'd the ANTECEDENT, the OTHER the CONSEQUENT; thus, ſuppoſe A be compar'd to B, A is call'd the ANTECEDENT, and B the CONSEQUENT; A being (in this Compariſon) ſaid to be either GREATER THAN, LESS THAN, or EQUAL TO B; or A the DOUBLE, TRIPLE, HALF, *&c.* of B.

The FIRST Way of comparing HOMOGENEAL Quantities (*viz.* as to their DIFFERENCE) is done by SUBTRACTION; thus,

If from 12 be taken 10, there remain 2; therefore *that* is GREATER THAN *this*, or *this* LESS THAN *that* by 2 Units. Thus again,

If from 3 Pound-weight be taken 2 Pound-weight, there remains 1 Pound-weight; but if after *Subtraction* NOTHING

NOTHING remains, the Quantities are EQUAL to each other. But,

The SECOND Way of comparing HOMOGENEAL Quantities (*viz.* as to their RATIO) is by DIVISION; for the *Quotient* of the *Diviſion* ſhews the RATIO of the *Dividend* to the *Diviſor*: Thus, If 12 be divided by 6, the QUOTIENT will be 2; hence the RATIO which the Number 12 has to the Number 6, is call'd DOUBLE: And thus, 4 Pound-weight is the *Double* of 2 Pound-weight, becauſe if 4 Pound-weight be divided by 2 Pound-weight, the QUOTIENT is 2, *i. e.* SO OFTEN *this* Weight is contain'd in *that*.

And therefore,

Where the Quotients are equal to one another, there the Quantities are in the ſame RATIO *or* PROPORTION.

Again, all RATIOS of Quantities, of *what Kind ſoever*, are HOMOGENEAL to each other: For,

Altho' a LINE and a WEIGHT, for Inſtance, are Things plainly HETEROGENEAL, and have NO RATIO to each other, yet the RATIO of a LINE to a LINE, and *that* of a WEIGHT to a WEIGHT, are plainly HOMOGENEAL.

Thus for Inſtance,

The RATIO of a LINE of 2 Feet to a LINE of 1 Foot, and of a 2 POUND Weight to a Weight of 1 POUND, are HOMOGENEAL; nay, the RATIO is the very SAME, *viz.* DOUBLE, for, after *Diviſion*, the QUOTIENT in both is 2. But,

All DIFFERENCES of Quantities, of *what Kind ſoever*, are not HOMOGENEAL to each other:

For

For the *Difference* between a 4 Pound Weight, and a 2 Pound one, is 2 Pound; the *Difference* between a Line of 4 Feet, and one of a Foot, is a Line of 3 Feet: But now there can be no Compariſon made between theſe *Differences* (either as to their *Difference*, or as to their *Ratio*) ſince a Line of 3 Feet, and a 2 Pound Weight, are Quantities plainly *heterogeneal*: And the Cauſe of this Difference ariſing from the COMPARISONS made of Quantities, as to their RATIOS and DIFFERENCES, is as follows;

When the COMPARISON is made as to the *Difference*, the Reſidue or Exceſs is always *of the ſame Nature* with the Things compar'd (for the *Whole* and its *Part* are *homogeneal* to each other); and therefore, if *theſe* two compar'd ones be *heterogeneal* to *thoſe* two, the *Difference* of *theſe* will alſo be *heterogeneal* to the *Difference* of *thoſe*; and thence theſe *Differences* are incapable of *Compariſon*. But when the COMPARISON is made as to the *Ratio*, the *Ratio* which is gain'd, often *differs from the Nature* of the Things *compar'd*, and paſſes into a *Numeral* Nature, let the Things which are *compar'd* be of what Kind they will; and therefore, howſoever *heterogeneal* any two Things *compar'd* together, are to any other two Things *compar'd* together, yet their *Ratios* are *homogeneal*, and thence may be *compar'd* to each other.

But to proceed:

From what has been ſaid, may be underſtood what it is to COMPARE *homogeneal* Quantities TO EACH OTHER, whether as to their EXCESS or DIFFERENCE, or to their RATIO or PROPORTION, *viz.*

'Tis THERE enquir'd *how much* THIS is *greater than* THAT, or the contrary; which is perform'd by SUBTRACTION; HERE 'tis enquir'd *how often* THIS contains, or *how much* THIS is *of* THAT, or THAT *of* THIS; which is anſwer'd by DIVISION.

In

In the FORMER the EXCESS or DIFFERENCE is produc'd, in the LATTER, the QUOTIENT or RATIO.

But now,

From this DOUBLE *Comparison*, whether of *Numbers*, or of *any other* HOMOGENEAL *Quantities whatsoever*, there ariſes a TWO-FOLD PROGRESSION, *one* of which is call'd ARITHMETICAL, *the other* GEOMETRICAL; (NAMES, I believe, choſen merely *at Pleaſure*, ſince EITHER of them has the LIKE *Regard* to *both* ARITHMETIC and GEOMETRY).

That is call'd ARITHMETICAL *Progreſſion* where *ſeveral Numbers*, or *other Quantities*, HOMOGENEAL to each other, proceed with EQUAL DIFFERENCES (whether *Exceſſes* or *Defects*); as 2, 4, 6, 8, *&c.* and 3, 5, 7, 9, *&c.* and contrary-wiſe 9, 7, 5, 3, *&c.*

That is call'd GEOMETRICAL *Progreſſion* where *ſeveral Numbers*, or *other Quantities*, HOMOGENEAL to each other, proceed in the SAME (or EQUAL) RATIO or PROPORTION; as 2, 4, 8, 16, 32, *&c.* and 3, 9, 27, 81, 243, *&c.* and 8, 12, 18, 27, *&c.* and contrary-wiſe 32, 16, 8, 4, *&c.*

For as ARITHMETICAL *Progreſſion* regards the EQUALITY of the DIFFERENCES; ſo GEOMETRICAL, you may call an EQUALITY, IDENTITY or SIMILITUDE of RATIO's; whence, *there*, THE COMMON EXCESS or DIFFERENCE, *here*, THE COMMON RATIO or PROPORTION, determines the SPECIES of the *Progreſſion*.

But now *theſe Progreſſions* (whether *Arithmetical* or *Geometrical*) are either CONTINUED or DISJUNCT, (DISCRETE, DISCONTINU'D, INTERRUPTED); CONTINU'D, when the Quantities, plac'd one after another, proceed with EQUAL, *either* DIFFERENCES or RATIO's;

DISJUNCT, when indeed the *ſame* DIFFERENCE or RATIO returns, but is not *continually* preſerv'd: In the FORMER, every intermediate Term is both *antecedent* and *conſequent* of the COMMON, whether *Difference* or *Ratio*; in the LATTER 'tis otherwiſe: Thus, 3, 5, 7, 9, 11, or 11, 9, 7, 5, 3; are in *arithmetical Progreſſion* CONTINU'D, becauſe in the *former* Caſe every Number exceeds its *Antecedent*; in the *latter*, its *Conſequent*, by 2.

But 3, 5; 9, 11; or 9, 11; 5, 7; are in *arithmetical Progreſſion* DISJUNCT, or DISCONTINU'D; for the *ſecond* exceeds the *firſt*, and the *fourth* the *third*, with the SAME Exceſs; but the *third* does not *ſo* exceed the *ſecond*. And in like manner 5, 3; 11, 9; 17, 15; 19, 17; are in the ſame *arithmetical Progreſſion*, but DISCONTINU'D.

After the ſame Manner 2, 4, 8, 16, 32, *&c.* or 32, 16, 8, 4, *&c.* are in *geometrical Progreſſion* CONTINU'D, becauſe in the *former* Caſe the *Conſequent* is to its *Antecedent* all along in a DOUBLE RATIO; or as 2 to 1 in the *latter*, in a SUBDUPLE RATIO, or as 1 to 2.

But now, 2 : 4 : : 3 : 6 : : 4 : 8, are, indeed, in a *geometrical Progreſſion*, but a DISCONTINU'D or INTERRUPTED *one*, becauſe the *ſecond* exceeds the *firſt*, and the *fourth* the *third*, and the *ſixth* the *fifth*, in the SAME RATIO; but the *third* does not *ſo* exceed the *ſecond*, or the *fifth* the *fourth*.

In like Manner it is ſaid of 4 : 1 : : 12 : 3, which Numbers are indeed GEOMETRICALLY *proportional*, but not CONTINUALLY *ſo*.

And now (which let be ſaid once for all) when EITHER *Numbers* OR *Quantities*, in the following Pages, are (as they often are) THUS *denoted*, A : B : : C : D.; thoſe *Numbers* or *Quantities* muſt be underſtood to be PROPORTIONALS (*geometrical* ones I mean)

mean) at leaſt DISCONTINU'D, *i. e.* WHATEVER *Ratio* A has to B, THE SAME C has to D.

But A, B, C, D, ÷ ſignifies the *Quantities* denoted to be CONTINUAL *geometrical* PROPORTIONALS.

And ARITHMETICAL *Progreſſion*, when needful, may be THUS denoted, A, B; C, D; or if CONTINU'D, THUS, A, B, C, D, $\frac{\cdot}{;}\frac{c}{\cdot}$

Thus have I, in ſome meaſure, explain'd the DOUBLE *Compariſon* of *Quantities* to each other, and have alſo ſhewn the NATURE of both ARITHMETICAL and GEOMETRICAL PROGRESSION, which hereafter ſhall be treated of more at large.

What has been already ſaid concerning them, is ſufficient for our preſent Purpoſe, *viz.* to enable us to treat, IN A RATIONAL WAY, of that moſt uſeful Rule commonly call'd the GOLDEN RULE; which to do is the Deſign of the next Chapter.

CHAP. X.

WHAT has been ſaid in the foregoing Chapters, being rightly underſtood, the following RULE muſt be eaſily intelligible as to its THEORY, as well as PRACTICE.

This Rule has divers NAMES, as

The { GOLDEN-RULE, RULE OF THREE, and RULE OF PROPORTION.

It is call'd (and not undeſervedly) the GOLDEN Rule, becauſe of its *excellent* and *univerſal* Uſe.

It is called the Rule of THREE, becauſe in it THREE Numbers are *given* or *known*, from whence a FOURTH, in *Geometrical Proportion* at leaſt *diſcontinu'd*, though *unknown*, may be diſcover'd: For the ſame Reaſon its call'd the Rule of PROPORTION.

This Rule is either DIRECT or RECIPROCAL (or INVERSE); and theſe again, either SIMPLE or COMPOUNDED.

RULE. In the *Simple* Rule of *Three* DIRECT, if the *third* Term be multiply'd by the *ſecond* (or *this* by *that*) and the Product be divided by the *firſt*, the Quotient will exhibit the *fourth* Term ſought.

DEMON-

DEMONSTRATION.

Let the four proportional Terms be ſignify'd by the Letters a, b, c, d: Now, by the foregoing Rule, the three firſt, *viz.* a, b, and c, being given, the fourth Term ſought d, will be $= \frac{b\,c}{a}$. And that it will certainly be ſo, may be prov'd, thus,

Since it is by the Hypotheſis $a : b :: c : d$, and thence (by Inverſion) $b : a :: d : c$, that is $\frac{b}{a} = \frac{d}{c}$ if both be $\times c$, there will be produc'd $\frac{b\,c}{a} = d$, or $a)\ b\,c\ (d$: Q. E. D. Or thus,

Becauſe by the Hypotheſis it is $a : b :: c : d$, that is $\frac{a}{b} = \frac{c}{d}$ if both be $\times b\,d$, there will be produc'd $a\,d = b\,c$, dividing both which by a, it will be $d = \frac{b\,c}{a}$ Q. E. D.

Or, which may be more perſpicuous to the Learner, thus,

Since it is $a : b :: c : d$, that is (by the Definition of PROPORTIONALS, or of the *ſame* or *equal* RATIO's, given in the laſt Chapter) $\frac{a}{b} = \frac{c}{d}$: And ſince (by Ax. 4.) if equal things be multiply'd by equal things, the Products will be equal, therefore if both $\frac{a}{b}$ and $\frac{c}{d}$ be multiply'd into the ſame Quantity b, the former Product will be a, and the latter $\frac{b\,c}{d}$ which two Products are $=$ each other.

Again,

Again, If each of theſe two *equal* Products be multiply'd by the *ſame* Quantity *d*, we ſhall have on the *one* Side the new Product *a d*, and on the *other*, the new one *c b*; which Products (by the foremention'd Axiom) muſt alſo be = each other: But now, *a* and *d* are the *firſt* and *laſt* Terms of the Proportion firſt mention'd, and *b* and *c* are the *ſecond* and *third* Terms in the *ſame* Proportion, therefore this Concluſion is fairly gain'd, *viz. Of four geometrical Proportionals, the Product of the Extreams, is equal to the Product of the Means.* Now, laſtly, (by Ax. 5.) if equal things be divided by equal things, the Quotients will be equal: If, therefore, both the laſt *equal* Products, *viz. a d* and *c b*, be divided by the ſame Quantity *a*, the *one* Quotient will be *d*, the *other* $\frac{c\,b}{a}$ or $\frac{b\,c}{a}$ which Quotients muſt be equal to each other: Therefore univerſally.

If of four geometrical Proportionals (whether *continu'd* or *interrupted*) *the Product of the ſecond and third be divided by the firſt, the Quotient will give the fourth ſought.* Q. E. D.

This being prov'd, I now proceed to give a familiar Example or two in this Rule, making ſome uſeful Obſervations as I go along.

1. If 3 Pounds of *any* Wares coſt 5 *s.* how many Shillings will 15 Pounds of the ſame Wares coſt?

Here now, becauſe in the Queſtion 'tis ſuppos'd, that the 15 Pounds of Wares are at the *ſame Rate per* Pound as the 3 Pounds are; therefore it will be,

As the *Wares* to the *Wares*, ſo the *Price* to the *Price*; and ſo,

As

As 3 (the Number of Wares bought) to 15 (the Number to be bought) ſo 5 (the Number of the given Price) to the Number (if I may ſo expreſs myſelf) of the Price ſought.

Call the Price ſought Q: Then,

p. p. s. s.
3 : 15 :: 5 : Q. *or*,

Becauſe all that are here employ'd are NUMBERS, and therefore HOMOGENEAL, it will be *alternately*, (and none but HOMOGENEAL Quantities can be *alternately* compar'd).

As the *Wares* to the *Price*, ſo the *Wares* to the *Price*, *viz.*

p. s. p. s.
3 : 5 :: 15 : Q.

Now this is ſolv'd, according to the preſent Rule, by multiplying the ſecond Term into the third (or, which is the ſame thing, the third into the ſecond) whence we have ($5 \times 15 =$) 75; and, by dividing by the firſt, we have ($\frac{75}{3} =$) $25 = Q$: And thus the Price ſought is 25 Shillings.

See the WORK.

15
5
—
3) 75 (25 = Q.

2. If 3 Days Wages come to 2 Shillings, what will 12 Days Wages come to? *i. e.*

Da. *s.* *Da.* *s.*
3 : 2 : : 12 : Q.

By multiplying 2 into 12, you have 24, which divided by the firſt Number 3, gives 8 = Q; therefore 12 Days Wages is 8 Shillings. Thus,

12
2
—
3)24(8 = Q.

But if it happen that the *homologous* Terms propos'd in the Queſtion, are not of the *ſame* Denomination, thoſe Terms muſt be reduc'd to the *ſame* Denomination with each other, before you can proceed to the Operation.

For Inſtance,

Suppoſe a Queſtion of this kind be propos'd, *viz.*

3. If 3 OUNCES of Silver be worth 15 *Engliſh* Shillings, what are 4 POUNDS of Silver worth?

Since of the four Terms which are here ſuppoſed proportional, *two* have reſpect to WEIGHT, and *two* to VALUE; and ſince *thoſe* which have reſpect to WEIGHT, are not of the *ſame* Denomination with each other; but the ONE ſpeaks of OUNCES, and the OTHER of POUNDS; therefore *theſe* Terms muſt be reduc'd into the *ſame* Denomination, before we can *work* as by the foregoing Rule we are order'd; and, therefore, inſtead of 3 Ounces, we muſt either ſubſtitute ¼ of a Pound (for ſince a Pound of Silver contains 12 Ounces, 3 Ounces will be ¼ of a Pound); and then the Queſtion will be thus ſtated;

As

As $\frac{1}{4}$ *of a Pound, to* 4 *Pounds; ſo* 15 *Shillings to the Value ſought.*

Or otherwiſe,

Inſtead of 4 Pounds, we muſt ſubſtitute 48 Ounces (for ſince a Pound contains 12 Ounces, 4 Pounds will contain 48 Ounces $= 4 \times 12$) and then the Queſtion will be thus ſtated;

As 3 *Ounces, to* 48 *Ounces; ſo* 15 *Shillings, to the Value ſought:* Now, by either of theſe Methods will be produc'd the Value ſought = 240 Shillings, *i. e.* 12 Pounds of *Engliſh* Money; as is manifeſt from the following Operations.

p. p. s. s.
$\frac{1}{4} : 4 :: 15 : Q.$
4
— s.
$\frac{1}{4})60(240 = Q.$
l.
$12 = Q.$

Oz. Oz. s. s.
$3 : 48 :: 15 : Q.$
15
240
48
— s.
$3)720(240 = Q.$
l.
$12 = Q.$

But now, it may be neceſſary to ſay ſomething more fully concerning the *rightly* ordering the Terms of any Proportion: And here Care muſt be taken, that each of the three given Quantities be not promiſcuouſly put in *any* Place, but every one in its *own*, i. e. The four Quantities (*viz.* the three given ones, and the fourth ſought) muſt be *ſo* diſpos'd, that the *firſt* may be *rightly* ſuppos'd to have the SAME RATIO to *one* of the remaining ones, as the *other* remaining one has to the *fourth* ſought; for otherwiſe, when ſtated, they'll *not* be PROPORTIONALS.

Thus in the firſt Example; The *firſt* Term is neither 5 nor 15, but 3; the *ſecond* Term, at pleaſure, either 5 or 15, *i. e.* it will be,

As 3 *Pounds to* 15 *Pounds, ſo* 5 *Shillings to* 2. *Shillings.* For,

The *ſame* RATIO as the WEIGHT has to the WEIGHT, the *ſame*, 'tis *rightly* ſuppos'd, the VALUE has to the VALUE. Or it will be,

As 3 *Pounds to* 5 *Shillings; ſo* 15 *Pounds to* 2. *Shillings.* For,

As the *former* WEIGHT is to its given VALUE, *ſo*, it's ſuppos'd, the *latter* WEIGHT is to its VALUE ſought.

Let this be ſufficient concerning the *right ordering* the Terms of any Proportion.

And now, when the Terms are thus *rightly* diſpos'd, 'tis the ſame thing (as I have already ſaid) whether the *third* Term be multiply'd by the *ſecond*, or *this* by *that*; not only becauſe (as was juſt now ſaid) it is all one whether of the two is put in the *ſecond* or *third* Place; but chiefly, becauſe of two Numbers to be multiply'd together, 'tis all one whether of the two we make the *Multiplier* or *Multiplicand*; as has been before taught.

But to proceed,

'Tis all one whether we begin theſe Operations with MULTIPLYING or DIVIDING: For,

Whether we *multiply* the *ſecond* and *third* Terms together (as is commonly done) and *divide* their *Product* by the *firſt*: Or,

Whether

Whether we *divide* EITHER the *ſecond* or *third* by the *firſt*, and then *multiply* the *Quotient* ariſing thence, by the *remaining* Term (whether it happen to be the *ſecond* or *third*) 'tis the ſame thing; For,

The *fourth* Term ſought is EITHER Way the ſame: Thus,

If it be A : B : : C : D, it will be univerſally $\frac{B\,C}{A} = \frac{C\,B}{A} = \frac{B}{A}\,C = \frac{C}{A}\,B = D$ the fourth Term ſought.

Or thus in NUMBERS,

If, for Inſtance, it be 4 : 12 : : 8 : 24, it will be $\frac{12 \times 8}{4} = \frac{8 \times 12}{4} \left(= \frac{96}{4}\right) = \frac{12}{4} \times 8 \left(= 3 \times 8\right) = \frac{8}{4} \times 12 \left(= 2 \times 12\right) = 24$. *i. e.* The *fourth* Term in the Proportion, will be = the *Product* of the *ſecond* Term into the *third*, divided by the *firſt*; Or,

To the *Product* of the *third* Term into the *ſecond* divided by the *firſt*; Or,

To the *Quotient* of the *ſecond* Term divided by the *firſt*, multiply'd by the *third*; Or,

To the *Quotient* of the *third* divided by the *firſt*, multiply'd by the *ſecond*: And therefore,

'Tis left to every Man's Liberty to *order* his *Operation* as ſeems moſt commodious to him. But however,

'Tis agreed in the Uſe of this *Rule*, rather to *multiply* firſt; for, by doing ſo, the Trouble of FRACTIONS is the better ſhunn'd: For,

If, for Example's ſake, a given *Proportion* be

2 : 5 :: 8 : Q.

And I would *divide* the *ſecond* by the *firſt*, by this Method the *Quotient* will be $2\frac{1}{2}$, and therefore the *third* Term 8 muſt be *multiply'd* by $2\frac{1}{2}$ (a Number encumber'd with a FRACTION) thus,

$$\begin{array}{r} 2)\,5\,(2\tfrac{1}{2} \\ 8 \\ \hline 20 \end{array}$$

But if the *Product* of the *middle* ones 5 and 8 (*viz.* 40) be *divided* by the *firſt*, 2, each Operation will produce a whole Number.

$$\begin{array}{r} 2 ; 5 :: 8 : Q. \\ 5 \\ \hline 2)40(20 \end{array}$$

Both Ways, indeed, in the End, yield the ſame Succeſs: For,

$8 \times 2\frac{1}{2} = 20$, and $\frac{40}{2} = 20 = Q.$ the Thing ſought. But the laſt Method has the leaſt Trouble in it. Thus again,

If the Proportion offer'd be 2 : 5 : : 7 : Q, and I would *divide* the *ſecond* by the *firſt*, i. e. 5 by 2, which *quotes* $2\frac{1}{2}$, and then *multiply* the *third* by this $2\frac{1}{2}$. Or,

If I would *divide* the *third* by the *firſt*, i. e. 7 by 2, which *quotes* $3\frac{1}{2}$, and then *multiply* the *ſecond*, *viz.* 5 by this $3\frac{1}{2}$, the Operation either Way is more incumber'd, than if, after firſt having *multiply'd* the *middle* Terms

t

Terms 5 and 7 into each other, I then *divide* their *Product*, *viz.* 35, by the *firſt*, *viz.* by 2.

$$2)5(2\tfrac{1}{2}$$
$$7$$
$$17\tfrac{1}{2}$$

$$2)7(3\tfrac{1}{2}$$
$$5$$
$$17\tfrac{1}{2}$$

$$5$$
$$7$$
$$2)35(17\tfrac{1}{2}$$

Yet take which Method you will, there is at laſt produc'd $17\frac{1}{2}$, the fourth Number ſought; and altho' at laſt here alſo is produc'd a Number with an adjoin'd *Fraction*, yet it is ſuch a one as will create no farther Difficulty, there being no following Operation to be made more perplex'd by ſuch annex'd *Fraction*.

But however,

When 'tis ſeen that one of the *middle* Terms CAN be *divided* by the *firſt*, SO AS TO LEAVE NO FRACTION, it may be convenient to *divide* firſt; for thus the Operation will be perform'd in leſs Numbers.

Thus for Inſtance,

Suppoſe the *Analogy* or *Proportion* be as before, *viz.*

2 : 5 :: 8 : Q.

It

It may be convenient to *divide* (tho' NOT the *ſecond*, YET) the *third* by the *firſt*, and to *multiply* the *Quotient* into the *remaining* one.

Thus, the *third* Term 8, *divided* by the *firſt*, 2, *quotes* 4, which *multiply'd* into the *ſecond*, 5, produces 20, the *fourth* Number ſought.

2) 8 (4
5
—
20

Sometimes alſo, to render the Operation more expeditious, a precedent *Diminution*, or *leſſening* of the Terms, will be neceſſary, and this is done by *dividing*, by a COMMON DIVISOR, either the *firſt* and *ſecond* Term, or the *firſt* and *third*, where it's ſeen it can be done; or, indeed, ſometimes by doing *both*: Thus,

Let the Proportion be 6 : 9 : : 8 : Q.

If the *firſt* and *ſecond* be *divided* by 3,

3) 6 (2 3) 9 (3

And the *Quotients* 2 and 3 be ſubſtituted in their Stead, this Proportion is got,

2 : 3 : : 8 : Q.

But, if the *firſt* and *third* be *divided* by 2,

2) 6 (3 2) 8 (4

And their *Quotients* 3 and 4 be taken in their Stead, the Proportion 3 : 9 : : 4 : Q. is gain'd; And,

If,

If, laſtly, *both* be done, *i. e.* ſuppoſe both the *firſt* and *ſecond* be *divided* by 3, and the *firſt* now abbreviated, and the *third* be *divided* by 2.

3)6(2 3)9(3
2)2(1 2)8(4

The folllowing Proportion is gain'd, *viz.*

1 : 3 :: 4 : Q.

Take which of theſe Methods you will, you'll ſee that Q, the fourth Term ſought, will be found = 12.

In like manner in SYMBOLS.

If there be propos'd the *Analogy*

ABC : ADE :: BEF : Q.

Dividing the *firſt* and *ſecond* by A, you have it thus,

BC : DE :: BEF : Q. Or,

Dividing the *firſt* and *third* by B, you'll get

AC : ADE :: EF : Q. Or again,

By doing *both*, you'll get

C : DE :: EF : Q.

By each of which Proportions, the *ſame* fourth Term is produc'd, or at leaſt *ſuch* fourth Terms as have *the ſame* VALUE, *i. e.* The fourth Term Q is =

$\frac{ADE \times BEF}{ABC} = \frac{DE \times BEF}{BC} = \frac{ADE \times EF}{AC} = \frac{DE \times EF = DEqF}{C}$.

Some-

Sometimes the Terms are conveniently prepar'd by MULTIPLICATION, thus,

If either the *firſt* and *ſecond*, or *firſt* and *third*, be *multiply'd* by a COMMON MULTIPLIER, or, indeed, if *both* be done, by which Means the Trouble of the FRACTIONS is eaſily avoided: Thus,

Let this *Analogy* be propos'd,

$\frac{5}{2} : 4 :: 3 : Q.$

Here we may conveniently ſubſtitute the *Double* of the *firſt* and *ſecond*, or of the *firſt* and *third*; Thus,

$5 : 8 :: 3 : Q.$ Or, $5 : 4 :: 6 : Q.$

For either of theſe Proportions anſwer the ſame End with the given one.

Thus again, if the *Proportion* offer'd be

$3) \frac{5}{2} : \frac{2}{3} :: \frac{1}{2} : Q.$

We may ſubſtitute in its Room, either of the following ones, *viz.* either

$\frac{5}{2} : 2 :: \frac{1}{2} : Q.$

(Here the *firſt* and *ſecond* Terms are *multiply'd* by 3) Or,

$\frac{5}{3} : \frac{2}{3} :: 1 : Q.$

(Here the *firſt* and *third* are *multiply'd* by 2).

Or, laſtly,

$5 : 2 :: 1 : Q.$

(Here

(Here both the *first* and *second* are *multiply'd* by 3, and the *first* and *third* by 2).

The like in Symbols: thus, Inſtead of the following *Analogy*, *viz.*

$$\frac{Aq}{BC} : \frac{DE}{BF} :: \frac{GH}{CF} : Q,$$

May be ſubſtituted (*multiplying* the *firſt* and *third* Terms into C) the following one, in its Stead, *viz.*

$$\frac{Aq}{B} : \frac{DE}{BF} :: \frac{GH}{F} : Q.$$

Or, *Multiplying* the *firſt* and *third* into B, the following one may be us'd,

$$\frac{Aq}{C} : \frac{DE}{F} :: \frac{GH}{CF} : Q.$$

Or rather, *Both* being done, *i. e.* the *firſt* and *third* being *multiply'd* into C, and then, this *firſt*, ſo *multiply'd*, and the *ſecond* being *multiply'd* into B, the following Proportion is gain'd, *viz.*

$$Aq : \frac{DE}{F} :: \frac{GH}{F} : Q.$$

And now, by *multiplying* the *firſt* and *ſecond* into F, we get this,

$$AqF : DE :: \frac{GH}{F} : Q.$$

And laſtly, by *multiplying* the Now *firſt*, and the *third* into F, is gotten the following Proportion, *viz.*

$$AqFq : DE :: GH : Q.$$

Thus is the firſt given Proportion clear'd of its fractional Terms.

The common Foundation of which preparative Operations is *this*,

There is the *ſame* RATIO of *Multiples* to their *Equi-Multiples*, as of *Simples* to *Simples*. As thus,

The ſame *Ratio* as 3×2 has to 4×2, the very ſame has 3 to 4: Alſo 'tis

$$\tfrac{3}{2} : \tfrac{4}{2} :: 3 : 4$$

Alſo $3A : 4A :: 3 : 4$

And $BA : CA :: B : C$

And $\frac{B}{A} : \frac{C}{A} :: B : C$

And the like in others.

Before we go on, it may be neceſſary to clear one Difficulty, which is as follows:

Analogies, or *Proportions*, are ſometimes fram'd amongſt *heterogeneal* Quantities (as thus ſuppoſe) As the *Line* A to the *Line* B; ſo the *Weight* α to the *Weight* β) and it may be ask'd, how can *one* of theſe be *multiplied* or *divided* by the *other*? As alſo, what, for Inſtance, *to multiply the Weight* α *into the Line* B, or, *to divide it by the Line* A, means? For *what* will that PRODUCT be whoſe *Length* is a LINE, and *Breadth* a WEIGHT? or *what* that QUOTIENT where it's ſought *how often* the *Weight* α contains the *Line* A?

In

In *anſwer*, we ſay, That a *Line* here is *not* compar'd to a *Weight*, (which are *heterogeneal* Quantities, and thence not poſſibly to be compar'd with each other) but that the RATIO of the two *Lines* is compar'd with the RATIO of the two *Weights*; which *Ratios* are *homogeneal* to each other; as ſuppoſe *double*, *triple*, *ſeſquialter*, &c. *viz.* taking the *Denomination* from the *Quotient* of the *Antecedent* divided by the *Conſequent*; but that *Quotient*, as has been before ſhewn, is a NUMBER, or at leaſt ſomething of a *numeral* Nature, and *homogeneal* to real Numbers; as ſuppoſe, a *broken* Number, or a *ſurd* one, or the like.

So that when 'tis ſaid, $A : B :: \alpha : \beta$, we mean no more, than that the RATIO of A to B, or $\frac{A}{B}$ is $=$ the RATIO of α to β, or $\frac{\alpha}{\beta}$. Or, which comes to the ſame, that the *Quotient* of the Quantity A $\div$ the *homogeneal* one B, is $=$ the *Quotient* of the Quantity $\alpha \div$ its *Homogeneal* (however *heterogeneal* to the former ones) Thus,

If A be a *Line* of 2 Feet, B a *Line* of 1 Foot; *that* ſhall be the *Double* of *this*, or as 2 to 1. Again,

If α be a *Weight* of 4 Pounds, and β a *Weight* of 2 Pounds; *that* will be the *Double* of *this*, or as 2 to 1. For both B)A(2, and $\beta)\alpha(2$: And therefore $\frac{A}{B} = \frac{\alpha}{\beta} = 2 = \frac{2}{1} = \frac{4}{2}$, &c.

Since, then, we have regard here to nothing but the RATIO, or the QUOTIENT, we may, inſtead of the *propos'd* Quantities, ſubſtitute *others*, of any kind, which are in the *ſame* RATIO, or which exhibit the *ſame* QUOTIENT, however *heterogeneal* they are to thoſe *propos'd*.

pos'd. The ARITMETICIAN therefore, in his *Operations*, inſtead of the *Quantities propos'd*, of what Kind ſoever, whoſe RATIO *only* is conſider'd, ſubſtitutes NUMBERS (or what is of the *Nature* of *Number*) which are *in the ſame* PROPORTION.

Thus, for Inſtance,

Inſtead of the *Lines* A and B of two Feet, and one Foot, he ſubſtitutes any two Numbers *in the ſame* RATIO, as 2, 1; or 4, 2; or 6, 3, *&c.* Thus, $\frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{A}{B}$ And, by ſubſtituting *equal* Quantities, inſtead of *thoſe* propos'd, the Quantity is by no means *chang'd.*

Thence it's the ſame thing whether he ſays, As the *Line* of two Feet A, to *that* of one Foot B, or as the *Number* 2 to 1; or as 4 to 2, *&c.* the *Quotient*, and thence alſo the *Ratio* (which is all we regard here) being on both Sides *equal.* And although, inſtead of theſe *Numbers*, he ſometimes again ſubſtitutes *Symbols*, or retains thoſe the *Lines* had, yet he takes theſe, as it were, for *Numbers*, and treats them like *Numbers*; nor does he ſuppoſe himſelf to be converſant ſo much with *Lines* and *Weights*, as with *Numbers* in the *ſame* RATIO's with thoſe *Lines* and *Weights.*

Thus in the propos'd Example,

If A, B, be imagin'd to ſignify 4, 2; (*viz.* becauſe $\frac{A}{B} = \frac{4}{2}$) α and β, if you will, remaining as *Symbols* of the WEIGHTS: It will be the *ſame* thing to ſay, $A : B :: \alpha : \beta$, as to ſay, $4 : 2 :: \alpha : \beta$: And therefore the *Rectangle* of the *ſecond* and *third* $B \times \alpha$ is $2 \times \alpha$, or the *Double* of the *Weight* α: And $\frac{B \times \alpha}{A}$ is $\frac{2 \times \alpha}{4}$ or the *fourth* of that *Double.*

But

But now, to *double* a *Weight*, and then to *divide* that *Double* into four Parts, has nothing abſurd in it.

But of this enough for our preſent Purpoſe.

Now, altho', as has been at large ſhewn, it's manifeſt, that the *Rule of Three* DIRECT may be *ſeveral Ways* WROUGHT, yet the *firſt* being the moſt *general*, and moſt *practis'd*, as has likewiſe been ſhewn, not without ſome *Reaſon*, ſo I ſhall make uſe of it in the *Solution* of the following *Queſtions*.

QUESTION I.

If the *Intereſt* of 100 Pounds for one Year be 5 Pounds, what is the *Intereſt* of 530 Pounds, for the ſame Time?

OPERATION.

100 : 6 :: 550
6
——— *Anſwer*
100)3300(33 Pounds.

QUESTION II.

If 4 Pounds of any *Wares* coſt 8 Shillings, what will one hundred Weight (112 Pounds) coſt at that Rate?

p. s. p.
4 — 8 — 112
8
——— *Anſwer*
4)896(224 Shill.
16
0

Note,

Note, Since the Number 1 neither *multiplies* nor *divides*, ſo as to *alter* the Number *multiplied* or *divided* by it, if in any *Queſtion* in this *Rule* of *Three*, the *firſt* Number of the *three* given ones be the Number 1, the *Work* may be *done* by *Multiplication* only; and if the *laſt* of the *three* be the Number 1, the *Work* is *perform'd* by *Diviſion* only.

N. B. In whatever *Queſtion* in *this* RULE there happens a *Remainder* after *Diviſion*, ſuch *Remainder* is *Part* of the *firſt* DIVIDEND, and is ſtill to be *divided* by the *former* DIVISOR, *i. e. ſuch* REMAINDER is the NUMERATOR of a *vulgar* FRACTION, to which the DIVISOR is the DENOMINATOR: Which FRACTION is *Part* of ONE ſuch INTEGER as the QUOTIENT *found* conſiſts of.

This is plain, with a little Conſideration; but, that the *Learner* may want no *Help*, let him obſerve well what follows under the following *Example*.

QUESTION III.

Suppoſe 15 Pounds of *Cochineal* coſt 24 Pounds, what will 82 Pounds coſt at that Rate?

P.	*l.*	P.
15	— 24 —	82

328
164

15)1968(131 *l.*
46
18
3

The

The *Anſwer* is as you ſee 131 *l.* and $\frac{3}{15}$ of a Pound.

Here you ſee a *Pound*, or 20 Shillings, is divided, or ſuppos'd to be ſo, into 15 equal Parts, and this FRACTION, $\frac{3}{15}$, ſignifies 3 of thoſe Parts, which, it's plain, are leſs in Value than a *Pound*, but *how much* leſs, or what the real *Value* of theſe 3 Parts is in *Money*, the *Learner* does not ſo eaſily ſee: For the *preſent* then, he may obſerve what follows;

The POUND, *by Cuſtom*, is divided into 20 equal Parts, and each of thoſe Parts we call a *Shilling*: Now, what we have to do, is, to find how many Parts of 20 (*i. e.* how many *Shillings*) anſwer to 3 Parts of 15, which is done by ſaying thus,

15 : 3 :: 20

3

15)60(4 Parts or Shill.

By this *Work* we find that 4 Shillings is = $\frac{3}{15}$ of a Pound.

And by the *ſame* Work we ſee, that if the former *Remainder* 3 be × 20, and that *Product* be ÷ the old *Diviſor*, 15, the *Quotient* 4 (*i. e.* 4 Shillings) will be the *Value* of $\frac{3}{15}$ of a POUND in our *Engliſh Money*. Therefore,

In Caſes of this Kind, we may take the following for a RULE always to be depended on.

RULE.

Multiply the Remainder by the next INFERIOR *Denomination to* THAT *the* QUOTIENT *conſiſts of, and then divide*

divide ON, *by the old Diviſor, and the Quotient will give you the* VALUE *of ſuch* REMAINDER.

Take one *Example* more.

QUESTION IV.

If 40 Pieces of Broad Cloth coſt 750 *l.* 15 *s.* what, at that Rate, will one Piece coſt?

Pieces		*l.*	*s.*	Piece
40	———	750 :	15	——— 1

Bring the ſecond Number (*viz.* 750 *l.* 15 *s.*) into the loweſt Denomination mention'd (*viz. Shillings*) and then it will ſtand thus,

P.	*s.*		P.	*s.*	
40)	15015	::	1	(375	and $\frac{15}{40}$ of a Shill.
	3			*l.* *s.*	
	2			or 18 — 15	$\frac{15}{40}$ of a Shill.
	—				
	15				

The *Value* of which FRACTION ($\frac{15}{40}$) in *Money*, may be found by the foregoing *Directions*, thus,

40 : 15 :: 12 :
12

40)180(4 $\frac{2}{4}$ = $\frac{1}{2}$
2

So that the *Price* of *one* Piece of Broad Cloth, at the foregoing *Rate*, is found to be *l.* 18 15 4 $\frac{1}{2}$.

QUE-

QUESTION V.

If 8 Pounds of Tobacco coſt 14*s.* what will half a hundred weight (*viz.* 56 Pounds) coſt at the ſame Rate?

Anſwer. 4*l.* 18*s.* For,

P.	*s.*		*P.*	*l.*	*s.*
8	14	::	56	4 —	18

QUESTION VI.

If 14 Shillings will buy 8 Pounds of Tobacco, how much will 4*l.* 18*s.* buy after the ſame Rate?

Anſwer. 56 Pounds. For,

s.	*P.*		*l.*	*s.*	*P.*
14	8	::	4 —	18	56

QUESTION VII.

If half a hundred Weight of Tobacco be worth 4*l.* 18*s.* how much may be bought for 14*s.* at that Rate?

Anſwer. 8*l.* For,

l.	*s.*	*P.*		*s.*	*P.*
4 —	18	56	::	14	8

QUESTION VIII.

Suppoſe 4*l.* 18*s.* will buy 56 Pounds of Tobacco, what will 8 Pounds of the ſame Tobacco coſt?

Anſwer. 14*s.* For,

P.	*l.*	*s.*		*P.*	*s.*
56	: 4 —	18	::	8	: 14

See more *Examples* in the APPENDIX.

I ſhall now ſay a Word or two concerning the Proof of this *Rule* of *Three*, i. e. *Shew* how we may, if any *Doubt* ſhould ariſe, be aſſur'd whether the *Operation* be true or no; and this may be done *ſeveral Ways*.

Some order it *thus*,

They *multiply* the *firſt* Term into the *fourth*, and the *ſecond* into the *third*, and if *each Multiplication* produces the *ſame Product*, the Thing is *rightly* done.

This is the Method commonly us'd, and 'tis a juſt one.

Thus, for Example,

Suppoſe the *Proportion* be 3 : 5 :: 9 : a fourth Number ſought, which is found, and which we think is truly found by the foregoing Directions = 15.

In order, now, to *prove* whether this Number 15 be *truly* the *fourth* Number which was *ſought*, *multiply* the *firſt* Number by the *laſt*, *i. e.* 3 by 15, it makes 45; and the two *middle* ones *multiply'd* together, *i. e.* 5 and 9, make *alſo* 45; therefore the *Numbers* 3, 5, 9, and 15

15 are *Proportionals*, and the *Work* was *rightly* done; for it has been already *prov'd*, That,

Of four directly proportional Terms, the PRODUCT *of the* MEANS *is* = *the* PRODUCT *of the* EXTREMES.

Therefore we may ſurely conclude, that *theſe four Terms, the* PRODUCT *of whoſe* MEANS *is* = *that of its* EXTREMES, *are directly proportional*: Therefore, as was juſt now ſaid, the *Numbers* 3, 5, 9, 15, are certainly *directly Proportionals*, and therefore the Number 15 *truly anſwers* the *Queſtion* propos'd.

Secondly,

The PROOF of theſe *Operations* may be order'd thus,

If the *fourth* Term found be made now the *firſt* Term, and the *Operation* be *inverted*, *viz.* ſo as it may be,

As *that* which *was* the *fourth* to the *third*, ſo *that* which *was* the *ſecond* to the thing *ſought*.

If now, after having work'd this *Proportion* truly, *that* Number be *produc'd*, which in the *former Proportion* was the *firſt* Term, the *former* OPERATION is *rightly* perform'd; if *otherwiſe*, there is a *Fault* committed *ſomewhere*.

See the WORK.

15 : 9 :: 5 : Q.

5

15)45(3

0

By which it's plain the *former* Work is *true*.

But, laſtly,

Whether the *Number* 15 be the *fourth* Number, *anſwering* in *Proportion* to the *three* given ones 3, 5, and 9, I ſhould *rather* prove thus, *viz.* By ſaying,

$$5 : 3 :: 15 : Q. = 9.$$ Or thus,

By ſaying,

$$9 : 3 :: 15 : Q. = 5.$$ For,

By *this* Method the *Numbers* 5 and 9, which *before* were MULTIPLIERS, become *now* DIVISORS.

The *Truth* of this kind of *Operations* may, indeed, on Occaſion, be *prov'd* ſeveral *other* Ways; but what I have already ſaid, is, may-be, more than ſufficient concerning theſe PROOFS.

Before I *end* this *Chapter*, I ſhall ſhew the *Learner* ſomething concerning the *Compariſon* of *Hours* under *different* MERIDIANS; then give him ſome *Caution* in relation *to Quantities* NOT *Proportional*.

Firſt then,

What o' Clock is it at *NAPLES*, when 'tis 8 at *LONDON*?

Now, to anſwer QUESTIONS of *this Kind*, we muſt *fore-know* from GEOGRAPHY, that *NAPLES* is ſituate 15 Degrees (a *little more* or *leſs*) *Eaſtward* from *LONDON*, and therefore the SUN comes to the *Meridian* of *NAPLES*, *ſooner* than it comes to *that* of *LONDON*; but the *Queſtion* is, *How much* ſooner?

To

To *anſwer* this, it muſt alſo be known, that the SUN paſſes over his *whole* diurnal Circle (ſpeaking after the common Manner) in (about) 24 Hours, *i. e.* in *this* time is return'd to the ſame MERIDIAN from which it ſet out; and further it muſt be known, that there are 360 Degrees in the *whole* Circumference of a Circle; Now *theſe* Things being *premis'd*, the ANALOGY or PROPORTION will be as follows, *viz.*

If the SUN in 24 Hours paſſes over 360 Degrees, how many Hours is he paſſing over 15 Degrees?

And, ſince the SUN moves with the *ſame equable* CELERITY, and therefore paſſes over *Spaces* DIRECTLY *proportional* to the *Times*, 'twill be as 360 Degrees to 15 Degrees; ſo 24 Hours to the Hours ſought, *viz.* 1 Hour, as is manifeſt from the adjoin'd *Operation*.

Deg. *Ho.* *Deg.* *Ho.*
360 : 24 :: 15 : Q. = 1

15

120
24

360)360(1 = Q.

Deg. *Deg.* *Ho.* *Ho.*
360 : 15 :: 24 : Q. = 1.

15

120
24

360)360(1 = Q.

Therefore the SUN comes to the *Meridian* of *NAPLES* an *Hour* ſooner than to *ours* at *LONDON*; and therefore *their* NOON is *one Hour* ſooner than *ours*; for *there* it is NOON, or 12 o' *Clock*, when at *LONDON* 'tis but 11; and therefore to anſwer the Queſtion put, 'tis 9 o' *Clock* at *NAPLES*, when 'tis 8 at *LONDON*. Again,

Suppoſe it be asked, *What time of the Day* it is at *ACAPULCO*, in *MEXICO*, when 'tis NOON at *LONDON*?

Firſt

First we must know, that *ACAPULCO* lies about 106 Degrees to the *Westward* of *LONDON* (a *little more* or *less*) and therefore the SUN comes to the *Meridian* of *ACAPULCO*, *later* than it comes to *that* of *LONDON*; but 'tis here sought HOW MUCH *later*? To find *which*, the *Analogy* or *Proportion* will be as follows,

As 360 Degrees, the SUN's *whole diurnal Course*, to 106 Degrees, the Difference of *Longitude* between the Places; so is 24, the Hours in the *whole* Day, to the Time *sought*, = 7 Hours 4 Minutes.

See the WORK.

Deg.		*Deg.*		*Hou.*		*Ho.*		*Ho.*	*Min.*
360	:	106	: :	24	: Q. =	$7\frac{1}{15}$	=	7	4
		24							
		424							
		212							
								H.	*M.*
36|0)		254|4	(	$7\frac{24}{360}$	=	$7\frac{1}{15}$	=	7	4
		252							
		24							

Therefore the SUN comes to the *Meridian* of *ACAPULCO* abot 7 Hours 4 Minutes *later* than to that of *LONDON*; and therefore *their* NOON is 7 Hours 4 Minutes *later* than *ours*; and therefore 'tis but 56 Minutes past 4 in the *Morning* at *ACAPULCO*, when 'tis NOON at *LONDON*.

But to go on,

The

The *Learner* muſt take great Care, leſt he be *deceiv'd*, and take thoſe Things *to be* PROPORTIONAL which are *not ſo*.

For Example's Sake,

Suppoſe a *Weight*, moving by its own GRAVITY, deſcends 20 Feet in 2 Seconds of Time, and 'tis requir'd to find *how many* Feet it will fall in 10 Seconds?

Here, now, if the *third* Number be × the *ſecond*, or the *ſecond* by the *third*, and that *Product* be ÷ the *firſt*, the *fourth* will be 100; but this Number does *by no means anſwer* the *Queſtion*.

See the WORK.

Sec. *F.* *Sec.*
2 : 20 : : 10
10

2)200(100 = Q.

For altho' it be true, that this *Weight* will fall 100 Feet in 10 Seconds, if it was to move with the *ſame* Velocity with which it deſcended 20 Feet in 2 Seconds (becauſe the Number 100 is a *fourth Proportional* to the *three given* ones, 2, 10 and 20) yet this *can't* be a *true* Anſwer, becauſe

A *falling Weight* does *not* proceed with an *equable* Motion, but with an *accelerated* one; its *Motion* being *ſo much* the *ſwifter*, by *how much* the *longer* it is in *Motion*. Again,

If a Veſſel that will hold (ſuppoſe 12 Gallons) be fill'd with Water, and a Hole at the Bottom be made, and

and thro' it there runs out 3 Gallons in 2 Minutes of time, and 'tis demanded *in what time* ALL the Water will run out?

Here again, the *Learner* muſt take great *Care*, leſt he be *deceiv'd*; for, if the *Queſtion* be *thus* ſtated, *viz.*

If 3 Gallons run out in 2 Minutes, in *how many* Minutes will 12 Gallons run out?

It will be *anſwer'd*, according to the *Tenour* of the *foregoing* RULE, that ALL the Water will run out in 8 Minutes: For,

Gall. Min. Gall. Min.
3 : 2 :: 12 : 8

And, indeed, *this* would be *true*, ſhould the Water *continue* to run out with the *ſame* VELOCITY with *which* it *began* to run; but it is quite *otherwiſe*; for,

The CELERITY of the Water *running out* continually *decreaſes*, and by *how much* the *leſs* remains of Water *to run out*, by *ſo much* the *ſlower it runs out*, i. e. *It runs out* with a *greater* VELOCITY or SWIFTNESS when 'tis *near* FULL (for Inſtance) than it does when it is but *half* FULL, becauſe the *inferior* Water *running out*, is in the *former Caſe* preſs'd by a *greater* Weight of *ſuperior* Water, than it is in the *latter*.

But now,

By *what* DEGREES, *either the* VELOCITY of a FALLING *Weight* INCREASES, or the VELOCITY of *Water*, thus *running* out, DECREASES, is not ſo *much* my Buſineſs HERE to enquire, as it is to endeavour to make the *Learner throughly* underſtand, that this RULE of three DIRECT, under *Explanation*, is not employ'd but with *Quantities truly* PROPORTIONAL. FOR,

By

By *it* a *fourth* Quantity is *ſought*, which has the *ſame* RATIO to a *third* given, as the *ſecond* has to the *firſt*; or to *which* a *third* given has the *ſame* RATIO as the *firſt* has to the *ſecond*. But now,

Tho' an ENQUIRY, by *what Degrees* the *Velocity* of a *falling* WEIGHT *increaſes*, and *that* of WATER *running out decreaſes*, may ſeem foreign to the main Deſign of this *Tract*, yet having propos'd Examples of this Kind, the *Learner* will excuſe me if I go a little out of my Way, and juſt mention the following *Helps*, which he may take to his Aſſiſtance on ſuch Occaſions: Firſt then,

GALILÆUS (in *Prop.* 2. concerning MOTION *naturally accelerated*) has prov'd, That *the Spaces deſcrib'd, and the Diſtances run thro' by the falling Weight, will always be as the Squares of the Times ſpent in falling*. Wherefore,

If a heavy Body, deſcending from its Place of Reſt, fall any Length in a given Time, in *twice* that Time it will deſcribe *four* times that *Length*; in *thrice* that Time, *nine* times that *Length*, *&c.* Whence the foregoing *Queſtion* may be eaſily ſolv'd, and the *true Anſwer* found to be 500 Feet: Thus,

As 4, the *Square* of the *firſt* time, to 100, the *Square* of the *latter* time, ſo 20 Feet, the *Deſcent* in the *firſt* time, to the *Deſcent* in Feet in the *latter* time, = 500. For,

4 : 100 :: 20 : 500. Again,

If a *Veſſel* that will hold (ſuppoſe 12 *Gallons*) be fill'd with Water, and a *Hole* at the *Bottom* of the Veſſel be made, thro' which there runs out 3 *Gallons* in 2 *Minutes* of time, and 'tis demanded *in what time* ALL the Water will run out?

Here again, the young *Arithmetician* muſt take great Care, leſt *he* be deceiv'd: For,

If the *Queſtion* be *thus* ſtated, *viz.*

If 3 *Gallons* run out in 2 *Minutes*, in *how many* Minutes will 12 *Gallons* run out?

It will be *anſwer'd*, according to the *Tenour* of the *foregoing common* RULE of *Proportion*, that ALL the Water will run out in 8 *Minutes*: For,

Gall. Min. *Gall. Min.*
3 : 2 :: 12 : 8

And, indeed, this would be *true*, ſhould the Water *continue* to run out with the *ſame Velocity* with *which* it *began* to run; but it is quite *otherwiſe*; for the *Celerity* of the Water running out, continually *decreaſes*; and by *how much* the LESS remains of Water *to run out*, by *ſo much* the SLOWER it *runs out*; *i. e.* it runs out with a *greater Velocity* or *Swiftneſs* when 'tis *near* FULL (for Inſtance) than it does when 'tis but *half* FULL; becauſe the *inferior* Water running out, is, in the *former* Caſe, preſs'd by a *greater* Weight of *ſuperior* Water than it is in the *latter*.

TORRICELLIUS has demonſtrated in a little *Treatiſe* concerning the *Motion of Water*, That,

If there be a *Veſſel* (for Inſtance, a *Parallelipiped*) full of Water, in the *Bottom* whereof is made a *Hole*, thro' *which* the WHOLE Quantity of Water will run in exactly 10 *Minutes*: He has, I ſay, *prov'd*, that this Water ſhall *deſcend* in the *Veſſel* thro' UNEQUAL *Spaces*, in EQUAL *Times*, in ſuch Sort, that if the Height of the Veſſel be divided into 100 Parts, the Water running out will ſink 19 of theſe Parts in the *firſt* Minute, 17 of them in the *ſecond*, 15 in the *third*, 13 in the *fourth*, 11 in the *fifth*, 9 in the *ſixth*, 7 in the *ſeventh*, 5 in the *eighth*, 3 in the *ninth*, and 1 in the *tenth*, or *laſt* Minute.

At

At *this Rate* we ſee there runs *nineteen* times as much Water out in the *firſt* of the *ten* Minutes, as runs out in the *laſt*.

Now here 'tis plain, that the *forementioned* Numbers 19, 17, 15, 13, 11, 9, 7, 5, 3, and 1, are the DIFFERENCES of the SQUARES of the *Times*; for the WHOLE *given time*, in this EXAMPLE, is TEN *Minutes*, the SQUARE whereof is 100; and the SQUARE of 9 is 81; The DIFFERENCE of *which* SQUARES is 19, the *Number* of *Parts* ſunk in the *firſt* Minute. Again,

The SQUARE of 8 is 64, the DIFFERENCE between *which*, and the *laſt* SQUARE 81, is 17, the *Number* of *Parts* ſunk in the *ſecond* Minute: And ſo on, as in the following

TABLE.

Parts *ſunk.*	Times *Min.*	Times *ſquar'd.*	Differ. of Squar. of *Times.*
0	0	0	
19	1	1	1
17	2	4	3
15	3	9	5
13	4	16	7
11	5	25	9
9	6	36	11
7	7	49	13
5	8	64	15
3	9	81	17
1	10	100	19

By which TABLE we may eaſily gain the following Help, to *ſolve* the *foregoing*, and *ſuch-like* QUESTIONS: Thus,

Since the VESSEL, or WHOLE *Quantity*, in the EXPERIMENT *above-mention'd*, is *divided* into 100 EQUAL *Parts:*

Suppoſe always the *given* WHOLE (be it what it will) to be *divided* into 100 EQUAL *Parts* ALSO: And then find WHAT *Part* of 100 THAT *given* Quantity is, that *runs out* in a *given* Space of *Time.*

In the *foregoing* EXAMPLE the *given* WHOLE is 12 *Gallons*; the *given Quantity*, that runs out in a *given* Space of *Time*, *viz.* 2 *Minutes*, is 3 *Gallons*: Wherefore, by the common *Rule* of *Proportion*,

Whole Gall. : *Whole Parts.* :: *Gall.*

12 : 100 :: 3, the *Given* Part of

12 (in 2 Minutes) to 25, the like Part of 100 in the *ſame* Time (ſuppoſe).

Thus, according to the *Velocity* of the Water's running out, in this (ſuppos'd) *Example*, the Water ſinks 25 of the 100 *equal* Parts in 2 *Minutes*; But,

In the EXPERIMENT *above*, the Water was found to ſink 36 (= 19 + 17) of the 100 *equal* Parts, in the *ſame* Space of Time (*viz.* 2 *Minutes*) whence its manifeſt, that the Water runs out (in this *ſuppos'd* Example) with *leſs* Velocity than it did in the EXPERIMENT (for if the VELOCITIES were *equal*, the PARTS ſunk in *both*, in the *ſame* Space of Time, would have been *equal*).

Therefore,

Therefore, the WHOLE 100 *Parts* (12 Gallons) in this (ſuppos'd) *Example*, will be *longer* running out, than the WHOLE 100 *Parts* in the EXPERIMENT were.

On the contrary, *thoſe* would run out in *leſs* time than *theſe*, if the Water had been found to run with *greater* Velocity in this (ſuppos'd) *Example*, than it did in the EXPERIMENT. But,

Be *that* found as it will (*viz.* be the VELOCITIES *equal* or *unequal*) the foregoing EXPERIMENT may ſerve as a *ſtanding* TABLE, or SCALE (or *whatever* ELSE you pleaſe to call it) by the Help of which (after the *Preparation* above) the *foregoing*, and *ſuch-like* QUESTIONS, may be *ſolv'd*, by ſaying *thus*,

As 25, the *Parts* of 100, run out in 2 *Minutes* (according to the ſuppos'd *Example*) to 36, the *Parts* of 100 (anſwering, by the EXPERIMENT, to the *ſame* Space of Time) ſo is 100 (the *whole* Quantity, according to the ſuppos'd *Example*, run out in a *certain*, tho' *unknown*, Space of *Time*) to 144, the *Quantity* or *Number* of Parts ſunk, according to the EXPERIMENT, in the *ſame* Space of Time: Which Quantity (144) anſwers, by the foregoing TABLE, if produc'd, to 12 *Minutes* of Time, = the *Time* SOUGHT. For,

Tho' in the TABLE, the largeſt Number of *Parts* be 100 (= the *Square* of the whole Time there mention'd, *viz.* 10 Minutes) yet the *Time* anſwering to theſe 144 *Parts*, may be eaſily *found*; for this TABLE may be carried on, for *Uſe*, as far as you pleaſe, by only carrying on the *equal* TIMES as far as you pleaſe; thus,

By adding to the 10 *equal* Spaces of *Time* mention'd in the EXPERIMENT, *two* more *equal* Spaces, *viz.* 11 and 12, and *ſquaring* them, in order to get the DIFFERENCES of their *Squares*, *viz.* 23 the DIFFERENCE between

tween the *Squares* of 12 and 11; and 21 the DIFFERENCE of the *Squares* of 11 and 10: *Which* DIFFERENCES (23 and 21) added to the *foregoing* SERIES of DIFFERENCES 19, 17, 15, 13, *&c.* make 144, exactly *anſwering* to 12 *Minutes* of Time. Thus then,

If 3 *Gallons* run out in 2 *Minutes*, 12 *Gallons* will be found to run out in 12 *Minutes*, and NOT in 8 *Minutes*. Again,

Since *theſe* DIFFERENCES of the *Squares* are a SERIES of Numbers in ARITHMETICAL PROGRESSION; and ſince the SUM of this SERIES is always = the WHOLE Quantity of Water, it may be eaſily deduc'd from *known* PROPERTIES of ARITHMETICAL *Progreſſionals*, that the ROOT of this SUM, whether *under* or *above* 100, is always = the TIME *ſought*, thus,

In the EXAMPLE above, the SUM is found to be 144, whoſe ROOT is 12, as before. And,

If the SUM be *under* 100, the TIME is found, by the foregoing TABLE, by bare *Inſpection*.

Let this be ſufficient here concerning Quantities NOT *proportional*.

More of *this Nature* you'll find in the APPENDIX.

I now proceed to *explain* the *Single Rule* of *Three Inverſe*.

CHAP.

CHAP. XI.

THE RECIPROCAL *Golden Rule*, or the *Single Rule of Three* INVERSE, is when from *three* Quantities *given*, a *fourth* RECIPROCALLY *proportional* is *ſought*.

Now, we ſay *Quantities* are RECIPROCALLY *proportional*, when, *of the compar'd* RATIO's (*viz.* of the *firſt* to the *third*, and of the *ſecond* to the *fourth*) *the one is the* INVERSE *of the other*.

As $\frac{2}{4}$ $\frac{2}{1}$, or, $\frac{2}{3}$ $\frac{3}{2}$. *Thus*, for Example,

4 : 18 :: 6 : 12 will be RECIPROCALLY *proportional*, becauſe $\frac{4}{6}$ are $= \frac{2}{3}$, and $\frac{18}{12} = \frac{3}{2}$. And the like in others.

Now THIS *Rule* differs from the DIRECT *Rule*, which I have already explain'd, *thus*,

As, *there*, Regard was had to the *Equality* of the RATIO's or QUOTIENTS; ſo, *here*, we regard the *Equality* of the RECTANGLES or PRODUCTS, *viz.* *There*,

As the *firſt Term* to the *ſecond*, ſo the *third* to the *fourth* ſought; and thus the *ſame* QUOTIENT comes forth from *dividing* the *firſt* by the *ſecond*, as there does from *dividing* the *third* by the *fourth*: but, *here*, the RECTANGLE or PRODUCT of the *firſt*, *multiply'd by* the *ſecond*, is *equal to* the RECTANGLE or PRODUCT of the *third multiply'd by* the *fourth* ſought.

For

For Example's Sake,

If a *Field* 40 Rods *long*, and 4 *broad*, contain an ACRE of Land; *how broad* muſt *that* Field be *which* is but 20 Rods *long*, to contain *alſo* an ACRE of Land?

Here 'tis manifeſt, ſince in *both*, the *ſame* Quantity of Land is conſider'd, that the GREATER the *Length* of the Field is, the LESSER its *Breadth* muſt be; and the contrary: That the EXCESS of the *one*, may make good the DEFECT of the *other*: And therefore,

As the *Length* of the *former* to its *Breadth*, ſo the *Length* of the *latter* (not DIRECTLY, for ſo would be produc'd a SIMILAR *Figure*, NOT an EQUAL *one*, but) RECIPROCALLY to its *Breadth*. Or,

As the *Length* of the *ſecond*, to the *Length* of the *firſt*, ſo the *Breadth* of the *firſt*, to the *Breadth* of the *ſecond*: For,

This is to be RECIPROCALLY *proportional*:

If therefore the *Length* of the *former* be called A, and its *Breadth* B, the *Length* of the *ſecond* α, and its *Breadth* β, it will be,

$$A : \alpha :: \beta : B. \text{ Or, } \beta : B :: A : \alpha.$$

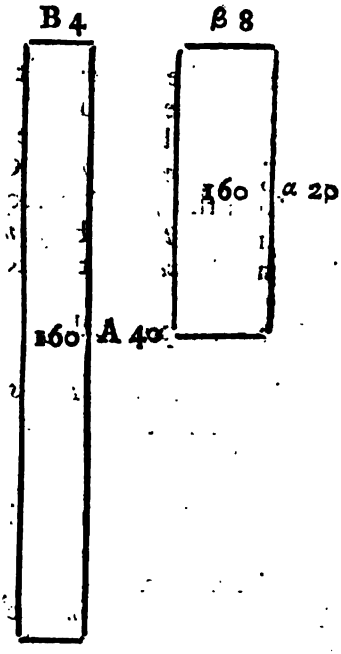
That $A \times B$, may be $= \alpha \times \beta$.

The *Practice* of *this Rule* then is *this*,

If of the *three* Terms, the *Product* of the *firſt* into the *ſecond* be *divided by* the *third*; the *fourth* RECIPROCALLY *proportional* will be *gain'd*. And thus,

In

In the *Example* given, ſince A, the *Length* of the *former* Field × its *Breadth* B, makes the *Rectangle* A B, *dividing* by α, the *Length* of the *ſecond*, there will be produc'd its *Breadth* ſought $= \frac{A B}{\alpha}$:



Or in Numbers,

Since the *Length* of the *former* 40, × its *Breadth*, 4, is 160 = 40 × 4: *Dividing* by 20, the given *Length* of the *ſecond*, the *Breadth* of *this* will be found 8 = $\frac{160}{20}$.

This PRAXIS then *differs* from *that* of the *foregoing* DIRECT *Rule*, in THIS, *viz.*

THERE the *firſt* Term was the *Diviſor*; HERE the *third*.

The DEMONSTRATION of *this* RULE is *this*:

Becauſe from the HYPOTHESIS, the RECTANGLE of the *firſt* into the *ſecond*, is = the RECTANGLE of the *third* into the *fourth*, i. e. A × B = $\alpha \times \beta$, or A B = $\alpha\beta$; it will be, by *dividing on both Sides* by α, $\frac{A B}{\alpha} = \beta$.

But now the young *Arithmetician* may *diſtinguiſh* the *Queſtions* to be ſolv'd by THIS Rule, from *thoſe* to be ſolv'd by the *preceding* DIRECT one, *thus*,

If from the *Nature* of the *Queſtion*, by *how much* the GREATER the *firſt* Term is, *ſo much* the GREATER the *fourth* MUST be; it is the *Buſineſs* of the DIRECT *Rule*: but if, by *how much* the GREATER the *third* is, *ſo much* the LESS the *fourth* is requir'd to be; or by *how much* the LESS the *third*, by *ſo much* the GREATER the *fourth*; it is the *Buſineſs* of the INVERSE *Rule*.

For Example's Sake,

If any one in 12 *Hours* travels 20 *Miles*, HOW MANY *Miles* will he travel in 6 *Hours* at the *ſame Rate* of travelling?

Becauſe here it is manifeſt, that in

The { LONGER / SHORTER } *time* he'll travel the { MORE / FEWER } *Miles*;

By *how much* the { GREATER / LESS } the *third* Term is (the *Number* of *Hours*) by *ſo much* the { GREATER / LESS } will *alſo* the *fourth* be (the *Number* of *Miles* ſought) and therefore the *fourth* Term ſought, 10, is found by the DIRECT *Golden Rule*, thus,

H. *M.* *H.*
12 : 20 :: 6 : Q = 10.
6

12)120(10 = Q.

But on the *other Hand*,

If

If a *Work* be perform'd by 12 *Men* in 20 *Days*, in How MANY *Days* will 6 *Men* perform the SAME *Work*, at the *ſame Rate* of working?

Here now,

Becauſe it's manifeſt, from the *Nature* of the *Queſtion*, that FEWER *Men* CAN'T do the *ſame Work*, at the *ſame Rate* of working, under MORE *Days*;

And therefore,

By *how much* the LESS the *third* Term is (the *Number* of *Men*); *ſo much* the GREATER the *fourth* Term MUST be (*viz.* the *Number* of *Days* requiſite): Therefore the *fourth* Term, 40, will be found by the INVERSE *Rule*, thus,

M. *D.* *M.*
12 : 20 :: 6 : Q. = 40
12

6)240(40 = Q.

After the *ſame* Manner,

If a Piece of *Tapeſtry* 9 *Feet* WIDE, and 20 LONG, be to be *lin'd* with a courſe Thing 3 *Feet* WIDE, *how many Feet* LONG ought *this* to be to do the Buſineſs?

Here,

Becauſe its manifeſt, that by *how much* the LESS this *Lining* has of BREADTH, *ſo much* the MORE it *muſt* have in LENGTH, to *make up* its *Want* of BREADTH; The LENGTH *ſought* will be found by the INVERSE *Rule* to be 60 *Feet*: thus,

Feet wide. *Feet long.* *Feet wide.*
9 : 20 :: 3 : Q. = 60
9

3)180(60 = Q.

Now the *Abbreviation* of the *Work* given in the DIRECT *Rule* (whether as to the ORDER of performing the *Operations*, or to the *precedaneous* REDUCTION of the *Terms*) take Place *here* alſo, MUTATIS MUTANDIS. But *theſe* I ſhall not *repeat here*, but rather leave them to the Sagacity of the *Learner* to apply them. Again,

What was *there* ſaid concerning the PROOFS of THAT *Rule*, the *Learner* may, if he think it worth his while, without any Difficulty, MUTATIS MUTANDIS, apply *here*.

Laſtly,

The *Learner* may obſerve, that a RECIPROCAL *Proportion* may be *turn'd* to a DIRECT *one*, if *that* Term which is the *third* in the RECIPROCAL *one*, be made the *firſt* in a DIRECT *one*; thus,

If in a RECIPROCAL Proportion it be

9 : 20 :: 3 : Q. = 60

It will be in a DIRECT *one*,

9 : 20 :: 9 : Q. = 60

9 : 20 :: 3 : Q. = 60
9

3)180(60 = Q.

3 : 20 :: 9 : Q. = 60
9

3)180(60 = Q.

I ſhall

I ſhall now inſert a few *Queſtions* to exerciſe the Learner in *this Rule*, and then proceed to the COMPOUND *Golden Rule*.

QUESTIONS.

I. *How much* Silk of *half Yard* WIDE, will *line* a Coat that has *two Yards* $\frac{3}{4}$ of Cloth *ſeven Quarters* WIDE?

Anſwer. 9 *Yards* 3 *Quarters* $\frac{1}{2}$.

II. If I lend my Friend 250 *Pounds* for 5 *Months*, *how much* muſt he lend me for 9 *Months*, to repay my Kindneſs?

Anſwer. *l.* 138 $\frac{8}{9}$, or, 138*l.* 17*s.* 9*d.* 1*f.* $\frac{1}{9}$.

III. If 100 Workmen in 12 Days finiſh a Piece of Work, *how many* Workmen are ſufficient to do the *ſame* in 3 *Days*, at the *ſame Rate* of working?

Anſwer. 400 Workmen.

IV. *How many* Yards of *Matting*, that is 2 Feet *wide*, will cover a Floor that is 24 Feet *long*, and 20 Feet *broad*?

Anſwer. 240 Feet.

See more *Queſtions* in the APPENDIX.

CHAP.

CHAP. XII.

THAT which is call'd, *The* COMPOUND *Golden Rule* (or by *ſome*, *The Double Rule of Three*) conſiſts of TWO or MORE *Analogies*, either *Direct* or *Reciprocal*; and therefore requires the *Operation* of the SINGLE *Rule* to be *repeated* (at leaſt VIRTUALLY) but the PROCESS or WORK of the COMPOUND *Rule*, is built upon the *ſame* PRINCIPLES as THAT of the SINGLE *Rule* is.

So that without any *farther* PREPARATION, I ſhall, at once, proceed to *ſhew* the *Method* of *working*, by ſome EXAMPLES.

EXAMPLE.

If the *Expences* of 4 *Scholars* in 3 *Months* be 20 *l.* How MANY *Pounds* will ſerve 6 *Scholars*, at this *Rate*, 12 *Months*, or a *Year*?

Scho.	*l.*	*Scho.*
4	: 20 : :	6 :
Mon.		*Mon.*
3	:	12 : Q.

'Tis manifeſt, that in this *Queſtion* there are found 2 *Analogies*; for *both* PERSONS are *compar'd* to PERSONS, and TIME to TIME.

This

This COMPOUND *Analogy* therefore being *reſolv'd* into *two* SIMPLE *ones*, will ſtand *thus*,

If 4 *Scholars* expend 20 *l.* How MANY *Pounds* will ſerve 6 *Scholars* the *ſame* Time?

Scho.	*l.*	*Scho.*	*l.*
4 :	20 : :	6 : q.	= 30

6

l.

4)120(30 = Q.

Thus, ſetting *aſide* a little the Conſideration of the *different* TIMES in the *Queſtion* (and this *Operation* over) it will appear, that 30 *l.* will be expended by 6 *Scholars*, in the *ſame* Time wherein 4 *Scholars* ſpent 20 *l. i. e.* from the *Hypotheſis*, in 3 *Months*.

Again, (*Reſuming* the Conſideration of the TIME)

If the *Expences* of *theſe* 6 *Scholars* (as was juſt now found) in 3 *Months*, be 30 *l.* How MANY *Pounds* will they *expend* in 12 *Months*?

(Where the *fourth* Term, found by the *former Analogy*, becomes the *ſecond* in *this ſecond Analogy*, that at Length the *Term ſought* may be *gain'd*)

Mon.	*l.*	*Mon.*	*l.*
3 :	30 : :	12 : Q.	= 120

30

l.

3)360(120 = Q.

Thus, the *Work* being *perform'd*, Q. will be found = 120, the *principal Number ſought*.

Thus

Thus the *propos'd* COMPOUND *Analogy* is *reſolv'd* into *theſe* TWO SIMPLE *ones*:

Compound Analogy.	Simple Analogies.
Scho. l. Scho.	*Scho. l. Scho. l.*
4 : 20 :: 6	4 : 20 :: 6 : q
Mon. Mon.	*Mon. l. Mon. l.*
3 12 Q.	3 : q :: 12 : Q.

The *ſame* thing exactly would come forth, if, *firſt*, the TIMES were *compar'd*, and *then* the PERSONS.

Suppoſe, *firſt*,

If their (the 4 propos'd *Scholar's*) *Expences* in 3 *Months*, be 20 *l.* HOW MANY *Pounds* will the *ſame expend* in 12 *Months*, at the *ſame* Rate of *Expence*?

The *Anſwer* will be 80 *l.*

And then,

If (in 12 *Months*) the *Expences* of 4 *Scholars* (as is manifeſt from the *former Analogy*) *be* 80 *l.* HOW MANY *Pounds* (in the *ſame* time) *will* 6 *expend*?

The *Anſwer* will be, (as is manifeſt from the *Work*) 120 *l.*

Mo. l. Mo. l.	*Sch. l. Sch. l.*
3 : 20 :: 12 : 9 = 80	4 : 9 = 80 :: 6 : Q. = 120
12	6
3)240(80 = 9.	4)480(120 = Q.

viz.

viz. Reſolving the COMPOUND *Analogy (If in* 3 *Months* 4 *Men expend* 20 *Pounds, in* 12 *Months* HOW MANY *Pounds will* 6 *Men ſpend)* into *two* SIMPLE *ones,* ONE *comparing* the TIMES, the OTHER the PERSONS.

Compound Analogy.	Simple Analogies.
Mo. *l.* *Mo.*	*Mo.* *l.* *Mo.* *l.*
3 : 20 :: 12	3 : 20 :: 12 : q = 80
Scho. *Scho.*	*Scho.* *l.* *Scho.* *l.*
4 6 : Q.	4 : q = 80 :: 6 : Q. = 120

The REASON of the *whole* PROCESS is *plain* enough of *itſelf*, ſince it is *nothing but* the SINGLE *Operation* TWICE *perform'd*; nor does it want any NEW *Demonſtration.*

But now,

What has been *done* by TWO *Operations* of the SINGLE *Golden Rule*, may likewiſe be *perform'd* by ONE of the COMPOUND *one*: For,

If the *firſt* Terms of *both* SIMPLE *Analogies*, *multiply'd* together, be made the *firſt* Term of the COMPOUND *one*; And, in like manner, the *third* Terms of the SIMPLE *ones*, *multiply'd* together, be made the *third* of the COMPOUND *one*; And the *other* Term, *homologous* to *that ſought*, be made the *ſecond.*

Thus, in the *foregoing* EXAMPLE,

If 4 *Men, in* 3 *Months, ſpend* 20 *Pounds, how many Pounds will* 6 *Men ſpend in* 12 *Months?*

For the *firſt* Term of the COMPOUND *Analogy*, take $12 = 4 \times 3$ (*viz.* which is made by *multiplying* the *firſt* MEN, into the *Number* of the *firſt* MONTHS); For the *third*, take $72 = 6 \times 12$ (which is made by *multiplying* the *Number* of the *ſecond* MEN, into the *Number* of *their* MONTHS); And for the *ſecond* take 20, *viz.* the *Number* of POUNDS, *i. e.* The *Term* HOMOLOGOUS to *that ſought*:

Then the *Whole* PROCESS will ſtand *thus*,

$$\begin{array}{ccccc} 4 & : & 20 & : : & 6 \\ \times\ 3 & & & & \times\ 12 \\ \hline 12 & : & 20 & : : & 72 : Q. = 120 \\ & & & & 20 \\ & & & & \hline \end{array}$$

$$12)1440(120 = Q.$$

The REASON or DEMONSTRATION of this COMPOUND *Operation*, may be as follows;

In the foregoing Example

$$\text{Let} \left\{ \begin{matrix} \tau \\ T \\ \mu \\ M \\ P \end{matrix} \right\} \text{be} = \left\{ \begin{matrix} 3 \\ 12 \end{matrix} \right\} \text{Months.} \quad \left\{ \begin{matrix} 4 \\ 6 \end{matrix} \right\} \text{Men.} \quad 20 \text{ Pounds.}$$

Then, as before ſhewn, at the *Beginning* of this EXAMPLE,

It will be,

Firſt, $\mu : P :: M : q$

Secondly, $\tau : q :: T : Q.$

Then

Then $q = \frac{PM}{\mu}$.

$Q = \frac{Tq}{\tau}$.

Therefore, putting $\frac{PM}{\mu}$ (in the former of the two EQUATIONS) inſtead of q (in the latter) it will be,

$Q = \frac{TPM}{\mu\tau}$ which was the Thing to be *prov'd*.

But to *return*,

After the *ſame* Manner, as in the *foregoing* EXAMPLE, we muſt *manage*, if the *propos'd* ANALOGY be *yet* MORE *Compounded*: See the following

EXAMPLE.

Suppoſe the WEIGHT of a *Beam*, whoſe LENGTH is 4 *Feet*, BREADTH 3 *Feet*, HEIGHT 2 *Feet*, be 240 *Pounds*, WHAT WILL the WEIGHT of *another Beam* be, whoſe LENGTH is 10 *Feet*, BREADTH 4 *Feet*, HEIGHT 1 *Foot*?

Suppoſe ω *Pounds*.

Then I ſay, That $\frac{10 \times 4 \times 1 = 40}{4 \times 3 \times 2 = 24} \times 240$ are $= \omega = 400$, as is manifeſt by *reſolving* the COMPOUND *Analogy* into 3 SIMPLE *ones*, as follows,

4	: 240	::	10	4	: 240	::	10 : q = 600
3			4	3	: 600	::	4 : Q = 800
2			1	2	: 800	::	1 : ω = 400

For *firſt*, $q = \frac{240 \times 10}{4}$.

Secondly, $Q = \frac{240 \times 10 \times 4}{4 \times 3}$.

Then laſtly, $\omega = \frac{240 \times 10 \times 4 \times 1}{4 \times 3 \times 2}$, or, $\frac{240 \times 10 \times 4}{4 \times 3 \times 2}$.

Q. E. D.

But let theſe be ſufficient concerning the COMPOUND *Rule* of PROPORTION, when the SIMPLE *compoſing* ANALOGIES are DIRECT.

But it may happen, that ONE of the *compoſing* ANALOGIES may be DIRECT, and *the* OTHER RECIPROCAL.

For *Example*'s Sake,

If 36 POUNDS of *Bread* be ſufficient for 3 *Soldiers* for 6 *Days*, HOW MANY *Days* will 180 POUNDS ſuffice 9 *Soldiers* ?

Here now, is ONE *Proportion* DIRECT, *viz.*

If 36 *Pounds are ſufficient for* 6 *Days, how many Days will* 180 *Pounds ſuffice* ?

Suppoſe $q = 30$ *Days*.

And *the* OTHER RECIPROCAL, *viz.*

If for 3 SOLDIERS *the Bread be ſufficient for* 30 *Days*, $= q$, HOW MANY *Days will it ſerve* 9 SOLDIERS ?

Surely for FEWER *Days*, becauſe there are MORE *Soldiers*, *viz.* for 10 *Days*.

In

In the *former* ANALOGY, therefore, the DIRECT *Rule* muſt be uſed; in this *latter*, the RECIPROCAL. Thus,

$36 : 6 : 180 \qquad 36 : 6 :: 180 : q = \frac{6}{36} \times 180 = 30$

$3 \quad 9 : Q \qquad 3 : q :: 9 : Q = \frac{3}{5} \times q = \frac{3}{5} \times 30 = 10$

But if, inſtead of theſe *two* SIMPLE *Operations*, you would ſubſtitute *one* COMPOUND one, the *Product* of the *two* DIVISORS (as ſuppoſe of the *firſt* Term of the DIRECT *Proportion*, and the *third* of the RECIPROCAL *one*) will be the COMPOUND DIVISOR (and therefore, in a DIRECT *Analogy*, the *firſt* Term): And the *Product* of the MULTIPLIERS, the COMPOUND MULTIPLIER (and ſo to the *ſame Analogy* the *third* Term to be *multiply'd* into the *ſecond*) *i. e.* in the *Queſtion propos'd*,

$$36 \times 9 : 6 :: 180 \times 3 : Q = \frac{180 \times 3 \times 6}{36 \times 9} =$$

10, as before.

And, indeed, *how many* compoſing ANALOGIES *ſoever* there be, *whether* DIRECT or RECIPROCAL, the *following* RULE is UNIVERSAL.

RULE.

From all the DIVISORS (*viz.* the *firſt* Terms of the DIRECT *Analogies*, and the *third* of the RECIPROCAL) *continually* MULTIPLY'D, the DIVISOR is *made*; from *all* the REST *continually* MULTIPLY'D, the DIVIDEND is *gain'd*, and the QUOTIENT will *exhibit* the TERM *ſought*.

I ſhall

I ſhall now inſert a few *Queſtions* to *exerciſe* the *Learner* in this COMPOUND *Golden Rule*, and then go to the RULE of FELLOWSHIP.

QUESTIONS.

I. If the *Carriage* of 60 *Hundred Weight*, 20 *Miles*, coſt 14 Pounds 10 Shillings, *what will* 15 *Hundred Weight* coſt *carrying* 30 *Miles*, after *that Rate* of *Carriage?*

Anſwer. 5*l.* 8*s.* 9*d.*

II. If 48 *Shillings* be the *Wages* of 6 *Men* for 6 *Days*, *what* will the *Wages* of 48 *Men* amount to in 30 *Days*, at *that Rate?*

Anſwer. 96*l.*

III. If 18 *Rods* of *Ditching* be *done* by 3 *Men* in 6 *Days*, *how many* Rods ſhall be *wrought* by 8 *Men* in 4 *Days*, at *that Rate* of *Working?*

Anſwer. 32 *Rods.*

IV. If 10 *Cannon* in *one* Day ſpend 40 *Barrels* of *Powder*, *how many Barrels* will 24 *Cannon* ſpend in 30 *Days*, at *that Rate?*

Anſwer. 2880 *Barrels.*

V. If 36 *Buſhels* of *Provender* ſerve 12 *Horſes* 20 *Days*, *how long* will 15 *Buſhels* ſerve 24 *Horſes* at *that Rate?*

Anſwer. 5 *Days.*

VI. If

VI. If 4 *Barrels* of *Beer* ſerve 42 *Perſons* 16 *Days*, How MANY *Days* will 28 *Barrels* ſerve 36 *Perſons* at the *ſame Rate* of *Drinking*?

Anſwer. 130 *Days.*

See more *Queſtions* in the APPENDIX.

Thus much concerning the *Golden* COMPOUND *Rule.*

Now follows the RULE call'd FELLOWSHIP.

CHAP.

CHAP. XIII.

SINCE the RULE of FELLOWSHIP, or PARTNERSHIP in TRADE, is *nothing but* the *Golden Rule* oft' *repeated*; I need not ſay much concerning it, nor trouble the *Learner* with NEW *Demonſtrations* of its *Operations*.

At once then,

By *this* RULE is found *each Partner's* SHARE of the GAIN or LOSS, in *Proportion* to his SHARE in the *common* STOCK.

EXAMPLE.

Three Traders enter *Partnerſhip*, and the *firſt* (which we'll call A) puts into the *common* STOCK 60 *Pounds*; the *ſecond* (B) 50 *Pounds*; and the *third* (C) 40 *Pounds*; (and thence the *whole* STOCK is 150 *Pounds* = 60 + 50 + 40).

Now, let the *whole* GAIN by *Trade* be 30 *Pounds*, HOW MUCH of *the* 30 *Pounds* ought *each Perſon* to *have?*

To *anſwer* this, there's *nothing* requir'd *but* to *repeat* the PRAXIS of the RULE of THREE DIRECT, as *often* as in the Nature of the Thing is requiſite.

Thus, by ſaying,

If

If the *whole* STOCK 150 *l. gain* 30 *l. what will* A's *Part* of this *Gain* be? or, *What will* A's *Part* of the STOCK, *viz.* 60 *l. gain? What will* B's, or 50 *l. gain?* And, *What will* C's, or 40 *l. gain?*

And *working* as has been taught, you'll find 12 *l.* to be A's *Part*, 10 *l.* B's *Part*, and 8 *l.* C's *Part.* For,

Whole Stock. *Whole Gain.*

A's, B's, C's } *Stock* { =60, =50, =40 } = 150 : 30 :: { 60 : 12 = A's, 50 : 10 = B's, 40 : 8 = C's } *Gain.*

Z = 150

And *this* is *what* is call'd the SINGLE *Rule* of FELLOWSHIP or PARTNERSHIP; or, The RULE of *Fellowſhip* or *Partnerſhip* WITHOUT TIME.

But now,

If beſides the *Diverſity* of STOCKS, there happens a *Diverſity* of TIME, *i. e.* If beſides the *particular* SHARES themſelves, the TIMES, of *continuing* their SHARES in, be alſo *conſider'd*, THEN *this Rule* is call'd the COMPOUND *Rule* of FELLOWSHIP or PARTNERSHIP, or the RULE of *Fellowſhip* or *Partnerſhip* WITH TIME.

And *as before* the SIMPLE, *ſo here* the COMPOUND *Golden Rule Direct* muſt be *repeated.*

Let an *Example* be as follows,

Three *Traders* enter *Partnerſhip*, and the *firſt* (A) puts into the *common* STOCK 60 *l.* and keeps it in 4 *Months*; the *ſecond* (B) 50 *l.* for 3 *Months*; the *third* (C) 30 *l.* for 2 *Months*: Let the GAIN of the WHOLE

1

STOCK be 30 *l.* How MUCH of *this* GAIN muſt *each Trader* HAVE?

Since now, *here* is a COMPOUND RATIO, *viz.* a *Ratio* COMPOUNDED of the *Ratio* of the SHARE, and the TIME to the TIME; *each* SHARE muſt be *multiply'd* into *its* TIME thus,

$$60 \times 4 = 240;\ 50 \times 3 = 150;\ 30 \times 2 = 60$$

And the NUMBERS thus *produc'd taken* TOGETHER will conſtitute the FIRST Term 450 (= 240 + 150 + 60); and the SAME *taken* SEVERALLY, ſo *many* THIRD Terms; and the WHOLE *Gain* itſelf the SECOND Term.

And thus,

The FIRST muſt have 16 *l.* the SECOND 10 *l.* and the THIRD 4 *l.* For,

Share. Time.

$$\left.\begin{matrix} 60 \times 4 = \\ 50 \times 3 = \\ 30 \times 2 = \end{matrix}\right\} = 450 : 30 :: \left\{\begin{matrix} 240 : 16 = A's \\ 150 : 10 = B's \\ 60 : 4 = C's \end{matrix}\right\} \textit{Gain.}$$

And after the *ſame* Manner may ALL *other* QUESTIONS of *this Nature* be *manag'd*, which muſt, after what has been ſaid, be plain to every one that conſiders this Rule to be nothing but the *preceding* COMPOUND GOLDEN RULE *repeated.*

But if the *Learner* ſhould want ſtill ſome Aſſiſtance, in order to his underſtanding the REASON of *multiplying* the STOCKS by the TIMES; he may conſider, that if the STOCKS and TIMES are *all* EQUAL, the GAINS are alſo EQUAL; and in *ſuch* Caſe, ALL *we have to do*, is to DIVIDE the GAIN by the NUMBER of PARTNERS. But,

If

If the TIMES are *only* EQUAL, it's plain, the SHARES of the GAIN are *proportional* to the *particular* STOCKS.

And if ONLY the STOCKS are EQUAL, theſe SHARES are *proportional* to the UNEQUAL TIMES.

And, *conſequently*,

If BOTH are UNEQUAL, the SHARES muſt be *proportion'd* to BOTH, *multiply'd together*.

If what has been *here* ſaid be not ſufficient, the *Learner* may conſider *farther*, that, for *Inſtance*,

If MY *Stock* be *double* HIS; but HIS continu'd in *Company*, *double* the TIME of MINE, 'tis plain, HE claims EQUAL *Gain* with ME.

And *ſo* it is by the *foregoing* RULE: For,

MY *double Stock*, multiply'd by the SAME *Time* with HIS, makes a DOUBLE *Product*; but if multiply'd by the HALF of HIS, will make an EQUAL *Product*; and *ſo*, in *any other* PROPORTION of the STOCKS and TIMES, *the multiplying* them together, *increaſes* EACH STOCK in PROPORTION to its TIME.

And, *conſequently*,

The SHARES of the GAIN found by uſing *theſe* PRODUCTS, are *truly* PROPORTION'D to *both* STOCKS and TIMES.

Such as are not contented with the *foregoing* ILLUSTRATIONS of the *preceding* RULE, may (at leaſt, after they underſtand ſomething of ALGEBRA) peruſe the *following*

DEMONSTRATION.

Let $\left|\begin{matrix} A \\ B \\ \alpha \\ \beta \\ T \\ \tau \end{matrix}\right|$ be = the $\left|\begin{matrix} \text{First} \\ \text{Second} \\ \text{First} \\ \text{Second} \\ \text{First} \\ \text{Second} \end{matrix}\right\}$ $\begin{matrix} \textit{Person's} \text{ Stock.} \\ \textit{Person's} \text{ Gain.} \\ \textit{Person's} \text{ Time.} \end{matrix}$

Then, by the *common* RULE of PROPORTION it will be

$$A : \alpha :: B : \frac{B\alpha}{A} \textit{ i. e.}$$

As the *Stock* A, to its *Gain* α (in the Time T) ſo is the *Stock* B to its *Gain* (in the ſame *Time* T) $= \frac{B\alpha}{A}$.

But now, the *Stock* B did not continue in *Partnerſhip* the *Time* T, but τ.

Again, therefore, by the ſame *common* RULE of PROPORTION, it will be

$$T : \frac{B\alpha}{A} :: \tau : \frac{B\alpha\tau}{AT} = \beta =$$ the *Gain* of B in the *Time* τ.

From which EQUATION ($\beta = \frac{B\alpha\tau}{AT}$) is deduc'd the following ANALOGY,

$$AT : B\tau :: \alpha : \beta. \textit{ i. e.}$$

As the FIRST *Person's* STOCK, *multiply'd* into its TIME, to the SECOND *Person's* STOCK into its TIME; ſo

ſo is the FIRST *Perſon's* GAIN on account of his STOCK and TIME, to the SECOND *Perſon's* GAIN, by his STOCK and TIME:

Then by *Compoſition* (ſee *Defin.* 14th of the 5th of EUCLID) it will be

$$AT + B\tau : B\tau :: \alpha + \beta : \beta.$$

Which, *Alternately*, or *by Permutation*, will be

$$AT + B\tau : \alpha + \beta :: B\tau : \beta. \text{ i. e.}$$

As the Sum of the PRODUCTS of *each Perſon's* STOCK *into its* TIME, *to the whole* GAIN; *ſo is each Perſon's* STOCK *multiply'd by its* TIME, *to his proportional Share of that* GAIN. Q. E. D.

Much more has been ſaid under *this* RULE than was, I believe, needful; however, I hope to have the *Learner's* PARDON, for whoſe *Service*, leſt any thing ſhould be *wanting*, it was *intended*.

I ſhall now return, and inſert, for the *Learner's* EXERCISE, a few *Queſtions* in the *foregoing* RULE, both SIMPLE and COMPOUND.

QUESTIONS.

I. One dies indebted to 6 ſeveral Perſons, A, B, C, D, E, and F, *viz.* To A, 180*l.* To B, 90*l.* To C, 80*l.* To D, 50*l.* To E, 64*l.* 10*s.* And to F, 36*l.* 10*s.* Now, *what* he left, *amounts* to but 310*l.* whereof *each Creditor* agrees to take his *proportional Part*: *What Part* of it muſt *each Creditor* have?

Anſwer.

		l.	*s.*	*d.*	*f.*
Anſwer. A	must have	111	7	6	2 $\frac{26}{167}$
B		55	13	9	1 $\frac{13}{167}$
C		49	10	0	0 $\frac{160}{167}$
D		30	18	9	0 $\frac{109}{167}$
E		39	18	2	1 $\frac{129}{167}$
F		22	11	8	1 $\frac{73}{167}$

II. Three *Perſons*, A, B, and C, had a *common* STOCK of 500*l.* and loſt by *Trade*, 36*l.* A's Loss was 30*l.* B's 36*l.* And C's 70*l.* *what* was *each* Perſon's STOCK *at firſt?*

	l.	*s.*	*d.*	*f.*
A's was	110	5	10	2 $\frac{6}{17}$
B's was	132	7	00	2 $\frac{14}{17}$
C's was	257	7	00	2 $\frac{14}{17}$

III. Three Perſons enter'd *Partnerſhip*; The *firſt* put into the *common* STOCK 20*l.* for 6 *Months*; The *ſecond* 40*l.* for 3 *Months*; And the *third* 60*l.* for 2 *Months*; Their *whole* GAIN was 36*l.* *what* muſt *each* Perſon's *Share* of *this* GAIN *be?*

Anſwer. 12*l.*

IV. Three Perſons A, B, and C, are *Partners* in *Trade*; A puts into the *common* STOCK 100*l.* for 6 *Months*; B puts in 75*l.* for 8 *Months*; C puts in 150*l.* for 4 *Months*; and they *gain'd*, in ALL, 100*l.* *what* muſt *each* Perſon have of *this* GAIN?

Anſwer. 33*l.* 6*s.* 8*d.*

V. Three *Traders*, A, B, and C, have *company'd* together; A put into the *common* STOCK *January* the *firſt*, 1000*l.* B, *March* the *fifth*, put in 500*l.* C, *July* the *twelfth*,

twelfth, put in 800 *l.* And, at the *Year's End*, they found they had *gain'd* 180 *l.* *what* will *each* Perſon's *Share* of the GAIN be?

Anſwer.

	l.	*s.*	*d.*	*f.*
A's	100 —	11 —	11 —	1 $\frac{579}{2177}$
B's	41 —	9 —	6 —	3 $\frac{2037}{2177}$
C's	37 —	18 —	5 —	2 $\frac{1738}{2177}$

Three Perſons, A, B, and C, were *Partners* in *Trade*, *January* 1. A *depoſited* 1200 *l.* and *April* 1. he *drew out* 200 *l.* *March* 1. B *put in* 600 *l.* and *Auguſt* 1. *put in* 250 *l.* *more.* C, *July* 1. *depoſited* 500 *l.* and *October* 1. he *took out* 100 *l.* And, at the *Year's End*, they found they had *gain'd* 300 *l.* *what* muſt *each* Perſon's *Share* be of *this* GAIN?

Anſwer.

		l.	*s.*	*d.*	*f.*
A's	*Share is*	167 —	12 —	6 —	2 $\frac{178}{411}$
B's		96 —	9 —	0 —	2 $\frac{106}{411}$
C's		35 —	18 —	4 —	3 $\frac{167}{411}$

See more *Queſtions* in the APPENDIX.

But now, before I *end* this *Chapter*, I ſhall juſt take notice, that,

Beſides the RULES of FELLOWSHIP, there are *other* RULES founded on the GOLDEN *one*; and *ſuch* are the RULES of ALLIGATION and FALSE POSITION: But,

As for the *former* of *theſe*, I might wholly *omit* it, as *imperfect*, becauſe the *Queſtions* falling under one *Part* of it at leaſt, are *indetermin'd* ones:

And as for the *latter* of *theſe*, I might wholly *omit* IT alſo; becauſe, after SPECIOUS ARITHMETIC, or ALGEBRA, has been *introduc'd*, it may be *ſpar'd* without any *Damage*: For,

Such

Such Queſtions as are uſually *anſwer'd* by *this* GUESSING RULE, or RULE of FALSE (as it is often call'd) will ALL of them, and MANY OTHERS, be more *eaſily* and *univerſally* SOLV'D by any one who has but a little *Skill* in ALGEBRA.

But, however,

That I may *omit nothing* that may be of any *Service* or *Pleaſure* to the LEARNER, I ſhall, in the *following* CHAPTERS, ſay ſomewhat concerning them.

CHAP.

CHAP. XIV.

THE RULE of ALLIGATION (ſo call'd, from the *Ligatures* which *practical Arithmeticians* make uſe of in their *Operations*) is *perform'd* by the Help *chiefly* of the GOLDEN RULE:

This *Rule* is uſually divided into *two Parts*, and *diſtinguiſh'd* by the *Terms* or *Names*,

MEDIAL and ALTERNATE;

The PRAXIS of both which I ſhall now give a *brief Account* of:

1. ALLIGATION MEDIAL is employ'd,

To find the *mean Rate* or *Value* of a *Mixture*, the *particular* different *Quantities* of the Things *mix'd* and their reſpective *Rates* being given.

To find which Rate (after the reſpective Quantities are all reduc'd into *one Denomination*, if they are not *ſo* given)

The RULE is,

As the *Sum* of the *given Quantities*,

To the *Sum* of their *total Values* or *Amounts*, at their reſpective *given Rates*;

So is 1, or *any other Part* of the *Mixture*,

To the *mean Rate* of ſuch *Part*.

This RULE is very evident (at leaſt muſt appear ſo after what has been offer'd and *demonſtrated* under the RULES of PROPORTION and FELLOWSHIP is underſtood) and, I think, wants no farther *Explanation*; however, take the following one:

Let A be a *given Quantity* of one kind of Commodity valu'd at any *Rate* whatſoever; and B a different *given Quantity* of another kind of Commodity valu'd at *any other Rate* whatſoever:

Then by the common RULE of *Proportion*,

As 1 is to the *given Rate* of the *firſt* Thing to be *mix'd*; ſo is A, or the *given Quantity* of that Thing, to its *total Value* or *Amount*, which call Q:

And as 1 is to the *given Rate* of the *ſecond* Thing to be *mix'd*; ſo is B, or the *given Quantity* of that Thing, to its *Amount*, or q. But,

If A coſts Q; and B coſts q; then A added to or mix'd with B (that is the *whole Mixture*) will coſt Q + q (*i. e.* the *whole Value* of the *Mixture*).

And conſequently,

Since the *Value* of the *whole Mixture* may be always thus obtain'd, the Value of 1, or *any other Part* of it, *i. e.* the *mean Rate requir'd*, may be always found by the common RULE of PROPORTION, thus,

As the *whole Mixture*, or *Sum* of the *given Quantities*,

To

To the *Sum* of their *Amounts*, at their reſpective *given Rates*;

So is 1, or *any other Part* of the *Whole*,

To the *mean Rate* requir'd. Q. E. D.

To apply which, take the following

EXAMPLE.

If with 10 Buſhels of *Wheat*, at 4*s.* the Buſhel, there are *mix'd* 40 Buſhels of *Rye*, at 3*s.* 50 Buſhels of *Barley*, at 2*s.* and 20 Buſhels of *Oats*, at 1*s.* the Buſhel; what may a Buſhel of this *Mixture* be ſold for, ſo as neither to *gain* nor *loſe* by it?

In order to *anſwer* THIS and SUCH *like* QUESTIONS,

1. *Reduce all* the *Meaſures* into *one* (the *leaſt* mention'd) *Denomination*; and againſt *each Quantity* ſet down the *Price* or *Rate* of them, as follows,

	B.	*d.*	*d.*
	10	48	480
	40	36	1440
	50	24	1200
	20	12	240
Sum of *all* the *Simples*	= 120		3360 = *Total Value.*

Then ſay, by the RULE of THREE *direct*,

B. *d.* *B.* *d.*
120 : 3360 :: 1 : 28 (= 2*s.* 4*d.*)

Thus it's found, that the *Price* of a Buſhel of the *fore-mention'd Mixture* is 2*s.* 4*d.*

By the *ſame* RULE, the *Value* of any *other Quantity* of *that* COMPOSITION is *juſtly determin'd.*

The PROOF of *all Operations* in this Sort of *Mixtures* is done by *comparing* THE TOTAL VALUE of THE WHOLE MIXTURE, as *ſold at* the MEAN *Rate*, with THE TOTAL VALUE of ALL THE PARTICULAR QUANTITIES, *ſuppoſing them ſold at their* RESPECTIVE *Rates* ANNEX'D; For,

If thoſe SUMS are EQUAL, the *Work* is *true.*

PROOF.

Multiply the whole Quantity or Sum of all the Simples, 120, *by the common Price*, 2s. 4d. = 28d. *and it will (if the Operation be true) be = the Sum of the particular Prices* = 3360d. For,

As *that Part* of the *Compoſition* whoſe *Price* was *ſought*,

Is in *Proportion* to its *Value found*;

So is the *total Compoſition*,

To the *total Value*, thus,

B.	*d.*		*B.*
1	: 28	: :	120
	120		
	3360*d.*	=	*l.* 14 = the *total Value.*

Therefore 'tis concluded that the *Work* is *right.*

2. ALLI-

2. ALLIGATION ALTERNATE is a peculiar Method of *finding* the *particular Quantity* of *each Ingredient*, (*ſeveral* being given or propos'd to be *mix'd*) when the *particular Rate* or *Price* of *each Ingredient*, and the *mean Rate* of the *Mixture* are *given*: And

The CASES or *Varieties* that may happen to be *ſolv'd* by it, are *Three*.

CASE I.

Is, when *neither* the *whole Quantity* of the *Mixture* or *Compoſition*, nor *any one* of the *Quantities* of the *propos'd Ingredient*, is *given* or *limited*.

CASE II.

Is, when ſome *one* of the *ſeveral* Quantities of the *Ingredients propos'd* to be *mix'd* is *given* or *limited*: And,

CASE III.

Is when the *whole Quantity* of the *Compoſition* is *limited*.

The RULE or *Method* of *Solution* is THIS,

Obſerve *which* of the *particular Rates given* are *greater*, and *which* are *leſs*, than the *mean Rate*, and *alligate* or *link* a *greater* to a *leſs*, 'till *all* are *alligated* or *link'd* (the *Manner* of *which* may be ſeen under the ſeveral *Examples* following): Then,

Find the *Differences* between the *mean Rate* and *each* particular *Rate*, and place them *alternately*; *i. e.* Place the *Difference* of the *mean Rate* and a *greater Rate* againſt that *leſs Rate* to which ſuch *greater* is *alligated*; and

and place the *Difference* of the *mean Rate* and a *leſs Rate* againſt that *greater Rate* to which ſuch *leſs* is *alligated*, &c. 'till all the *Differences* are in *like manner* reſpectively *plac'd*; and *they* are the *particular Quantities* SOUGHT, or, at leaſt, *correſponding Proportionals* to them, *each* of that *Denomination*, or of that *Ingredient* which belongs to *that particular Rate* to *which* it is ſo annex'd. Now,

That *theſe Differences* ſo plac'd *are* the *Quantities* REQUIR'D, is *what is to be demonſtrated*: But,

That we may the more *clearly apprehend what* is to be *demonſtrated*, it may not be improper, firſt to *ſolve* an *Example* as follows,

EXAMPLE I.

How much *Canary*, at 2 *s.* 6 *d.* a *Quart*, and *Sherry*, at 1 *s.* 8 *d.* a *Quart*, muſt a *Vintner* mix together, to ſell *for* 1 *s.* 11 *d.* a *Quart*?

Here, the *particular Rates given* are 30 *d.* and 20 *d.* and the *mean Rate* 23 *d.* (after they are all *reduc'd* into *one Denomination*);

The *Difference* between the *mean Rate* and the *greater Rate* being 7; and the *Difference* between the *ſame* and the *leſſer Rate* being 3.

Which Differences plac'd *alternately*, as above directed, will ſtand thus,

d.			
d. 23	30	3 *Quarts of Canary*	= the Quantities requir'd.
	20	7 *Quarts of Sherry*	

Which may be *prov'd* by ALLIGATION MEDIAL: For,

3 Quarts

	d.	*d.*
3 Quarts of *Canary* at	30 a Quart, amount to	90
And 7 Quarts of *Sherry* at	20 a Quart, amount to	140
	Their Total amounts =	230

Then,

As 10 (= Z of the *Quantities*)

To 230*d.* (= Z of their *Amounts*);

So is 1,

To 23*d.* (= the *mean Rate*). Which was to be *prov'd.*

And now we ſhall *demonſtrate* the *above-given* RULE to be *univerſally true.*

DEMONSTRATION.

Let M repreſent the MIDDLE, or MEAN *Number*, or *Rate*, between a GREATER (G) and a LESS (L). Then the DIFFERENCE between the MEAN *Rate*, and the GREATER, will be G—M, and THAT between the MEAN *Rate* and the LESS, will be M—L; and, by the RULE of ALLIGATION ALTERNATE, *theſe Rates* and *Differences* would become THUS *link'd* and *plac'd*,

$$M \left\{ \begin{matrix} G \\ L \end{matrix} \right] \begin{matrix} M-L \\ G-M \end{matrix} \Big\}$$

Which DIFFERENCES, by the RULE, I ſay, are = the QUANTITIES REQUIR'D. Now,

If theſe DIFFERENCES are the TRUE QUANTITIES *requir'd*, *their total Values*, or *Amounts*, at *their reſpective* GIVEN RATES, muſt, in ONE SUM (by the HYPO-

HYPOTHESIS) be = *what* the SUM of *thoſe* QUANTITIES do *amount* to at the GIVEN MEAN RATE.

Firſt, therefore, *multiply theſe* QUANTITIES *by their particular* RATES *reſpectively*; that is, *multiply* the QUANTITY M—L, by *its* RATE G, *and it produces* GM—GL *for its* TOTAL VALUE: And,

Multiply the other QUANTITY G—M, *by its* RATE L, *and it produces* GL—LM *for its* TOTAL VALUE: And,

Theſe TOTAL VALUES (GM—GL, and GL—LM) being *added together*, the SUM *will be* GM—GL + GL—LM; or (by *exterminating* GL, ſince —GL + GL, is = 0) *the* SUM *of their* VALUES, *at their particular* RATES, is GM—LM. Again,

The SUM *of the above-mentioned* DIFFERENCES (*which are ſaid to be the* QUANTITIES REQUIR'D) is G—M + M—L; that is (by *exterminating* M) G—L; *which being multiply'd by the* MEAN RATE M, *produces the* SAME TOTAL VALUE *with* THAT *which they amounted to at the* PARTICULAR RATES; namely, GM—LM.

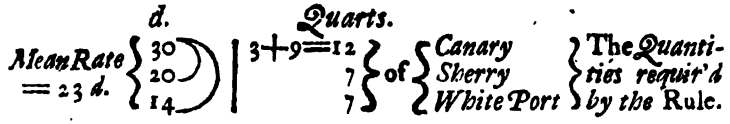
Therefore THOSE DIFFERENCES *are the* TRUE QUANTITIES REQUIR'D. Q. E. D.

EXAMPLE II.

Suppoſe a *Vintner* (as in the *former Example* he uſes but *two Sorts*) would uſe *three Sorts* of *Wine* in a *Mixture*, to be ſold for 1 *s.* 11 *d.* a Quart, *viz. Canary* at 2 *s.* 6 *d.* a Quart; *Sherry* at 1 *s.* 8 *d.* a Quart; and *White Port* at 1 *s.* 2 *d.* a Quart, how much of *each* muſt he uſe?

SOLU-

SOLUTION.



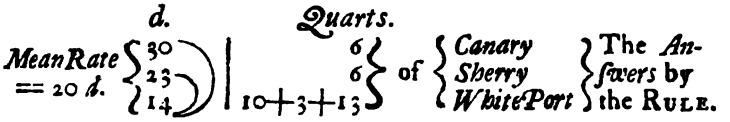
Thus the *Learner* may ſee the *Manner* of ALLIGATING or LINKING *three Rates*, when *but one* of them is *greater* than the *mean Rate*: And,

That *he* may alſo *ſee* how *three Rates* will become *link'd*, when *but one* of them is *leſs* than the *mean Rate*, let's *ſolve* the following *Example*:

EXAMPLE III.

How much *Canary*, at 2*s.* 6*d.*; *Sherry*, at 1*s.* 11*d.*; and *White Port*, at 1*s.* 2*d.* a Quart, muſt a *Vintner mix* together, to be ſold for 1*s.* 6*d.* a Quart?

SOLUTION.



The *Anſwers* found by ALLIGATION, in *each* of the foregoing *Examples* (where the *Number* of INGREDIENTS *propos'd to be mix'd*, is not above *three*) are the *only* ones that *can be found* by *that* METHOD: For,

Where there are but *three* things *given*, there can be *but* ONE *greater*, and TWO *leſs*, or ONE *leſs*, and TWO *greater*, than the MEAN: And ſince, by the RULE, a *Rate greater* than the *mean* one, muſt always

be *alligated* to a *leſs*, 'tis plain, that, when the *Number* of *Ingredients* does not exceed *three*, the *Alligations*, or *Linkings*, *can't* be *varied*, and conſequently *different* DIFFERENCES (*i. e. Anſwers*) *can't* be exhibited.

Nay, farther,

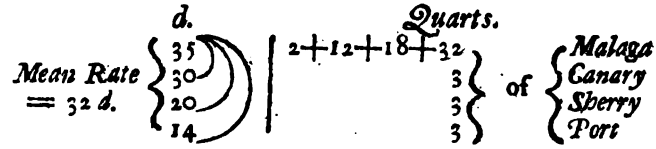
Let the *Number* of *Ingredients* be *what you will above* THREE, if there is *but one* of the *particular* GIVEN *Rates*, *greater* than the *mean Rate*, or *but one leſs* than the *mean Rate*, a *Variety* of *Linkings can't* be made, and therefore a *Variety* of *Anſwers can't* be obtain'd: As ſhall be ſhewn in the following *Examples*:

EXAMPLE IV.

How much old *Malaga*, at 2*s.* 11*d.* a Quart; *Canary*, at 2*s.* 6*d.* a Quart; *Sherry*, at 1*s.* 8*d.* a Quart; and *White Port*, at 1*s.* 2*d.* a Quart, muſt make a *Mixture*, to be ſold at 2*s.* 8*d.* a Quart?

Where there is *but one greater* than the *mean Rate.*

SOLUTION.



The *only Anſwers* by *Alligation.*

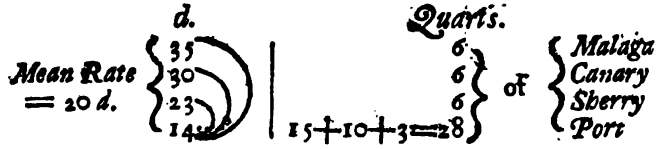
EXAMPLE V.

How much old *Malaga*, at 2*s.* 11*d.* a Quart; *Canary*, at 2*s.* 6*d.* a Quart; *Sherry*, at 1*s.* 11*d.* a Quart; and *White Port*, at 1*s.* 2*d.* a Quart, muſt be *mix'd*, to be ſold at the *mean Rate* of 1*s.* 8*d.* a Quart?

Where

Where there is *but one leſs* than the *mean Rate.*

SOLUTION.



The *only Anſwers* by *Alligation.*

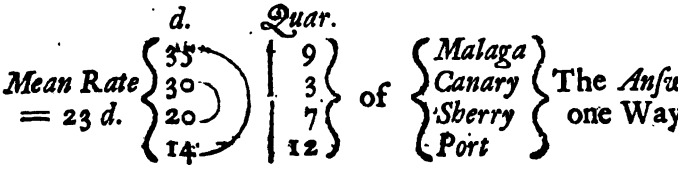
But, when there are *more than one* of the *particular Rates* GIVEN, *greater* than the *mean Rate*, the *Alligations* or *Linkings may be* VARIED; and, conſequently, a *Variety* of DIFFERENCES (or ANSWERS) *may* be *found*; *which* will be *more* or *leſs* in Number, as the *Things propos'd* to be *mix'd* are *ſo*, as in the following *Examples:*

EXAMPLE VI.

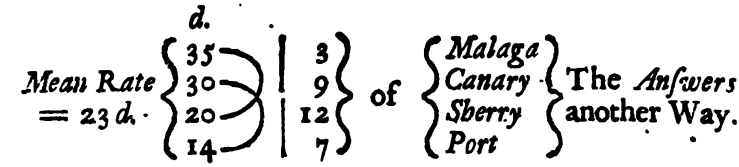
How much old *Malaga*, at 2*s.* 11*d.* a Quart; *Canary*, at 2*s.* 6*d.* a Quart; *Sherry*, at 1*s.* 8*d.* a Quart; and *White Port*, at 1*s.* 2*d.* a Quart, muſt be *put together*, to be ſold at 1*s.* 11*d.* a Quart?

Where there are *two* of the GIVEN RATES *greater*, and *two leſs*, than the *mean Rate.*

SOLUTION I.



SOLUTION II.



Tho' I thought it needleſs to inſert the PROOF, by ALLIGATION MEDIAL, under all the foregoing *Examples*, yet, of *this laſt Example*, I ſhall, for *Compariſon's* Sake, PROVE *both* the SOLUTIONS to be *true*, by ALLIGATION MEDIAL.

As to the *former* of them, then,

					d.		d.
9		*Malaga*			35		315
3	of	*Canary*	at		30	amount to	90
7		*Sherry*			20		140
12		*Port*			14		168
31 = the total Quantity.							713 = the total Value.

And, as to the *ſecond* SOLUTION,

					d.		d.
℔. 3		*Malaga*			35		105
9	of	*Canary*	at		30	amount to	270
12		*Sherry*			20		240
7		*Port*			14		98
31 = the total Quantity, as before.							713 = the total Value, as before.

Wherefore, ℔. d. ℔. d.
31 : 713 : : 1 : 23 = the *mean Rate given.*

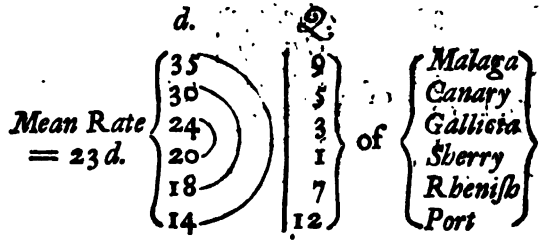
And,

And, for a greater *Variety* of SOLUTIONS, take another *Example*.

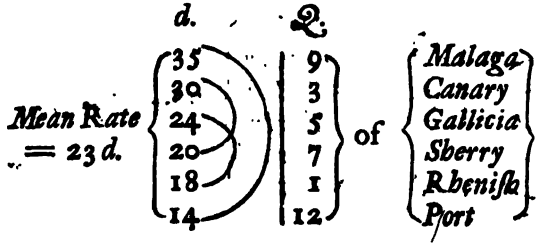
EXAMPLE VII.

Suppoſe it be requir'd to make a *Mixture* of *ſix Sorts* of *Wine*, *viz. Malaga*, at 2*s.* 11*d.* a Quart; *Canary*, at 2*s.* 6*d.* a Quart; *Gallicia*, at 2*s.* a Quart; *Sherry*, at 1*s.* 8*d.* a Quart; *Rheniſh*, at 1*s.* 6*d.* a Quart; and *White Port*, at 1*s.* 2*d.* a Quart, in *ſuch Proportion*, that the *Mixture* may be *ſold for* 1*s.* 11*d.* a Quart; *what Quantity* of *each Sort* muſt *compoſe* the *Mixture*?

SOLUTION I.



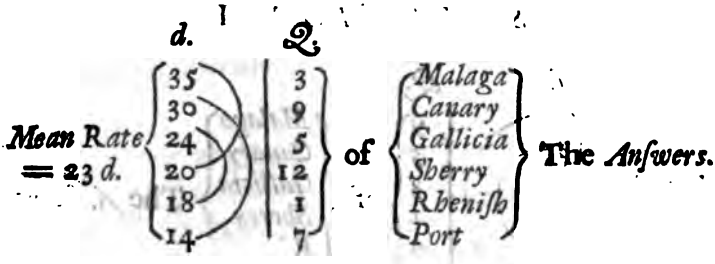
SOLUTION II.



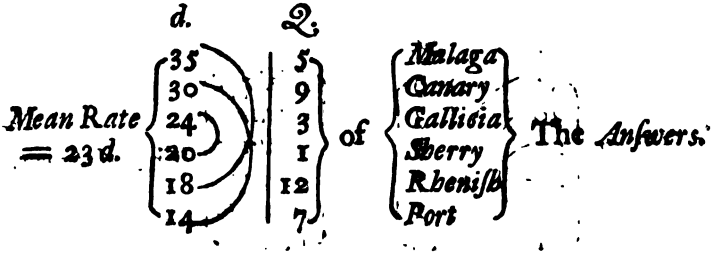
SOLU-

SOLUTION III.

SOLUTION IV.

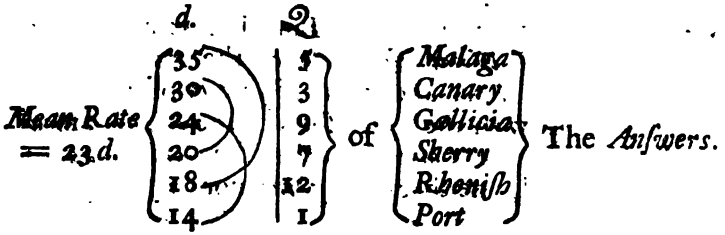


SOLUTION V.



SOLU-

SOLUTION VI.



Now, that *any one* of theſe *various Solutions* will *truly* ANSWER the *propos'd* QUESTION, may be PROV'D by ALLIGATION MEDIAL, as the two different *Solutions* of the *ſixth Example* were *prov'd* to be *true*.

SCHOLIUM.

Before I proceed to the SECOND and THIRD CASES, it may be *obſerv'd* by what has been (more abundantly than is uſual) ſaid under the FIRST CASE, that THIS RULE is IMPERFECT, and *cannot* give ALL the SOLUTIONS *poſſible*: For,

Though the *Anſwers* to *all* the *foregoing Queſtions*, found *thereby*, are ALL TRUE, yet it's very manifeſt, that *any Sett* of NUMBERS that have the SAME RATIO to *one another*, as the ANSWERS, under *any one* of the *preceding* SOLUTIONS, have to *one another*, will, every whit as well, *anſwer* the QUESTION; therefore the ANSWERS are INFINITE.

I now proceed to CASE II. wherein the *Quantity* of ONE of the *Ingredients propos'd* to be *mix'd*, is *limited*: As in the following *Example*.

EXAMPLE.

EXAMPLE I.

There are 15 Buſhels of *Wheat*, at 4 *s.* the Buſhel, to *mix* with *Barley* at 3 *s.* the Buſhel, *Rye* at 2 *s.* and *Oats* at 1 *s.* 6 *d.* *What Quantity* of the *Barley*, *what Quantity* of the *Rye*, and *what Quantity* of the *Oats*, muſt be *mix'd* with the 15 Buſhels of the *Wheat* aforemention'd, that the *Compoſition* may be *ſold* at *half* a *Crown* a Buſhel, neither to *gain* nor *loſe* by the *Bargain*?

Let the *Work* be firſt *manag'd*, as in the *ſixth Example* of CASE I. Thus, FIRST,

	d.	*B.*	
Mean Rate = 30 *d.*	48	12	*Wheat*
	36	6	*Barley*
	24	6	*Rye*
	18	18	*Oats*

of

Here, the *Differences* gain'd are not the *very Quantities* that the LIMITS of the QUESTION *require*; but are *correſponding* PROPORTIONALS to them (by what has been amply deliver'd under CASE I.) For,

If 12 Buſhels of *Wheat*, 6 of *Barley*, 6 of *Rye*, and 18 of *Oats*, are put together, *ſuch* a MIXTURE will be made, as upon Enquiry, may be found to be *worth* 2 *s.* 6 *d.* a Buſhel, which is the MEAN RATE *given*: *But the Quantity* of WHEAT to be *us'd* in the COMPOSITION in Hand, is (NOT 12, BUT) 15 *Buſhels* CERTAIN: Wherefore,

In order to *gain* SUCH QUANTITIES, of the *reſt* of the INGREDIENTS, *as* will (when *mix'd* with the 15 Buſhels of *Wheat*) ſufficiently *anſwer* the QUESTION *propos'd*, we muſt call to our Aſſiſtance the RULE of PROPORTION: Thus, then, we muſt go on and ſay,

12 :

$$\left.\begin{array}{l} 12 : 6 :: 15 : 7\frac{1}{2} \\ 12 : 6 :: 15 : 7\frac{1}{2} \\ 12 : 18 :: 15 : 22\frac{1}{2} \end{array}\right\} \text{ of } \left\{\begin{array}{l} \textit{Barley.} \\ \textit{Rye.} \\ \textit{Oats.} \end{array}\right.$$

And we find accordingly, that $7\frac{1}{2}$ Buſhels of *Barley*, $7\frac{1}{2}$ of *Rye*, and $22\frac{1}{2}$ of *Oats*, are the *Quantities requiſite* to be *mix'd* with the 15 Buſhels of *Wheat*, that the *Mixture* may be *ſold* at *Half a Crown* a Buſhel, *without* GAINING *or* LOSING *thereby*. But,

2dly. The *foregoing* RATES may be ALLIGATED as follows,

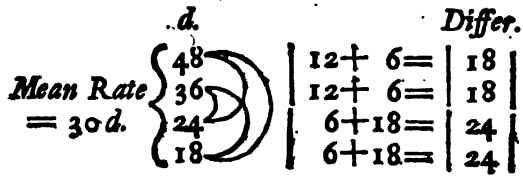
	d.	*B.*	
Mean Rate = 30*d.*	48	6	*Wheat.*
	36	12	*Barley.*
	24	18	*Rye.*
	18	6	*Oats.*

And thus we *gain* OTHER DIFFERENCES, which are of the *ſame Service* to us as the *former*, altho' they are *not* in the *ſame* PROPORTION to *one another*, as the *former* ones are: For,

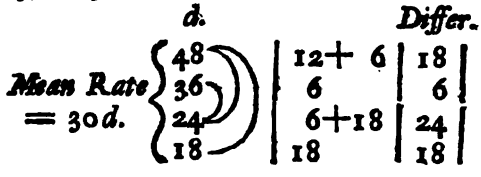
By the Help of the RULE of PROPORTION, as before, we may find, that this *new* SET of DIFFERENCES, or PROPORTIONALS, will *exhibit* 30 Buſhels of *Barley*, 45 of *Rye*, and 15 of *Oats*, to be *mix'd* with the 15 Buſhels of *Wheat*; *which* MIXTURE (THUS *proportion'd*) will *anſwer* the QUESTION *propos'd*, as ſufficiently as the *former* one does.

Now *theſe two Ways* of *forming* the MIXTURE, are ALL that *can* be *gain'd* by ALLIGATION *ſimply*, *i. e.* by *alligating* the *given Terms*, a *greater* with a *leſs*, according to the DIRECTIONS of the RULE, 'till *all* are *alligated*, without taking the Liberty of *repeating* or *compounding* the COMBINATIONS or ALLIGATIONS of the TERMS, after they are *all ſo alligated*: As *thus*,

†



Or, *thus*,



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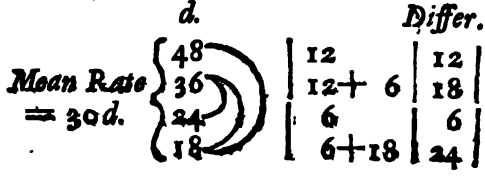
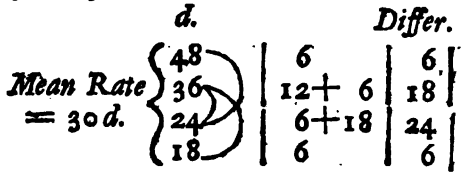
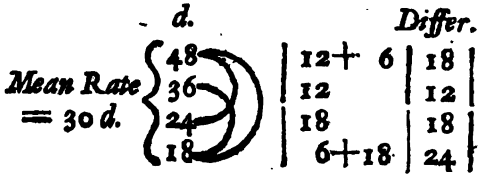


Or, *thus*,

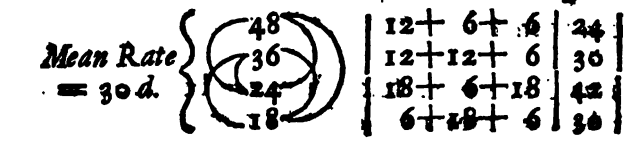


Or, *thus*,



Or,

Or, *thus*,



&c.

Not but that *any* of the Differences *gain'd* by Compounding the Combinations or Linkings (in *ſuch*, or *any other* Manner) are Corresponding Proportionals to the Quantities Requir'd, *as much* as *thoſe* gain'd by the *two firſt Ways* are, and will as ſufficiently contribute to *anſwer* the Queſtion propos'd.

And, by the by, the Various Ways of Compounding Alligations, may ſerve as an *Exerciſe* for the *curious* and *ingenious* Learner.

Here, farther, the *Learner* may *obſerve*, that, *beſides* the Innumerable *Anſwers* All *Queſtions of this kind* are *capable of*, *they* being Unlimited; Another *Imperfection* in This Rule moſt times happens in *this* Second Case, which does *not* in the First, and *that is*, That

Mere Alligation, without making *uſe* of the Rule of Proportion, is very *rarely* (I won't ſay *never*) *ſufficient* to *anſwer* the Propoſition.

EXAMPLE II.

When a *Tobacconiſt* would mix with 8 Pounds of *Tobacco*, rated at 2 *s.* a Pound; a *Sort* rated at 1 *s.* 8 *d.* a Pound; a *Sort* rated at 1 *s.* 2 *d.* a Pound; and a

Sort rated at 10 *d.* a Pound; *how much* of *each Sort* muſt he *uſe*, that the *Compoſition* may be afforded for 1 *s.* 6 *d.* a Pound, without *either* GAINING *or* LOSING by *ſo* MIXING them?

In *this* EXAMPLE, ſince there *can* be *but* TWO *different* ALLIGATIONS *made*, let us *make them* BOTH.

Thus, FIRST,

	d.	*Pounds.*
Mean Rate = 18 *d.*	24	8
	20	4
	14	2
	10	6

And, SECONDLY, *thus*,

	d.	*Pounds.*
Mean Rate = 18 *d.*	24	4
	20	8
	14	6
	10	2

Now here the *Quantities* GAIN'D by the FIRST ALLIGATION, *viz.* 4 Pounds, at 1 *s.* 8 *d.* 2 Pounds, at 1 *s.* 2 *d.* and 6 Pounds at 10 *d.* are *ſuch* as will, *without* the *Help* of the RULE of PROPORTION, ſufficiently *anſwer* the PROPOSITION, altho' *thoſe* GAIN'D by the SECOND ALLIGATION will *not*.

The *Anſwers* of ALL *Queſtions* of THIS CASE (as well as *thoſe* of the FIRST CASE) may be PROV'D by ALLIGATION MEDIAL.

And now, having largely handled the 2 FIRST CASES, I ſhall proceed to the THIRD and *laſt* CASE; and *that* is, when the *whole Quantity* of the *Compoſition* is *limited*.

EXAMPLE

EXAMPLE I.

Suppoſe a *Goldſmith* would make a *Wedge* of *Gold* to be 60 Ounces in Weight, and to be worth 3 *l.* 12 *s.* an Ounce, *how much* of *one Sort* (*better* than *Standard*, and) worth 3 *l.* 17 *s.* an Ounce; and *how much* of *another Sort* (*worſe* than *Standard*, and) worth 3 *l.* 5 *s.* an Ounce, muſt he *melt together*, to make a *Wedge* of the *Weight* and *Value propos'd*?

The *Work* (by the RULE) muſt be *firſt* manag'd thus,

	s.	*Oz.*	
Mean Rate = 72 *s.*	77	7	The *better.*
	65	5	The *worſe.*

The *Quantities* exhibited by the RULE.

But here (as in CASE II.) *they* are *not* SUCH *Quantities* as will *anſwer* the CONDITIONS of the QUESTION; for 7 Ounces of the *better*, *mix'd* with 5 Ounces of the *worſe*, make *but* 12 Ounces *in all*; whereas the *Wedge* is to weigh 60 Ounces. Wherefore,

To find *ſuch Quantities* as will *truly* ANSWER the PROPOSITION, we muſt (ſince the *Quantities* 7 and 5 *determin'd* by MERE *Alligation*, are *their* CORRESPONDING PROPORTIONALS) be here *alſo* obliged to *make uſe of* the RULE of PROPORTION, and ſay,

As 12 (the *Whole* of the *Quantities* found by ALLIGATION)

To 60 (the GIVEN *Whole* of the *particular* QUANTITIES *requir'd*):

So

So is $\left\{\begin{matrix}7\\5\end{matrix}\right\}$ The *proportional Part found* of the $\left\{\begin{matrix}\textit{Better}\\\textit{Worse}\end{matrix}\right\}$ *Sort*,

To $\left\{\begin{matrix}35\\25\end{matrix}\right\}$ The *Quantity* requir'd of the $\left\{\begin{matrix}\textit{Better}\\\textit{Worse}\end{matrix}\right\}$ *Sort*.

By which Means we find, that 35 *Ounces* of the BETTER *Sort*, and 25 of the WORSE, *melted together*, will *really* ANSWER the DESIGN of the QUESTION.

Here alſo we may *obſerve* an IMPERFECTION or DEFECT, which *generally* (tho' not *univerſally*) happens in *this* RULE of ALLIGATION, and muſt be *ſupply'd* by the RULE of PROPORTION.

EXAMPLE II.

How much *Canary*, at 2*s.* 6*d.* a Quart; *Sherry*, at 1*s.* 8*d.* a Quart; and *White Port*, at 1*s.* 2*d.* a Quart, muſt be *put together*, ſo that the *whole* COMPOSITION may contain 26 Quarts, and be worth 1*s.* 11*d.* a Quart?

	d.		*Quarts.*	
Mean Rate = 23*d.*	30	9 + 3 = 12		*Canary*
	20	7	of	*Sherry*
	14	7		*Port*

Which Quantities found by the RULE of ALLIGATION *alone*, do *(without any Defect)* ſufficiently *anſwer* the QUESTION. For, 12 + 7 + 7 = 26.

Here alſo the OPERATIONS may be PROV'D by ALLIGATION MEDIAL.

GENERAL SCHOLIUM.

Since ALL QUESTIONS *whatſoever*, that come under THIS RULE of ALLIGATION, will admit of *no more* SOLU-

SOLUTIONS than the GIVEN QUANTITIES will admit of ALLIGATIONS; and ſince THOSE are often times IMPERFECT, or *not ſo commodious* as the ENQUIRER's *Occaſions* and *Circumſtances* may *require*, let the *Learner* take notice, that, for a *Remedy* herein, he muſt have *Recourſe* from COMMON, to SPECIOUS ARITHMETIC, or ALGEBRA, whereby he may be enabled to *inveſtigate* and *diſcover all the Solutions poſſible*; as for *Inſtance* in

EXAMPLE III.

Suppoſe a *Vintner* would make a MIXTURE conſiſting of 40 *Gallons*, *viz.* Of *Canary*, at 7 *s.* *Sherry*, at 5 *s.* and *White Port*, at 4 *s.* a *Gallon*; *how* many *Gallons* of *each Sort* muſt he *put together*, that he may ſell the MIXTURE at 6 *s.* a *Gallon*, without GAIN or Loss by *ſuch Sale?*

SOLUTION.

Mean Rate = 6 *s.* { *s.* 7, 5, 4 } | 2 + 1 = { *Gall.* 3, 1, 1 } of { *Canary.* *Sherry.* *White Port.*

5

And then, 5 : 40 :: { *Gall.* 3, 1, 1 } : { 24, 8, 8 } of { *Canary.* *Sherry.* *White Port.*

Thus the *Vintner* by *this* RULE of ALLIGATION, and the *Help* of the RULE of PROPORTION, finds he muſt *put together* 24 *Gallons* of *Canary*, 8 *Gallons* of *Sherry*, and 8 *Gallons* of *White Port.* But now,

If *this* SOLUTION does *not anſwer* the *Vintner's* INTENT; that is,

Suppoſe he would have *his* MIXTURE *otherwiſe* PROPORTION'D; I then ſay, he muſt have *Recourſe* to ALGEBRA for *Aſſiſtance*: For,

'Tis

'Tis by an ALGEBRAIC *Proceſs* ONLY, that ALL the SOLUTIONS *poſſible* CAN be DISCOVER'D: *Which* wonderful *Advantage* I ſhall ſhew in an ALGEBRAIC SOLUTION of *this* EXAMPLE.

The *Learner*, I hope, will forgive this Step out of the Way, and take the SOLUTION in the *Manner following*.

Let $\left\{\begin{matrix} x \\ y \\ z \end{matrix}\right\}$ be = the *Gallons* of $\left\{\begin{matrix} \textit{Canary}, \\ \textit{Sherry}, \\ \textit{White Port}; \end{matrix}\right.$

Then $x + y + z = 40$

And $7x + 5y + 4z = 240$ } by the *Hypotheſis*.

Thence $z = 40 - x - y$

And $z = \frac{240 - 7x - 5y}{4}$

Therefore $160 - 4x - 4y = 240 - 7x = 5y$

$3x = 80 - y$

$y = 80 - 3x$

Therefore x muſt be *leſs* than $26\frac{2}{3}$ *Gallons*.

But ſince $z = 40 - x - y$

And ſince $y = 80 - 3x$

$z = 40 - x - 80 + 3x = 2x - 40$;

Therefore x muſt be more than 20 *Gallons*.

Thus having *diſcover'd* that the *Gallons* of *Canary*, that CAN POSSIBLY be US'D in the *above-mention'd* MIXTURE, *muſt* be *more* than 20, and *leſs* than $26\frac{2}{3}$, we from thence know *what Quantities* of the *Sherry* and *White*

White Port, CAN POSSIBLY be MADE USE OF *alſo*: For,

Since $z = 2x - 40$, take *what Quantity* of *Canary* you pleaſe betwixt 20 *Gallons*, and 26 $\frac{2}{3}$; *ſubtract* (in *this Example*) the *conſtant* NUMBER 40, from the *Double* of *ſuch Quantity* of *Canary*, and the *Remainder* will be z, or the *Quantity* of *White Port*, *that* muſt accordingly be US'D; wherefore the *Quantity* of *Sherry* is *conſequently* KNOWN.

Thus we have *obtain'd* a PERFECT *Solution* of *this* PROPOSITION, having DETERMIN'D ALL THE POSSIBLE MEANS OF FORMING THE MIXTURE.

I ſhall now inſert a few more *Queſtions* for the *Learner's* farther EXERCISE, and then go on to the RULE of FALSE.

QUESTIONS.

I. Of *three* Sorts of *rich Perfumes*, ſuppoſe *one* be worth 2 *s.* 6 *d.* a *Dram*; the *ſecond* be worth 3 *s.* 9 *d.* a *Dram*; and the *third* be worth 6 *s.* 3 *d.* a *Dram*; *what* will the *Value* of a *Dram* of the *Perfume* be, COMPOUNDED of 10 *Drams* of the *firſt* Sort, 8 *Drams* of the *ſecond*, and 12 of the *third*?

Anſwer. 4 *s.* 4 *d.*

II. A *Grocer* hath mix'd 4 *Sorts* of *Sugar*; the *firſt* Sort being *valu'd* at 11 *d.* the *ſecond* at 10 *d.* $\frac{1}{2}$. the *third* at 7 *d.* $\frac{1}{2}$. and the *fourth* at 4 *d.* a Pound; *what Quantity* of *each* Sort is there in the MIXTURE, ſuppoſing *ſuch Mixture* to be worth 9 *d.* a Pound?

	Pounds.		*Pounds.*		
Anſwer.	6	or	20	of the	1ſt
	20		6		2d
	8		6		3d
	6		8		4th

Sort.

III. A *Founder* hath a *Quantity* of *Silver* weighing 3 Pounds (*Avoirdupoiz Weight*) which he values at 4*l.* 8*s.* a Pound; *how* much *Iron*, at 1*d.* $\frac{1}{2}$. a Pound; and *Braſs* at 4*d.* $\frac{1}{2}$ a Pound, muſt be melted down together, with the 3 Pounds of *Silver*, that the MIX'D METAL may be worth 3*s.* 6*d.* a Pound?

lb.

Anſwer. { 39 } of { *Iron.* / *Braſs.* }
{ 39 }

IV. A *Diſtiller*, by *mixing ſeveral Sorts* of *Spirits*, would make the *Quantity* of a HOGSHEAD, to be *ſold* at a *Crown* a GALLON; *how many* GALLONS of the *firſt Sort*, at 6*s.* 8*d.* of the *ſecond*, at 5*s.* 10*d.* of the *third*, at 5*s.* 6*d.* of the *fourth*, at 4*s.* 4*d.* and of the *fifth*, at 3*s.* 6*d.* a GALLON, muſt he make uſe of to *anſwer* his *Purpoſe*?

Gall.

Anſwer. { 16 $\frac{1}{5}$ / 7 $\frac{1}{5}$ / 7 $\frac{1}{5}$ / 14 $\frac{2}{5}$ / 18 } of the { 1ſt / 2d / 3d / 4th / 5th } *Sort.*

In *all* 63 *Gallons*, = 1 *Hogſhead.*

See more *Queſtions* in the APPENDIX.

I ſhall now go on to the RULE of FALSE, or FALSE POSITION.

CHAP.

CHAP. XV.

THE RULE of FALSE, FALSHOOD, or FALSE POSITION, is *ſo* call'd from the *Method* taken to *diſcover* the *true* ANSWER to the QUESTION *propounded*, *viz.* From the SUPPOSING or GUESSING at a NUMBER for the ANSWER *requir'd*; *which Number*, for the moſt Part, happens to be FALSE: Now,

Sometimes the CONDITIONS of the PROPOSITION are *ſuch*, that ONE FALSE SUPPOSITION, *ſufficiently* enables us to *diſcover* the TRUE ANSWER, and ſometimes it does *not*; but we muſt neceſſarily GUESS *again*, or make a SECOND (tho' *falſe*) SUPPOSITION, to get *Help enough* for a TRUE SOLUTION. Whence,

This Rule of FALSE POSITION is uſually *divided* into TWO PARTS,

SINGLE and *DOUBLE*.

Of *which* I ſhall treat *diſtinctly*, and *firſt* of

POSITION SINGLE.

Where ONE *Falſe Poſition* ſufficiently enables us to *reſolve* the QUESTION *propos'd* by the following

RULE.

For the NUMBER *ſought*, take *what* NUMBER *you pleaſe*, and *try* according to the DATA and CONDITIONS of the QUESTION, whether your *ſuppos'd* NUMBER be *true* or *falſe*: if *true*, no more's to be done; but if *falſe*, you muſt obſerve *what* the *falſe* RESULT or CONCLUSION *is*, and then,

By the *common* RULE of PROPORTION, ſay,

As the FALSE CONCLUSION,

Is to the FALSE *ſuppos'd* NUMBER;

So is the NUMBER GIVEN,

To the TRUE NUMBER *ſought*.

The TRUTH of THIS RULE, is evident from the *meer* NATURE of PROPORTION, and therefore I need give no farther *Illuſtration* of it, than what may be had from the *Application* of it, to the SOLUTION of the *following*

EXAMPLE.

If three Merchants agree to buy a *Ship* for 1000 *l.* to be paid for amongſt them, in *ſuch Proportion*, that A's *Part* of the ſaid *Ship* be but $\frac{1}{6}$ of B's *Part*, and B's *Part* but $\frac{2}{3}$ of C's *Part*; *what muſt each Merchant pay?*

Let us *ſuppoſe* at Pleaſure, that A muſt *pay* 100 *l.* upon *which Suppoſition*, B muſt 600 *l.* and C 900 *l.* For,

By the CONDITIONS of the QUESTION, B is to *pay* 6 times as much as A, and but *2 thirds* of *what* C is to *pay*: Now,

If

If the SUPPOSITION of 100*l.* for A's *Payment* be TRUE, the RESULT muſt be, that *thoſe three Payments, viz.* A's 100*l.* B's 600*l.* and C's 900*l.* added together, make juſt 1000*l.* (being the *Number* of *Pounds propos'd* to be *expended*) but they do *not*: For,

100 *l.* + 600 *l.* + 900 *l.* are = 1600 *l.* *which* is a FALSE RESULT, or CONCLUSION;

Wherefore the POSITION of 100*l.* for A, is FALSE: But,

Since WHOLES are as their *like* PARTS, it will hold by the RULE of THREE DIRECT.

As 1600*l.* (the FALSE RESULT),

To 100*l.* (the FALSE *ſuppos'd* NUMBER);

So is 1000*l.* (the *whole* NUMBER GIVEN),

To 62*l.* 10*s.* (the TRUE NUMBER *ſought*). Whence,

It is *truly concluded*, that A muſt *pay* 62*l.* 10*s.* B, conſequently, muſt *pay* 375*l.* and C, 562*l.* 10*s.* For,

Theſe NUMBERS do not *only* AMOUNT to 1000*l.* in the WHOLE, but *bear* SUCH PROPORTION to ONE ANOTHER, As by the CONDITIONS of the PROPOSITION they *ought* to do.

This is, in ſhort, the WHOLE BUSINESS of the FIRST PART of the RULE of FALSE POSITION, *viz.* POSITION SINGLE.

Here the LEARNER may *obſerve*, that ALL QUESTIONS *that* are *reſolveable* by the *Help* of ONE FALSE SUPPOSITION *only*, muſt contain *ſuch certain* PROPOR-

PROPORTIONALS, or exhibit *ſuch* a RATIO, as will, by the *common* RULE of PROPORTION, *juſtly determine* the TRUE NUMBER *ſought*: But,

When the CONDITIONS of the PROPOSITION are SUCH as *afford* no SUCH *requiſite* PROPORTIONALS, we are *oblig'd* to *make uſe of* TWO SUPPOSITIONS, in order to SOLVE *it*; *which* leads me to the SECOND PART of *this* GUESSING RULE; namely,

POSITION DOUBLE.

For the SOLUTION of thoſe PROPOSITIONS that *require* a DOUBLE POSITION, the PROCESS *preparatory* to the *following* GENERAL RULE, muſt be always THIS:

Suppoſe TWO NUMBERS at *Pleaſure*, and *work* with *each* of them (as with the TRUE *one*) according as the CONDITIONS of the QUESTION *direct*; and if EITHER of the NUMBERS *thus arbitrarily taken*, happens to SOLVE the QUESTION, there needs *no more* to be *done*; but if NOT,

Obſerve whether the RESULTS *ariſing* from *them*, are GREATER or LESS than they *ought* to be, and make the ERRORS *accordingly* (for *Diſtinction* ſake) with *this* SIGN + *when too* GREAT, and with *this* — when *too* LITTLE:

This being done, make uſe of the following *general*

RULE.

Multiply the SUPPOSITIONS by the ERRORS *alternately*, i. e. *Multiply* the FIRST POSITION by the SECOND ERROR, and the SECOND POSITION by the FIRST ERROR: And if the ERRORS are BOTH *of the* SAME *kind*, to wit, BOTH of them EXCESSES, or BOTH DEFECTS, *divide* the DIFFERENCE of the PRODUCTS *by* the

the DIFFERENCE of the ERRORS; but if they are of DIFFERENT *Kinds*, to wit, ONE an EXCESS, and the OTHER a DEFECT, *divide* the SUM of the PRODUCTS by the SUM of the ERRORS, and the QUOTIENT will give the NUMBER SOUGHT.

DEMONSTRATION.

Whether the ERRORS are of the SAME or DIFFERENT *Kinds*, the RULE aſſigns a QUOTIENT for the NUMBER SOUGHT; *which* we'll call Q.

Now, if N be the *given* DIVISOR, NQ is the DIVIDEND; ſo that I ſhall proceed to DEMONSTRATE *what that* NUMBER *is*, *which* MULTIPLY'D *by the* NUMBER N, *produces* $NQ = P$.

1ſt. Let us *ſuppoſe* it to be $Q - R$, which *multiply'd* by N, *produces* $NQ - NR$, the *firſt falſe Reſult*.

Therefore the *firſt* ERROR is $P - NQ + NR$.

2dly. Let us *ſuppoſe* it to be $Q - S$, which *multiply'd* by N, *produces* $NQ - NS$, the *ſecond falſe Reſult*.

Therefore the *ſecond* ERROR is $P - NQ + NS$.

Now becauſe in BOTH *theſe* ERRORS the SIGNS are ALIKE, *i. e.* becauſe they are BOTH of the SAME KIND, let us SUBTRACT the LESSER ERROR (*which* is *done* by *changing all its* SIGNS) from the GREATER; and thus the DIFFERENCE of the ERRORS will be $P - NQ + NR - P + NQ - NS$. Or,

By *exterminating* the TERMS that are EQUAL, (ſince *they* mutually *deſtroy one another*) the DIFFERENCE of the ERRORS is $NR - NS$.

Multiply

Multiply Q — R (the *firſt Suppoſition*) by P — NQ + NS (the *ſecond Error*) or by NS only (ſince P is = NQ, and conſequently *cancel each other*) and the PRODUCT will be NSQ — NSR.

Alſo *multiply* Q — S (the *ſecond Suppoſition*) by P — NQ + NR = NR (the *firſt Error*) and *that* PRODUCT will be NRQ — NRS.

From the PRODUCT (NRQ — NRS) *ſubtract* the PRODUCT (NSQ — NSR) and the DIFFERENCE of *them* will be NRQ — NRS — NSQ + NSR, *i. e.* (by *exterminating* NRS) the DIFFERENCE of the PRODUCTS is NRQ — NSQ: But,

NRQ — NSQ (the DIFFERENCE of the PRODUCTS) *divided by* NR — NS (the DIFFERENCE of the ERRORS) is = Q (the NUMBER SOUGHT). Q. E. D.

Again,

Let the *firſt Suppoſition* be Q + R.

Which *multiply'd* by N, *produces* NQ + NR.

Therefore the *firſt* ERROR is NQ + NR — P.

And,

Let the *ſecond Suppoſition* be Q — S.

Which *multiply'd* by N, *produces* NQ — NS.

Therefore the *ſecond* ERROR is P — NQ + NS.

Here, becauſe the ERRORS are of DIFFERENT KINDS, *viz.* ONE an EXCESS, and the OTHER a DEFECT, let us ADD *them* together (*which* is *done* without *altering any* of the SIGNS) and *their* SUM will be NQ + NR — P + P — NQ + NS; or (by *exterminating* the EQUAL QUANTITIES P and NQ) the SUM of the ERRORS will be NR + NS.

Multiply

Multiply Q + R (the *firſt Suppoſition*) by P — NQ + NS (the *ſecond Error*) i. e. by NS (ſince P and NQ mutually *cancel one another*) and the PRODUCT will be NSQ + NSR.

Multiply likewiſe, Q — S (the *ſecond Suppoſition*) by NQ + NR — P = NR (the *firſt Error*) and the PRODUCT will be NRQ — NRS.

ADD the PRODUCTS together, and *their* SUM will be NSQ + NSR + NRQ — NRS; or (by *exterminating* NRS) the SUM of the PRODUCTS is NSQ + NRQ.

Wherefore Q (the NUMBER SOUGHT) is = NSQ + NRQ (the SUM of the PRODUCTS) *divided by* NS + NR (the SUM of the ERRORS). Q. E. D.

EXAMPLE.

A *Labourer* was *hir'd*, on *this Condition*, that *for every Day* he WORK'D, he ſhould *receive* 12*d.* but *for every Day* he was IDLE, he ſhould be *mulcted* 8*d.* When 90 Days were paſt, he had 3*l.* 5*s.* to *receive*; *how many Days* did he WORK, and *how many Days* was he IDLE?

Let the *firſt Suppoſition* be, that he WORK'D 85 Days, then he was IDLE 5 Days; now 85 Days WORK, at 12*d.* a Day, comes 1020*d.* and the 5 *Days* that he was IDLE, at 8*d.* a Day, comes to 40*d.* ſo that upon *that Suppoſition*, he would have to *receive* 1020*d.* — 40*d.* = 980*d.* *which* is MORE than his *juſt* WAGES (according to the *Conditions* of the PROPOSITION) *come to*, by 200*d.* for 3*l.* 5*s.* (*reduc'd* into *Pence*) are = 780*d.*

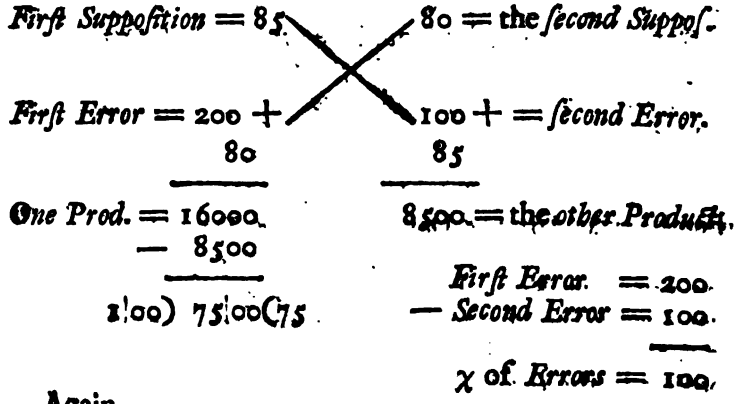
Let the SECOND SUPPOSITION be, That he WORK'D 80 Days, and then he muſt be IDLE 10 Days; in

THIS *Caſe* he *earns* 80 × 12 *d.* = 960 *d.* and *forfeits* 10 × 8 *d.* = 80 *d.* therefore he has to *receive* 960 *d.* — 80 *d.* = 880 *d.* *which* is alſo *too much* by 100 *d.*

Then *multiply* 85 (the FIRST POSITION) by 100 (the SECOND ERROR) and the *Product* will be 8500 likewiſe 80 (the SECOND POSITION) *multiply'd* by 200 (the FIRST ERROR) produces 16000: Now, ſince the ERRORS are BOTH of the SAME *Kind*, *viz.* Since BOTH are EXCESSES, the DIFFERENCE of *theſe* two PRODUCTS, *i. e.* 7500, *divided* by 100 (the DIFFERENCE of the ERRORS) will *quote* 75 for the *true* Number of *Days* that he WORK'D; conſequently he was IDLE 15 *Days*. For,

75 times 12 *d.* is = 900 *d.* (being *what* he *earn'd*) and 25 times 8 is = 120 *d.* (being his *Mulct*) and 900 *d.* — 120 *d.* = 780 *d.* = 3 *l.* 5 *s.* being *what* he *receiv'd*.

The *foregoing* OPERATION is cuſtomarily *perform'd* according to the *following* SCHEME.



Again,

If the *two falſe* POSITIONS were 50 and 60, then the FIRST ERROR would be 500 — and the OTHER 300 — BOTH of the SAME *Kind*, *viz.* *both* DEFECTS.

Here

Here alſo the DIFFERENCE of the PRODUCTS muſt be *divided* by the DIFFERENCE of the ERRORS.

Place, therefore, *theſe* POSITIONS and ERRORS, as *thoſe* above, and *work* according to the RULE, thus,

Firſt Poſition = 50 — 60 = the *ſecond Poſition*.

Firſt Err. = 500 — 300 — = the *ſec. Error*.

60 — 50

One Prod. = 30000 — 15000 = the *other Product*.

— 15000

200)150|00(75 The *Anſwer* as before.

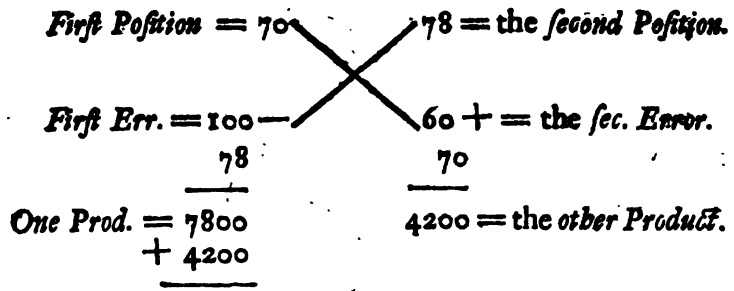
Firſt Error = 500
— *Second Error* = 300

$\frac{1}{2}$ of *Errors* = 200

And again,

If the *two falſe* POSITIONS were 70 and 78, you'd find the FIRST ERROR to be 100 —, and the OTHER to be 60 +; here then the ERRORS are of *different Kinds*, the *firſt* a DEFECT, and the *ſecond* an EXCESS.

Therefore when the POSITIONS and ERRORS are *properly plac'd*, we muſt, according to the RULE, *proceed* and *divide* the SUM of the PRODUCTS, by the SUM of the ERRORS, thus,



16|0)1200|0(75 The *Answer* as above.
0

First Error = 100
+ *Second Error* = 60

Σ of *Errors* = 160

Note, that ALL PROPOSITIONS *resolvable* by SINGLE *Position*, *may* be *resolv'd* by *Position* DOUBLE: And whether the SOLUTION be by ONE or TWO *false Positions*, it is no matter *how far* from the TRUE *Number* or *Answer* requir'd, the FALSE *Number* or *Numbers* are taken.

However, *one Number* may, according to the *Nature* of the PROPOSITION, be *more convenient* for its RESOLUTION than *another*.

All which may be seen by the several Solutions of the first of the Questions following.

QUESTIONS.

1. Suppose there be a STATUE that, with the *Water* issuing out of its *Eyes*, can fill a *Receiver* in 4 Hours; and with the *Water* issuing from its *Ears*, can fill the *same* in 5 Hours; and with *that* flowing from its *Mouth*, the *Receiver* would be fill'd in 20 Hours: In *how many* Hours

Hours would it be fill'd, by the *Water*'s flowing from the *Eyes*, *Ears*, and *Mouth* together?

Anſwer. 2 Hours;

As the *Learner* will *find*, by *obſerving what follows*: where he'll ſee the *Queſtion*, for his farther *Help*, throughly *explain'd*, and ſome uſeful Obſervations made as we go along.

Firſt, then,

Since by the CONDITIONS of this QUESTION, it is *apparent*, that the *Number* of *Hours ſought* is *leſs* than FOUR, let us SUPPOSE *it* to be ONE, in *which Time* the *Water flowing* from the *Eyes*, would (by the PROPOSITION) *fill* $\frac{1}{4}$ of the *Receiver*, and the *Water* flowing from the *Ears*, would *fill* $\frac{1}{7}$ of *it*, and the *Water flowing* from the *Mouth*, would *fill* $\frac{1}{20}$ of *it*; ſo that in ONE *Hour* (the *ſuppoſ'd Time*) the *Eyes*, *Ears*, and *Mouth together*, would *fill* $\frac{1}{4}$ of the *Receiver* $+ \frac{1}{7}$ of *it* $+ \frac{1}{20}$ of *it*. *Which Parts* added together, *ſhould* make 1, or juſt *once* the *Capacity* of the *Receiver*, if the SUPPOS'D *Time* (1 *Hour*) be the TRUE *Time* REQUIR'D: But *theſe Parts*, before they *can* be *added* together, *muſt* be *reduc'd* into *one Name* or *Denomination*, and then $\frac{1}{4}$ would become $\frac{5}{20}$, and $\frac{1}{7}$ would become $\frac{4}{20}$; now $\frac{5}{20}$ and $\frac{4}{20}$ and $\frac{1}{20}$, *added* together, make $\frac{10}{20} = \frac{1}{2}$: And thus we find, that the *Eyes*, *Ears*, and *Mouth together*, FILL *but* $\frac{1}{2}$ of the *Receiver* in an HOUR: Wherefore,

By the RULE of PROPORTION,

If $\frac{1}{2}$ the *Receiver* be *fill'd* in ONE *Hour*, the *Whole* will be *fill'd* in TWO *Hours*, *which* is the TRUE *Time* REQUIR'D: But,

Upon *this* SUPPOSITION of 1 *Hour* to be the TIME in *which* the *Eyes*, *Ears*, and *Mouth together*, would *fill* the

the *Receiver*, the *Learner* may *observe*, that we are unavoidably *encumber'd* with FRACTIONAL *Parts* in our OPERATION, *which Inconvenience* we ſhould, if poſſible, always *avoid*: Here then,

Since it's no matter *how far off* from the *Number required*, the FALSE *Number* be *taken* (as was obſerv'd *Page* 172) let us take 20 *Hours* for the TIME *wherein* the *Receiver* would be *fill'd* by the *Eyes*, *Ears*, and *Mouth together*, *which* is a *Number* that *any one* of tolerable *Judgment* may, at firſt Sight, deſcry to be a very *convenient one* (tho' notoriouſly *falſe*) for the SOLUTION of *this* QUESTION; For,

It's very evident, the *Receiver* will be *fill'd* by the *Eyes* 5 times in 20 *Hours*; by the *Ears* 4 times in 20 *Hours*; and by the *Mouth once* in 20 *Hours*; ſo that in 20 *Hours* (the SUPPOS'D *Number*) the *Eyes*, *Ears*, and *Mouth together*, will *fill* the *Receiver* 10 times, *i. e.* 5 times + 4 times + 1 time: Wherefore,

By the RULE of PROPORTION,

If the *Receiver* be *fill'd* 10 times in 20 *Hours*, it *muſt* be *fill'd* juſt *once* in 2 *Hours*, the TRUE *Number* SOUGHT.

From *which* TWO *Solutions* of *one* and the *ſame* PROPOSITION, it appears (as I have above *noted*) that *one Number* may be, according to the *Nature* of the PROPOSITION, *more convenient* for its RESOLUTION than *another*: For, I believe, *every* one will grant that the INTEGERS 5, 4, and 1, may be *more readily added* together, than the FRACTIONS $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{20}$ can be, and therefore the *falſe ſuppos'd Number* 20, is a *Number more convenient* for the *anſwering* this QUESTION, than the *other falſe ſuppos'd Number* 1 is.

I ſhall

I shall now give a SOLUTION of this QUESTION by POSITION DOUBLE, thus,

First Position = 20 — 40 = the *second Position.*

First Error = 9 + — 19 + = the *second Error.*

40 — 20

One Prod. = 360 — 380 = the *other Product.*

— 360

First Error = 9

Second Error = 19

χ of *Errors* = 10 — 10) 20(2 *Hours*, the *Answer as before.*

II. A *Gamester* at PLAY, *loſt Half* the *Number* of *Crowns* he had in his *Pocket*; and a *Quarter* of *them* that he had *left*, he gave to the *Poor*; and then he had 9 *Crowns remaining*; *how many Crowns* had he *at firſt*?

Anſwer. 24. For,

Let 16 be the SUPPOSITION, then he *loſt* 8 *Crowns*, gave 2 to the *Poor*, and had 6 *remaining*: Then,

By the RULE of THREE,

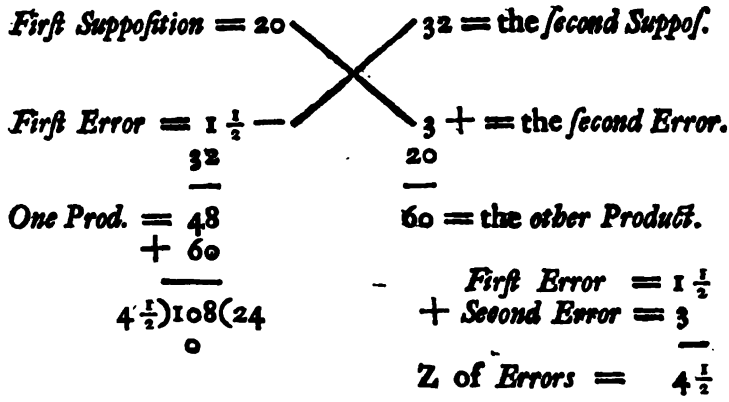
6 : 16 :: 9 : 24

9

6)144(24 *Crowns* the *true Anſwer.*

Again,

Again, by DOUBLE POSITION,



III. A *General* fighting a *Battle*, had the *third Part* of his ARMY *ſlain*; a *fifth Part taken Priſoners*; and the *reſt (viz.)* 7000, we *put* to *Flight*; *what* was the *Number* of the ARMY?

Anſwer. 15000. For,

Suppoſe 300 (if you will) to be the ARMY; a *third* of *which* is 100, a *fifth* 60, and 140 *remain*: Whence,

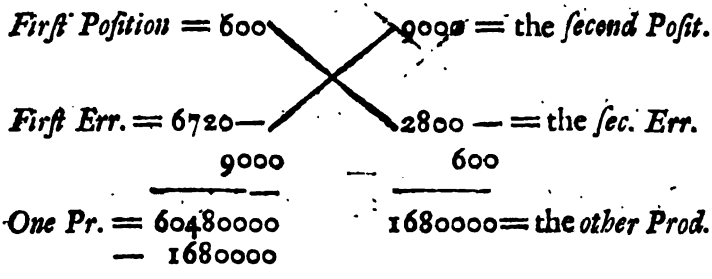
By the RULE of PROPORTION,

140 : 300 :: 7000 : 15000
7000

14|0)21000|0(15000 = the ARMY.
0

Or

Or *thus*, by DOUBLE POSITION;



First Error = 6700
— *Second Error* = 2800
χ of *Errors* = 3900

IV. Let a HOGSHEAD be *fill'd* with TWO *Sorts* of WINE; the ONE worth 6 *s.* and the OTHER 5 *s.* a GALLON; *I demand* HOW MUCH of EACH *Sort* of *Wine* the HOGSHEAD was *fill'd* with, it being (when full) worth 16 *l.* 14 *s.*?

Answer. $\left\{\begin{matrix}19\\44\end{matrix}\right\}$ GALLONS at $\left\{\begin{matrix}6\,s.\\5\,s.\end{matrix}\right.$

See the OPERATION.

First Position = 23 — 20 = the *ſecond Poſition.*

First Error = 4 + — 1 + = the *ſecond Error.*

20 — 23

One Prod. = 80 — 23 = the *other Product.*

— 23

3)57(19 *Gallons* at 6 *s.*
0 44 *Gallons* at 5 *s.*

63 = the HOGSHEAD.

First Error = 4
— *Second Error* = 1

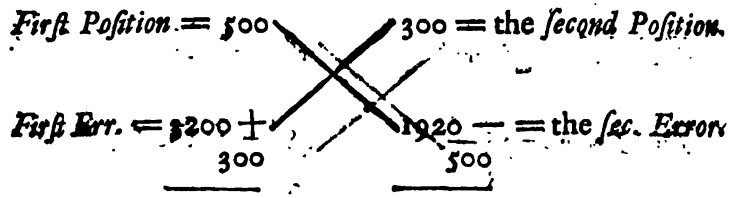
χ of *Errors* = 3

V. A *Gentleman* would *pay* a DEBT of 480 *l.* with GUINEAS (each 21 *s.*) and BROAD-PIECES (each 23 *s.*) without making uſe of *any other* PIECES of *Money*; and that the *Number* of GUINEAS that he *paid*, ſhould be 5 times the *Number* of BROAD-PIECES; *how many* then of EACH *Sort* muſt he *pay*?

Anſwer. { 375 GUINEAS.
75 BROAD-PIECES.

See

See the WORK.



512|0)1920000|0(375 GUINEAS.
3840 75 BROAD-PIECES.
2560
0

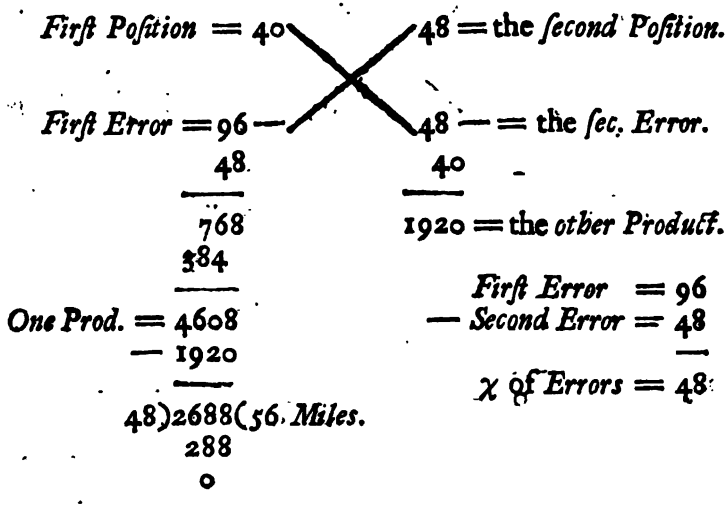
First Error = 3200
+ *Second Error* = 1920
χ of *Errors* = 5120

VI. Suppoſe Two SHIPS under the *ſame* MERIDIAN, ONE in the *Latitude* of 50 *Degrees* 10 *Minutes North*, and the OTHER in the *Latitude* of 45 *Degrees North*, ſet *Sail* on the *ſame Day*, and the *Northermoſt* SHIP *ſails* 12 *Miles* a *Day* MORE than the *other*, *after* 3 *Days Sailing*, without *departing* from the MERIDIAN, they MEET *one another*; HOW MANY *Miles* doth EACH SHIP *ſail* in a *Day*? and *what Latitude* are they *in*, when they *meet*?

Anſwer. { The *Northermoſt ſails* 56 *Miles* a *Day*.
The *Other* 44.

And when they *meet*, they are in the *Latitude* 47 *Degrees* 12 *Minutes Northerly*.

See the WORK.



I shall now insert a *few more* QUESTIONS for the *Learner's farther* PRACTICE, and then proceed to the RULE call'd PRACTICE.

QUESTIONS.

I. What is the NUMBER that the *given* NUMBERS 14 and 19 muſt be ſeverally *multiply'd* by, *ſo* that the DIFFERENCE between the *two* PRODUCTS may be 60?

Anſwer. 12.

II. A *Banker* hath a *Parcel* of GUINEAS in *three ſeveral* BAGS, *number'd* 1, 2, 3. In *thoſe number'd* 1 and 2 *together*, are 290 GUINEAS: In *thoſe number'd* 2 and 3 *together*, are 360 GUINEAS: And in *thoſe number'd* 3 and 1 *together*, are 350 GUINEAS: *How many* GUINEAS are there in EACH BAG?

Anſw.

Anſw. In *that number'd* { 1 / 2 / 3 } are { 140 / 150 / 210 } GUINEAS.

III. A *Gentleman*, having an ESTATE of 1560 *l.* a *Year*, would *lay up* YEARLY *above* 600 *l. viz. So much above* 600 *l.* as he *expends* in 8 *Weeks*; *how much* then may he *ſpend* a WEEK?

Anſwer. 16 *l.*

VI. The AGES of the FATHER, MOTHER, and SON, make *together* 100 *Years*; but the FATHER's and SON's *Years together*, make 24 *Years* MORE than the MOTHER's; and the MOTHER's and SON's *together*, make 8 MORE *than* the FATHER's; *what's* the AGE of EACH?

Anſwer. The { FATHER's / MOTHER's / SON's } AGE { 46 / 38 / 16 } *Years.*

V. A *Vintner* mixes 3 *Gallons* of CANARY with 5 of SHERRY, and being ſo *mix'd*, a *Gallon* is *worth* 6 *s.* 9 *d.* but, had he *put together as much* of ONE *Sort*, as of the OTHER, a *Gallon* would *then* have been *worth* 7 *s.* The PRICE of EACH *Wine per Gallon* is DEMANDED?

Anſwer. The { CANARY / SHERRY } is { 8 *s.* / 6 *s.* } a *Gallon.*

VI. Two *Merchants*, A and B, go *Partners* in a certain ADVENTURE, and SUCH is *each Man's* SHARE, that if to the SUM of *them* you *add* 160 *l.* the WHOLE will be *double* A's SHARE; but, if from the DIFFERENCE of *their* SHARES be taken away 60 *l.* the REMAINDER will be *half* B's SHARE: *what's* EACH *Man's* SHARE?

Anſwer.

Answer. { A's / B's } SHARE is { 360 / 200 } *Pounds.*

When I come to treat of SPECIOUS ARITHMETIC, or ALGEBRA, I shall make it evidently appear, that *Questions* of this Kind, are far more *expeditiously* solv'd ALGEBRAICALLY, than by *this* RULE of FALSE; and farther, that ALL QUESTIONS *capable* of SOLUTION, may be *answer'd* by ALGEBRA, when *but* a FEW, in *Comparison*, *can* be *solv'd* by *this* RULE of FALSE (*whether* SINGLE or DOUBLE).

See more *Questions* in the APPENDIX.

I shall now *proceed* to CHAP. XVI. which treats of the RULE commonly call'd PRACTICE.

CHAP.

CHAP. XVI.

THE RULE call'd PRACTICE, is *nothing elſe* but a COMPENDIUM or CONTRACTION of the *common* RULE of PROPORTION, when the FIRST TERM is 1, and is *ſo call'd, either* from the *general* USE and PRACTICE of it, *or* from its UTILITY and READINESS in the *Practice* and *Exerciſe* of TRADE: For,

By *this practical* RULE, the VALUE of *any Quantity* of GOODS or MERCHANDIZE, may be *more* EXPEDITIOUSLY *computed*, than it *can be* by the RULE of THREE, *when* the FIRST *Term* is an UNIT.

I don't think it *neceſſary* to *burthen* the *Learner* with ALL the *ſhort* RULES *whereby* DIFFERENT *Queſtions* may be *wrought*, it being morally *impoſſible* for *any one* to *retain* in his MEMORY, *ſo many* as MIGHT be *laid down*, but ſhall *inſtruct* him in the PLAIN and USUAL *Ways* of OPERATION by ALIQUOT *Parts*: And therefore,

Here I ſhall *previouſly inſert* a TABLE, conſiſting of the ALIQUOT *Parts* of a POUND, of a SHILLING, and alſo of an HUNDRED-WEIGHT.

N. B. The ALIQUOT *Part* of *any* NUMBER, is a *leſſer* NUMBER that * MEASURES a *greater*, or *that* is *contain'd* † a *certain Number of Times exactly*: As thus,

* *Vide Euclid. Def. 3. Lib. 7.*
† *Aliquoties.*

4 is

4 is an ALIQUOT *Part* of 12, becauſe it MEASURES 12 by 3, *i. e.* it is *contain'd* in 12 *exactly* 3 *times*; for 3 times 4 is 12:

But 5 is *not* an ALIQUOT *Part* of 12; for TWICE 5 is *leſs* than 12, and THREE *times* 5 is *more* than 12. Here follows

A TABLE of ALIQUOT *Parts*.

Of a POUND *Sterl.*			Of a SHILL.			Of an HUND. *Weight.*		
s.	*d.*		*d.*			*Q.*	*P.*	
10	0		6		$\frac{1}{2}$	2 or	56	$\frac{1}{2}$
6	8				$\frac{1}{3}$			
5	0		4		$\frac{1}{4}$			
4	0	*is the*	3	*is the*	$\frac{1}{5}$	1 or	28	$\frac{1}{4}$
3	4				$\frac{1}{6}$			*is the*
2	6		2		$\frac{1}{8}$			
2	0		$1\frac{1}{2}$		$\frac{1}{10}$		16	$\frac{1}{7}$
1	8				$\frac{1}{12}$			
1	0		1		$\frac{1}{20}$		14	$\frac{1}{8}$

The USE of this TABLE will appear in the SOLUTION of the *following* EXAMPLES.

And in order thereto, the *Learner* may *obſerve* the following

General DIRECTIONS, or RULES.

If the *given* PRICE (of any *given* QUANTITY) be an ALIQUOT *Part* of a SHILLING, you may DIVIDE the *given* QUANTITY or NUMBER, whoſe PRICE is *requir'd* by the DENOMINATOR or NAME of *ſuch* ALIQUOT *Part*, and the QUOTIENT will be in SHILLINGS, or in SHILLINGS

SHILLINGS, and *known* PARTS of a SHILLING, the PRICE *requir'd*; *which* SHILLINGS may (if there's Occaſion) be *reduc'd* to POUNDS by *dividing* by 20.

II. But when the *given* PRICE of 1 (of any *given* QUANTITY) is *not* ITSELF an ALIQUOT *Part* of a SHILLING, but conſiſts of *ſeveral* ALIQUOT *Parts* of a SHILLING, you may DIVIDE the *given* NUMBER, whoſe PRICE is *requir'd*, by the DENOMINATORS, or NAMES of *ſuch* ALIQUOT *Parts ſeverally*; and the SUM of the *ſeveral* QUOTIENTS will be in SHILLINGS, or in SHILLINGS, and *known* PARTS of a SHILLING, the PRICE *requir'd*. Again,

III. If the *given* PRICE of 1 (of any *given* QUANTITY) be an ALIQUOT *Part* of a POUND, you may DIVIDE the *given* NUMBER, whoſe PRICE is *requir'd*, by the DENOMINATOR or NAME of *ſuch* ALIQUOT *Part*, and the QUOTIENT will be the PRICE *ſought*, in POUNDS, or in POUNDS, and *known Parts* of a POUND. But,

IV. If the *given* PRICE of 1 (of any *given* QUANTITY) is not *exactly* an ALIQUOT *Part* of a POUND, but conſiſting of *ſeveral* ALIQUOT *Parts* of a POUND, you may DIVIDE the *given* NUMBER, whoſe PRICE is *requir'd*, by the *reſpective* DENOMINATORS or NAMES of *ſuch* ALIQUOT *Parts*, and the SUM of the *ſeveral* QUOTIENTS will be the VALUE *ſought*, *either* in POUNDS only, *or* in POUNDS, and ſome *known* PARTS of a POUND.

Theſe *general* DIRECTIONS, or RULES, may be *illuſtrated* by the OPERATIONS following.

EXAMPLE I.

What will 1440 *Ounces* come to, at 1 *d.* an *Ounce?*

Here 1 *d.* the *given* PRICE of the *given* NUMBER of (1440) *Ounces*, whoſe PRICE is *ſought*, is the $\frac{1}{12}$ *Part* of a SHILLING, as may be ſeen in the TABLE.

Divide therefore the *given* NUMBER (1440) by (12) the DENOMINATOR or NAME of *ſuch* ALIQUOT *Part* of a SHILLING, and the QUOTIENT will be 120 *s.* the PRICE *requir'd*, by RULE I. Which,

By *dividing* by 20, may be *reduc'd* into POUNDS; and thus we find, that the 1440 *Ounces*, at 1 *d.* an *Ounce*, amount to 6 *l.*

The *Manner* I ſhall *chooſe* to *make uſe* of in theſe *practical* OPERATIONS, is *this*,

By drawing *Lines*, I make *two* COLUMNS; in *one* of *them I place* the ALIQUOT *Parts*, *which I make uſe of*; and in the *other*, *I place* the DENOMINATORS or NAMES of *ſuch* PARTS, *viz.* my DIVISORS: For,

The ALIQUOT *Parts* being *ſo plac'd*, I can readily ſee *whether* they *do* (as they always *muſt*) *amount* to juſt the *given* RATE, thus,

d.	*Oz.*	*d.*
1	$\frac{1}{12}$	1440 at 1 an *Ounce*.
2|0)	12|0	*Shillings*; the *Anſwer* by RULE I.
		6 *l.*

EX-

1

EXAMPLE II.

What will 1455 *Ounces* come to, at 1 *d.* an *Ounce?*

d.		
1	$\frac{1}{12}$	1455 *Ounces* at 1 *d.*
20)	121	$\frac{3}{12}$ *Shill.* (= 121 *s.* 3 *d.*) by RULE I.
	6 *l.* 1 *s.* 3 *d.*	

Here it may be *obſerv'd*, that after *dividing* by 12, there *remains* $\frac{3}{12}$ of a SHILLING, becauſe the QUOTIENT (121) is SHILLINGS, by RULE I.

And, in like manner, *when* the QUOTIENT is POUNDS, the REMAINDER is *Part* of a POUND: Or,

Let the QUOTIENT be of *what* DENOMINATION *it will*, the REMAINDER (if *any*) will *always* be *Part* of *one* or the *ſame* DENOMINATION.

Now (*whatever* the DIVISOR be) to find the VALUE of the REMAINDERS, we muſt *proceed* as follows:

V. If the QUOTIENT be POUNDS, and there be a REMAINDER, *multiply* it *always* by 20, and *divide* by the *ſame* DIVISOR, and the QUOTIENT will be the VALUE of *ſuch* REMAINDER in SHILLINGS; and, if *any thing* ſtill *remains*, the QUOTIENT being SHILLINGS, it muſt be *multiply'd* by 12, and the PRODUCT *divided* by the *ſame* DIVISOR, and the QUOTIENT will be PENCE; and laſtly, *If any thing* then *remains*, it muſt (being *Part* of a PENNY) be *multiply'd* by 4, and the PRODUCT *divided* ſtill by the *ſame* DIVISOR, and the QUOTIENT will be FARTHINGS. And *how* thus to *multiply* and *divide* (as Occaſion requires) in *this practical*

RULE, the *Learner* may eaſily *inform* himſelf, by *examining* and *obſerving* the METHOD *us'd* in ſome of the *following* OPERATIONS.

EXAMPLE III.

What will 552 *Yards* come to, at 2 *d.* $\frac{1}{2}$ a *Yard*?

d.			
1 $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	552 at 2 *d.* $\frac{1}{2}$
1	$\frac{1}{2}$	1 2	
			69
			46
			2|0)11|5 The *Anſwer* in *Shillings*, by RULE II.
			l. 5 : 15

Here the *given* PRICE (2 *d.* $\frac{1}{2}$) is *not itſelf* an ALIQUOT *Part* of a SHILLING, but it conſiſts of ALIQUOT *Parts* of a SHILLING, as 1 *d.* $\frac{1}{2}$ and 1 *d.* are.

Now 552 *Yards*, at 1 *d.* $\frac{1}{2}$ a *Yard*, amount to 69 *s.* and at 1 *d.* a *Yard*, they amount to 46 *s.* *which two* SUMS *added together*, amount to 115 *s.* the PRICE *requir'd*, by RULE II.

EXAMPLE IV.

What will 686 *Ells* amount to, at 5 *d.* an *Ell*?

d.			
3	$\frac{1}{4}$	$\frac{1}{4}$	686 at 5 *d.*
2	$\frac{1}{6}$	$\frac{1}{6}$	
			171 : 6
			114 : 4
			2|0)28|5 : 10 = the *Anſwer* in *Shillings* and *Pence*, by RULES II. and V.
			l. 14 : 5 : 10

EX-

EXAMPLE V.

What will 687 *Ells* amount to, at 7 *d.* $\frac{1}{2}$ an *Ell*?

d.		
6	$\frac{1}{2}$	687 at 7 *d.* $\frac{1}{2}$
1 $\frac{1}{2}$	$\frac{1}{8}$	
		343 : 6
		85 : 10 $\frac{1}{2}$

2|0)42|9 : 4 $\frac{1}{2}$ = the *Anſwer* in *Shillings*, *Pence*, and *Farthings*, by RULES II. and V.

l. 21 : 9 : 4 $\frac{1}{2}$

And thus, by the *foregoing* EXAMPLES, have I *illuſtrated* the *firſt* and *ſecond* RULES.

I ſhall go on in *Order*, and *apply* the *third* and *fourth* RULES to PRACTICE, by *ſolving ſome other* EXAMPLES *proper* for *that Purpoſe*.

EXAMPLE VI.

What will 468 *Quarts* come to, at 1 *s.* 8 *d.* a *Quart*?

s.	*d.*	
1 :	8	$\frac{1}{1\ 2}$ 468 at 1 *s.* 8 *d.*

l. 39 The *Anſwer* in *Pounds*, by RULE III.

EX-

EXAMPLE VII.

What will 861 *Gallons* come to, at 3 *s.* 4 *d.* a *Gallon?*

s.	*d.*		
3 :	4	$\frac{1}{6}$	861 at 3 *s.* 4 *d.*

l. 143 : 10 *s.* The *Answer* in *Pounds* and *Shillings*, by RULES III. and V.

EXAMPLE VIII.

What will 452 *Gallons* amount to, at 6 *s.* 8 *d.* a *Gallon?*

s.	*d.*		
6 :	8	$\frac{1}{3}$	452 at 6 *s.* 8 *d.*

l. 150 : 13 : 4 The *Answer* in *Pounds*, *Shillings*, and *Pence*, by RULES III. & V.

EXAMPLE IX.

What will 672 *square Feet* come to, at 4 *s.* 2 *d.* a *Foot?*

s.	*d.*		
2 :	6	$\frac{1}{8}$	672 at 4 *s.* 2 *d.*
1 :	8	$\frac{1}{12}$	84
			56

l. 140 The *Answer* in *Pounds*, by RULE IV.

EX-

EXAMPLE X.

What will 777 *ſquare Yards* come to, at 5 *s.* 4 *d.* a *Yard?*

s.	*d.*		
3	4	$\frac{1}{2}$	777 at 5 *s.* 4 *d.*
2	0	$\frac{1}{10}$	
			129 : 10
			77 : 14
			l. 207 : 4

The *Anſwer* in *Pounds* & *Shillings*, by RULES IV. and V.

EXAMPLE XI.

What will 247 *Rods* amount to, at 8 *s.* 4 *d.* a *Rod?*

s.	*d.*		
5	0	$\frac{1}{4}$	247 at 8 *s.* 4 *d.*
3	4	$\frac{1}{6}$	
			61 : 15
			41 : 3 : 4
			l. 102 : 18 : 4

The *Anſwer* in *Pounds*, *Shill.* and *Pence*, by RULES IV. and V.

The ſeveral *foregoing* OPERATIONS do (I think) ſufficiently ſhew the *Uſe* of the RULES that are hitherto *laid down.*

As to a DEMONSTRATION of the *Truth* of *theſe* RULES, there *needs none*; it being *evident* enough from the *Nature* of PROPORTION, *which* hath been fully *handled* under *its proper* HEAD: For,

'Tis

'Tis plain by *common* PROPORTION, I ſay, that if 1 *Ounce* be rated at 1 *s.* 1440 *Ounces* will amount to 1440 *s.* that is, *any given* QUANTITY *whatſoever*, and *its* VALUE in *Shillings*, will *always* be the SAME in NUMBER, when 1 of *that given* QUANTITY is rated at 1 *Shilling*.

But if 1 *Ounce* be rated at 6 *d.* or the *Half* of a *Shilling*, then 1440 *Ounces*, will amount to 1440 *Six-Pences*, or *half* 1440 *s. which* the *given* QUANTITY (1440 *Ounces*) *halv'd*, or *divided by* 2, will *quote*, as RULE I. directs.

So likewiſe, if 1 *Ounce* be rated at 1 *Penny* (as in *Example* I.) or the *twelfth* Part of a *Shilling*, then 1440 *Ounces* will amount to 1440 *d.* or the *twelfth* Part of 1440 *s. which* the *given* QUANTITY (1440 *Ounces*) *divided* by 12 will give.

The *Caſe* is the very *ſame* (*mutatis mutandis*) in reſpect of *dividing* the *given* QUANTITY by ANY *given* ALIQUOT *Part* of a POUND, to get the PRICE of *ſuch* QUANTITY in POUNDS.

Beſides *thoſe* above offer'd, there are *other practical* RULES may be given, *which*, according to the *given Rate*, may ſometimes be more *conciſe* and *convenient*, than *thoſe* are; as by taking an ALIQUOT *Part* of an ALIQUOT already taken, inſtead of taking ALL the *Parts* out of a POUND, or out of a SHILLING, and the like, as I ſhall *ſhew* in *ſolving* ſome EXAMPLES following for *that Purpoſe*.

EX-

EXAMPLE XII.

What will 346 *Yards* come to, at 9 *d.* a *Yard*?

Here, inſtead of taking 6 *d.* the *Half* of a *Shilling*, and then 3 *d.* the *Fourth* of a *Shilling*, as by RULE II. I *may* take the *Parts* thus, *viz.* 6 *d.* the $\frac{1}{2}$ of a *Shilling*, and then I *may* uſe 3 *d.* as the $\frac{1}{2}$ of 6 *d.* inſtead of the $\frac{1}{4}$ of a *Shilling* (if I pleaſe); for the $\frac{1}{2}$ of 6 *d.* is all one as the $\frac{1}{4}$ of a *Shilling*, thus,

d.		*d.*	
6 $\frac{1}{2}$	346 at 9 *d.*	6 $\frac{1}{2}$	346 at 9 *d.*
3 $\frac{1}{2}$		3 $\frac{1}{4}$	
	173		173
	86 : 6		86 : 6
2|0)	25|9 : 6	2|0)	25|9 : 6
	l. 12 : 19 : 6		*l.* 12 : 19 : 6

EXAMPLE XIII.

What will 554 *Yards* come to, at 10 *d.* $\frac{1}{2}$ a *Yard*?

d.		*d.*	
6	$\frac{1}{2}$ 554 at 10 *d.* $\frac{1}{2}$	6	$\frac{1}{2}$ 554 at 10 *d.* $\frac{1}{2}$
3	$\frac{1}{2}$	3	$\frac{1}{4}$
1 $\frac{1}{2}$	$\frac{1}{2}$ 277	1 $\frac{1}{2}$	$\frac{1}{8}$ 277
	138 : 6		138 : 6
	69 : 3		69 : 3
2|0)	48|4 : 9	2|0)	48|4 : 9
	l. 24 : 4 : 9		*l.* 24 : 4 : 9

EXAMPLE XIV.

What will 418 *Yards* amount to, at 11 *d.* a *Yard?*

d.			*d.*		
6	$\frac{1}{2}\frac{1}{2}\frac{1}{3}$	418 at 11 *d.*	6	$\frac{1}{2}\frac{1}{2}\frac{1}{12}$	418 at 11 *d.*
3			4		
2		209	1		209
		104 : 6			139 : 4
		69 : 8			34 : 10
2|0)		38|3 : 2	2|0)		38|3 : 2
		l. 19 : 3 : 2			*l.* 19 : 3 : 2

I *may* (if I think fit) take *Parts* out of *Parts*, inſtead of taking them ALL out of a POUND, according to the 4th RULE; as in the *next* EXAMPLES.

EXAMPLE XV.

What will 483 *Yards* amount to, at 15 *s.* a *Yard?*

s.			*s.*		
10	$\frac{1}{2}\frac{1}{2}\frac{1}{5}$	483 at 15 *s.*	10	$\frac{1}{2}\frac{1}{2}\frac{1}{4}$	483 at 15 *s.*
5			5		
		241 : 10			241 : 10
		120 : 15			120 : 15
		l. 362 : 5			*l.* 362 : 5

EX-

EXAMPLE XVI.

What will 593 *Yards* amount to, at 16 *s.* a *Yard*?

s.		
10	½ ½ ½ ½	593 at 16 *s.*
5		
1	⅕	296 : 10
		148 : 5
		29 : 13
		l. 474 : 8

s.		
10	½ ½ ½ ½	593 at 16 *s.*
4	⅕ ⅕	
2	1/10	296 : 10
		118 : 12
		59 : 6
		l. 474 : 8

EXAMPLE XVII.

What will 199 *Yards* amount to, at 17 *s.* 6 *d.* a *Yard*?

s.	*d.*		
10	0	½ ½ ½ ½	199 at 17 *s.* 6 *d.*
5	0		
2	6	½ ½ ½	99 : 10
			49 : 15
			24 : 17 : 6
			l. 174 : 2 : 6

s.	*d.*		
10	0	½ ½ ¼ ⅛	199 at 17 *s.* 6 *d.*
5	0		
2	6		99 : 10
			49 : 15
			24 : 17 : 6
			l. 174 : 2 : 6

EXAMPLE XVIII.

What will 234 *Yards* amount to, at 18 *s.* a *Yard*?

s.	*d.*		
10	0	½ ½ ½ ½	234 at 18 *s.*
5	0	½ ½ ½	
2	6	½	117
	6	⅕ ⅕	58 : 10
			29 : 5
			5 : 17
			l. 210 : 12

s.		
10	½ ½ ½ ½	234 at 18 *s.*
4	⅕ ⅕ ⅕	
4	⅕	117
		46 : 16
		46 : 16
		l. 210 : 12

EXAMPLE XIX.

What will 355 *Yards* amount to, at 18 *s.* 4 *d.* a *Yard?*

s.	*d.*		
10	0	$\frac{1}{2}$	355 at 18 *s.* 4 *d.*
5	0	$\frac{1}{2}$	
3	4	$\frac{1}{3}$	177 : 10
			88 : 15
			59 : 3 : 4
			l. 325 : 8 : 4

s.	*d.*		
10	0	$\frac{1}{2}$	355 at 18 *s.* 4 *d.*
5	0	$\frac{1}{2}$	
3	4	$\frac{1}{6}$	177 : 10
			88 : 15
			59 : 3 : 4
			l. 325 : 8 : 4

EXAMPLE XX.

What will 380 *Yards* amount to, at 19 *s.* a *Yard?*

s.		
10	$\frac{1}{2}$	380 at 19 *s.*
5	$\frac{1}{2}$	
4	$\frac{1}{5}$	190
		95
		76
		l. 361

s.		
10	$\frac{1}{2}$	380 at 19 *s.*
5	$\frac{1}{2}$	
4	$\frac{1}{5}$	190
		95
		76
		l. 361

In the 9 *laſt* EXAMPLES, I have given the *Learner* the OPERATIONS *both Ways*, *viz.* as well by the 2d and 4th RULES, as by taking *one* PART out of *another*; but in *theſe particular* EXAMPLES, there's but *little* Trouble ſav'd, work which of thoſe Ways he will, tho', in *ſome* CASES, there is *more* ſav'd, as may appear by *ſome* of the OPERATIONS to be us'd in *other* EXAMPLES to be *given* ere long.

I ſhall,

I ſhall, in the mean time, ſhew how the 9 *laſt* EXAMPLES (ſince their reſpective *given* RATES are either *below* a SHILLING, by an ALIQUOT *Part* of a *Shilling*, or *below* a POUND, by an ALIQUOT *Part* of a *Pound*) *may* be *work'd* by a WAY ſtill *ſhorter* than *either* of the FORMER, as ALL OTHER EXAMPLES *of the ſame Property* may be.

The METHOD is THIS,

If the *given* RATE be *below* a SHILLING, by an ALIQUOT *Part* of a *Shilling*, *divide* the *given* QUANTITY by the DENOMINATOR of *ſuch* ALIQUOT *Part*, and *ſubtract* the QUOTIENT from the *ſame given* NUMBER, and the REMAINDER will *be* the ANSWER in SHILLINGS; and when the *given* RATE is *below* a POUND, by an ALIQUOT *Part* of a *Pound*, proceed in the *ſame Manner*, and the REMAINDER will be the ANSWER in POUNDS.

To illuſtrate *this*, I'll accordingly *ſolve three* or *four* of the *laſt* EXAMPLES, *viz.* The 13th, 14th, 18th, and 19th, which will be *ſufficient*.

Example XIII.

d.			
1 $\frac{1}{2}$	$\frac{1}{8}$	554 at 10 *d.* $\frac{1}{2}$	
		69 : 3	
2	0	48	4 : 9

l. 24 : 4 : 9

Example XIV.

d.			
1	$\frac{1}{12}$	418 at 11 *d.*	
		34 : 10	
2	0	38	3 : 2

l. 19 : 3 : 2

Example

Example XVIII.

s.		
2	$\frac{1}{10}$	234 at 18 *s.*
		23 : 8
		l. 210 : 12

Example XIX.

s.	*d.*		
1 :	8	$\frac{1}{12}$	355 at 18 *s.* 4 *d.*
			29 : 11 : 8
			l. 325 : 8 : 4

Here the METHOD is very *conciſe*, and very *plain*; for ſince 10 *d.* $\frac{1}{2}$ is *below* a SHILLING by 1 *d.* $\frac{1}{2}$, or the $\frac{1}{8}$ of a SHILLING, it's very evident, that if from the VALUE of the *given* QUANTITY, at 1 *s.* or 12 *d.* a piece, I *ſubtract* the VALUE of the *given* QUANTITY, at 1 *d.* $\frac{1}{2}$ (or the $\frac{1}{8}$ of 12 *d.*) a piece, the REMAINDER will be the VALUE *requir'd* of the *given* QUANTITY, at its *proper* RATE *given*, *viz.* at 10 *d.* $\frac{1}{2}$ a piece:

The SAME *holds good* in ALL CASES of *the like* NATURE.

When the *given* PRICE of 1 (of *any given* QUANTITY) is *between* 1 SHILLING and 1 POUND, it muſt conſiſt

Of { An { EVEN / ODD } } *Number* of SHILLINGS *alone.*
Of { SHILLINGS and PENCE *together*; Or,
Of { SHILLINGS, PENCE, and FARTHINGS, *together.*

In *theſe* CASES then, notwithſtanding the RULES *already exhibited*, the *following ones* may be ſometimes *as convenient as any* of *thoſe*, excepting when the *given* PRICE is juſt *any one* of the ALIQUOT *Parts* of a POUND, as 10 *s.* 6 *s.* 8 *d.* 5 *s.* 4 *s.* *&c.* in the TABLE mention'd, [illegible] ſtraight juſt before.

When

When the *given* PRICE then is an EVEN *Number* of SHILLINGS,

Multiply the *given* QUANTITY (whoſe VALUE is *requir'd*) by *ſuch given* PRICE, and *divide* the PRODUCT by 20, and the QUOTIENT is your ANSWER in POUNDS, the REMAINDER (if any) being SHILLINGS: Or,

Multiply the *given* QUANTITY by HALF *ſuch given* PRICE, and *divide* the PRODUCT by 10, and the QUOTIENT's your ANSWER in POUNDS, remembering here always to DOUBLE the REMAINDER (if any) for SHILLINGS; as in the *following* EXAMPLES.

EXAMPLE XXI.

What will 877 *Ells* amount to, at 8 *s.* an *Ell?*

877	Or thus,	877
8		4
20)7016		10)3508

l. 350 : 16 The *Anſwer.* *l.* 350 : 16 The *Anſw.*

For 'tis the *ſame* thing to *multiply any Number* by 4, and *divide* by 10, as to *multiply* the *ſame Number* by 8, and *divide by* 20; the *Remainder*, after this *Diviſion*, being ſo many *Twentieths* of a POUND, and conſequently *itſelf* ſo many SHILLINGS; but the *Remainder*, after that *Diviſion*, being ſo many *Tenths* of a POUND, or ſo many times TWO *Shillings*; wherefore it muſt be always *doubl'd* to be expreſs'd in SHILLINGS.

EX-

EXAMPLE XXII.

What will 573 *Yards* amount to, at 14 *s.* a *Yard*?

		Or thus	
573			573
14			7
2292			10)4011
573			*l.* 401 : 2 *Answer.*
2|0)802|2			
l. 401 : 2 *Answer.*			

But when the *given* PRICE is an ODD *Number* of SHILLINGS,

Omit the ODD 1, and with the EVEN *Number* of SHILLINGS *remaining*, proceed as I have just before taught, and to your QUOTIENT *add* $\frac{1}{20}$ of the *given* QUANTITY, and the SUM is the *Answer* requir'd; as in the *Examples* following:

EXAMPLE XXIII.

What will 199 *Yards* amount to, at 11 *s.* a *Yard*?

s.			*s.*		
1	$\frac{1}{20}$	199 at 11 *s.*	1	$\frac{1}{20}$	199 at 11 *s.*
		10			5
20)1990			10)995		
99 : 10			99 : 10		
9 : 19			9 : 19		
l. 109 : 9 The *Answer.*			*l.* 109 : 9 The *Answ.*		

EX-

EXAMPLE XXIV.

What will 221 *Yards* amount to, at 13 *s.* a *Yard*?

s.			*s.*		
1 | $\frac{1}{20}$	221 at 13 *s.*	Or thus,	1 | $\frac{1}{20}$	221 at 13 *s.*	
	12			6	
2 | 0	265 | 2		1 | 0	132 | 6	
	132 : 12			132 : 12	
	11 : 1			11 : 1	
l.	143 : 13	The *Answer.*	*l.*	143 : 13	The *Answ.*

EXAMPLE XXV.

What will 337 *Yards* amount to, at 17 *s.* a *Yard*?

s.			*s.*		
1 | $\frac{1}{20}$	337 at 17 *s.*	Or thus,	1 | $\frac{1}{20}$	337 at 17 *s.*	
	16			8	
	2022		1 | 0	269 | 6	
	337			269 : 12	
2 | 0	539 | 2			16 : 17	
	269 : 12		*l.*	286 : 9	The *Answer.*
	16 : 17				
l.	286 : 9				

And when the *given* PRICE conſiſts of *Shillings* and *Pence*; or, *Shillings*, *Pence*, and *Farthings* together,

You *may multiply* the *given* QUANTITY by the *Number* of *Shillings* in the *given* PRICE; and for the *reſt* of the *given* PRICE, work as directed by the 1ſt or 2d, or ſome other of the *forégoing* RULES; then *add* the QUOTIENT, or QUOTIENTS, and PRODUCT *together*, and *divide* the SUM by 20, and your QUESTION is *anſwer'd*.

Take the following *Examples*.

EXAMPLE XXVI.

What will 213 *Pounds Weight* coſt, at 1 *s.* 4 *d.* a *Pound*?

d.

4	$\frac{1}{3}$	213 at 1 *s.* 4 *d.*
		71

2|0)284

l. 14 : 4 The *Anſwer*.

EXAMPLE XXVII.

What will 33$\frac{1}{2}$ *Buſhels* coſt, at 9 *s.* 5 *d.* a *Buſhel*?

d.

3	$\frac{1}{2}$	337 at 9 *s.* 5 *d.*
2	$\frac{1}{6}$	9

3033
84 : 3
56 : 2

2|0)3173 : 5

l. 158 13 : 5 The *Anſwer*.

EX-

EXAMPLE XXVIII.

What will 279 *Groce* come to, at 13 *s.* 1 *d.* $\frac{1}{2}$ a *Groce*?

d.

1 $\frac{1}{2}$	$\frac{1}{8}$	279 at 13 *s.* 1 *d.* $\frac{1}{2}$
		13
		837
		279
		3627
		34 : 10 $\frac{1}{2}$
2	0	366 | 1 : 10 $\frac{1}{2}$

l. 183 : 1 : 10 $\frac{1}{2}$ The *Answer.*

EXAMPLE XXIX.

What will 454 *Quarters* coſt, at 16 *s.* 11 *d.* $\frac{1}{4}$ a *Quarter*?

d.

6	$\frac{1}{2}$	454 at 16 *s.* 11 *d.* $\frac{1}{4}$
3	$\frac{1}{2}$	16
2	$\frac{1}{4}$	2724
$\frac{1}{4}$	$\frac{1}{8}$	454
		7264
		227
		113 : 6
		75 : 8
		9 : 5 $\frac{1}{2}$
2	0	768 | 9 : 7 $\frac{1}{2}$

l. 384 : 9 : 7 $\frac{1}{2}$ The *Answer.*

And now, if the *given* PRICE of 1 (of *any given* QUANTITY) *exceeds* a POUND, you muſt *multiply* the *given* QUANTITY by the *Number* of POUNDS in the *given* PRICE, and the PRODUCT is the ANSWER *requir'd*, if there be *no* SHILLINGS, PENCE, and FARTHINGS in the *given* PRICE:

But if there *be*, work with them as directed by the 3d and 4th, or ſome other of the RULES *above given*; then *add* the QUOTIENT, or QUOTIENTS, to the *former* PRODUCT, and the SUM will be the ANSWER *deſir'd*.

As in the following OPERATIONS.

EXAMPLE XXX.

What will 155 *Chaldrons* come to, at 1 *l.* 6 *s.* a *Chaldron*?

s.		
5	$\frac{1}{4}$	155 at 1 *l.* 6 *s.*
1	$\frac{1}{20}$	38 : 15
		7 : 15
		l. 201 : 10 The *Anſwer.*

EXAMPLE XXXI.

What will 545 *Hogſheads* come to, at 14 *l.* a *Hogſhead*?

545 at 14 *l.*
14
2180
545
l. 7630. The *Anſwer.*

EX-

EXAMPLE XXXII.

What will 483 *Hundred Weight* amount to, 2*l.* 16*s.* 6*d.* a *Hundred Weight*?

s.	*d.*		
10	0	$\frac{1}{2}$	483 at 2*l.* 16*s.* 6*d.*
5	0	$\frac{1}{2}$	2
1	6	$\frac{1}{2}$	966
			241 : 10
			120 : 15
			24 : 3
			12 : 1 : 6
			l. 1364 : 9 : 6 The *Answer.*

If the QUANTITY whose VALUE or PRICE is *sought*, be a MIX'D *Number*, i. e. consisting of a WHOLE *Number*, and a FRACTION, let the VALUE of the INTEGRAL *Part* be first found, by some *one* or *other* of the *precedent* RULES: And,

For the VALUE of the FRACTIONAL *Part*, take the LIKE *Part* of the *given* RATE, and *add* it to the VALUE of the INTEGRAL, and the SUM will be the ANSWER *sought*; as may plainly appear by the *subsequent* EXAMPLES.

EX-

EXAMPLE XXXIII.

What will 187 $\frac{1}{2}$ *Hogsheads* amount to, at 15 *l.* 10 *s.* a *Hogshead?*

```
 s.                                          l.    s.
|10|½|187 ½ at 15l. 10s.   |½|½|15 : 10
      15                                  l. 7 : 15 = the Value of
     ----                                          the ½ Hogshead.
     935
    187
    ----
    2805
      93 : 10
       7 : 15 = the Value of the ½ Hogshead.
    ----------
  l. 2906 :  5 The Answer.
```

EXAMPLE XXXIV.

What 525 $\frac{3}{4}$ *Hundred Weight* come to, at 2 *l.* 18 *s.* 8 *d.* a *Hundred?*

```
 s.  d.                                      l.   s.   d.
|10  0|½|  525 ¾ at l.2 : 18 : 8   |2|½|2 : 18 : 8
|  6  8|⅓|    2                    |1|½|-----------
|  2  0|1/10|-----                        1 :  9 : 4
|      |    |1050                            14 : 8
           262 : 10                     ----------
           175 :                      l. 2 :  4 : 0 = the
            52 : 10                            Price of ¾.
             2 :  4 = the Price of ¾.
          ----------
        l. 1542 :  4 The Answer.
```

EX-

EXAMPLE XXXV.

H. *Q.* *lb.*
What will 325 : 1 : 14 come to, at 5 *l.* 5 *s.* a *Hundred*?

s.	*H.*	*Q.*	*lb.*		*l.*	*s.*	*Q.*	*lb.*		*l.*	*s.*	*d.*
5	$\frac{1}{4}$	325	1	14	at 5	5	1	00	$\frac{1}{4}$	1	6	3
		5						14	$\frac{1}{2}$		13	$1\frac{1}{2}$

1625
81 : 5
1 : 19 : $4\frac{1}{2}$ = the *Value* of 1 *Q.* 14 *lb.*

l. 1 : 19 : $4\frac{1}{2}$ = the *Value* of 1 *Q.* 14 *lb.*

l. 1708 : 4 : $4\frac{1}{2}$ The *Answer.*

N. B. When a *Bag*, *Box*, *Cask*, *Hogſhead*, &c. containing *any Commodity*, is *weigh'd*, the *Weight* of ſuch *Commodity* CONTAINED together with the *Thing* (as *Bag*, *Box*, &c.) CONTAINING the ſame, is call'd the GROSS *Weight*: And,

The *Weight* (or whatever is *allow'd* for the *Weight*) of the *Thing* CONTAINING the *Commodity*, is call'd * TARE, and the *Weight* of the *Commodity* ALONE is call'd the † NET *Weight*.

* *Tarâre, Ital.* to *ſubtract.*
† *Netto, Ital. Pure, Neat,*

I ſhall now conclude *this* RULE, with ſhewing the *Learner* the PRACTICAL Way of finding the TARE by ALIQUOT *Parts* of an *Hundred Weight*, in order to obtain the NET *Weight*:

Which I ſhall do by *ſolving* a few *Examples.*

EX-

EXAMPLE XXXVI.

What is the *Net Weight* of *H.* *Q.* *lb.* 27 : 2 : 16 *Groſs*, when 14 *lb. per Hundred* is allow'd for *Tare*?

lb.	*H.*	*Q.*	*lb.*	
14 1/8	27	2	16	*Groſs*, at 14 *lb. per Hundred.*
	3	1	23	*Tare.*
	24	0	21	*Net Weight.*

It is manifeſt from *common* PROPORTION, that *whatever* PART of 1 *Hundred Weight* the *given* ALLOWANCE for TARE is, the LIKE *Part* of the *given* GROSS *Weight*, is the *whole* TARE that is to be *deducted*: Thus, in the EXAMPLE *above*.

Since the *given* ALLOWANCE (14 *lb. per Hundred*) for TARE is the EIGHTH of an *Hundred Weight*, I *divide* the *given* GROSS *Weight* by 8, and the QUOTIENT is the *whole* TARE, *which* being deducted, the REMAINDER is the NET *Weight*.

But when the *given* ALLOWANCE for TARE is not an ALIQUOT *Part* of an *Hundred Weight*, you *may divide* it into *ſuch* PARTS, *as* may be us'd *ſeverally*, according to the *foregoing* RULES of *working* with PARTS of MONEY, and *proceed* with *them* accordingly; for the RULES hold good, as to the RATIONALE of *them*, let the NAME of the INTEGER be what it will, as MONEY, WEIGHT, or ANY THING ELSE.

This may be *illuſtrated* by the *following* OPERATIONS.

EX-

EXAMPLE XXXVII.

What is the NET-*Weight* of 124 : 3 *Groſs*, when 18 *lb. per Hundred* is *allow'd* for TARE?

lb.		*H.*	*Q.*	*lb.*	
16	$\frac{1}{7}$	124 :	3 :	0	*Groſs*, at 18 *lb. per Hundred.*
2	$\frac{1}{8}$				
		17 :	3 :	8	
		2 :	0 :	25 $\frac{1}{2}$	
		20 :	0 :	5 $\frac{1}{2}$	TARE.
		104 :	2 :	22 $\frac{1}{2}$	NET-*Weight.*

EXAMPLE XXXVIII.

What is the NET-*Weight* of 246 : 1 : 4 *Groſs*, when 21 *lb. per Hundred* is *allow'd* for TARE?

lb.		*H.*	*Q.*	*lb.*	
14	$\frac{1}{7}$	246 :	1 :	4	*Groſs*, at 21 *lb. per Hundred.*
7	$\frac{1}{2}$				
		30 :	3 :	4	
		15 :	1 :	16	
		46 :	0 :	20	TARE.
		200 :	0 :	12	NET-*Weight.*

EXAMPLE XXXIX.

What is the NET-*Weight of* H. 2. lb. 365 : 3 : 14 *Gross, when* 24 *lb. per Hundred is allowed for* TARE?

lb.		H.	Q.	lb.
16	1/2	365	3	14 *Gross, at* 24 *lb. per Hundred?*
8	1/2			
		51	3	26
		25	3	27
		77	3	25 TARE.
		285	3	17 NET-*Weight.*

EXAMPLE XL.

What is the NET-*Weight of* H. 2. 289 : 2 *Gross, allowing for* TARE, 22 *lb. per Hundred?*

lb.		H.	Q.	lb.
16	1/2	289	2	20 *Gross, at* 22 *lb. per Hundred.*
4	1/4			
2	1/2	41	1	12
		10	1	10
		5	0	19
		56	3	13 TARE.
		232	2	15 NET-*Weight.*

EX-

Or thus,

lb.		H.	Q.	lb.	
14	1/8	289	2	00	*Groſs*, at 22 *lb. per Hundred.*
7	2/7				
1	1/7	36	0	21	
		18	0	10 1/2	
		2	2	9 1/2	
		56	3	13	TARE.
		232	2	15	NET *Weight.*

The PROOF of the TRUTH of ALL *ſuch practical* OPERATIONS, is by the *common* RULE of PROPORTION, or by WORKING the SAME *Example* DIFFERENT *Ways*, as is in SOME of the EXAMPLES *before ſhewn.*

I ſhall now inſert a few more *Queſtions* for the *Learner's Exerciſe*, and which ſhall *end this* CHAPTER.

QUESTIONS.

I. *What* will 744 *Yards* amount to, at 9 *d.* a *Yard?*

Anſwer. 27 *l.* 18 *s.*

II. *What* will 284 *Yards* amount to, at 1 *s.* 3 *d.* a *Yard?*

Anſwer. 17 *l.* 15 *s.*

III. *What* will 238 *Yards* amount to, at 2 *s.* 7 *d.* a *Yard?*

Anſwer. 30 *l.* 14 *s.* 10 *d.*

IV. *What* will 331 *Ells* amount to, at 3 *s.* 11 *d.* an *Ell?*

Anſwer. 64 *l.* 16 *s.* 5 *d.*

V. *What* will 355 *Ells* amount to, at 7 *s.* 5 *d.* an *Ell?*

Anſwer. 131 *l.* 12 *s.* 11 *d.*

VI. *What* will 489 *Ells* amount to, at 11 *s.* 11 *d.* an *Ell?*

Anſwer. 291 *l.* 7 *s.* 3 *d.*

VII. *What* will 399 *Barrels* amount to, at 23 *s.* 5 *d.* a *Barrel?*

Anſwer. 467 *l.* 3 *s.* 3 *d.*

VIII. *What* will 557 *Barrels* amount to, at 1 *l.* 13 *s.* a *Barrel?*

Anſwer. 919 *l.* 1 *s.*

IX. *What* will 266 *Hogſheads* amount to, at 2 *l.* 19 *s.* 1 *d.* a *Hogſhead?*

Anſwer. 785 *l.* 16 *s.* 2 *d.*

X. *What* will 331 $\frac{1}{4}$ *Hogſheads* come to, at 13 *l.* 14 *s.* 4 *d.* a *Hogſhead?*

Anſwer. 4540 *l.* 4 *s.* 4 *d.*

H. 2. lb.

XI. *What* will 191 : 3 : 24 amount to, at 3 *l.* 17 *s.* an *Hundred?*

Anſwer. 739 *l.* 1 *s.* 3 *d.*

H. Q. lb.

XII. *What* will the *Net-Weight* of 511 : 2 : 14 *Groſs*, 36 *lb. per Hundred*, being deducted for *Tare*, amount unto, at 18 *s.* 4 *d. per Pound Net?*

Anſwer. 35643 *l.* 4 *s.* 2 *d.*

See more *Queſtions* in the APPENDIX.

Thus much at preſent concerning the RULE call'd PRACTICE.

Now follows the *Doctrine* of *Vulgar* FRACTIONS.

CHAP. XVII.

HAVING before *explain'd* the *Nature* of RATIOS, the *Nature* of FRACTIONS is eaſily *learnt*: For,

Since ARITHMETIC ſuppoſes UNITY, or ONE, to be the FIRST and LEAST *Number* (becauſe there *can't* be *fewer* than ONE, nor *ſeldomer* than ONCE) and ſince *all other Numbers* are made by a *Collection* of UNITS; therefore,

Whenever (which often happens) *any thing* is *leſs* than ONE; or when ONE is to be *divided* into PARTS; becauſe *theſe* PARTS, whatever they be, *can't* be *denoted* by any *True* Numbers, ARITHMETICIANS, as well as they can, are forc'd to *denote* them by *Broken* Numbers, as they are call'd, or FRACTIONS, thus,

Suppoſe any WHOLE is to be *biſſected*; the *Half* of that WHOLE is thus *denoted*, $\frac{1}{2}$: Again,

Let the *Number* 3 be to be *biſſected*; now, becauſe *its Half* is *greater* than UNITY, or ONE, and *leſs* than TWO; and becauſe between *theſe Numbers* (*viz.* 1 and 2) there is *no* intermediate *true* Number; therefore the *Half* of 3 is, as well as we can, thus *denoted*, $\frac{3}{2}$, or thus, $1\frac{1}{2}$, *&c.*

And thus FRACTIONS are *not* to be accounted as *true* Numbers; they *not* anſwering to the *Queſtion* How MANY: For *Example*,

Half,

Half, or ½ an *Hour*, can't ſo properly be ſaid to *anſwer* to How MANY *Hours*, as to How MUCH of an *Hour*; therefore the *Anſwer* given by *this*, and by *all other Intermediates* betwixt NONE and ONE, call'd FRACTIONS, may be affirm'd, properly ſpeaking, to belong not *ſo much* to *diſcrete Quantity* or NUMBER, as to *continu'd Quantity* or MAGNITUDE; for,

Altho' in ARITHMETIC there is *not* found *fewer* than ONE; yet in GEOMETRY is there found *leſs* than an INTEGER: And, indeed, as an ARITHMETICAL *Unit* is *Multipliable*, ſo a GEOMETRICAL *one* is *Diviſible ad Infinitum*.

But when ARITHMETIC wou'd, after ſome Manner or other, imitate the *Infinite Diviſibility* of GEOMETRY; it ſuppoſes UNITY, or ONE, as ſome INTEGER *diviſible*, into as many PARTS as you pleaſe; and at length, *thoſe* PARTS to be *numerable*; and thence, ſubject to the Conſideration of ARITHMETIC; for altho' the *Diviſion* of an INTEGER into PARTS, is rightly eſteem'd as ſomething GEOMETRICAL, yet the *Numeration* of *thoſe* PARTS into *which* it is *divided*, is plainly ARITHMETICAL.

By what has been ſaid, then, 'tis plain that in FRACTIONS, or BROKEN *Numbers*, TWO Things are always to be *ſignified*; As,

1ſt. The *Diviſion* of ONE, or an INTEGER, into PARTS (ſometimes *more*, ſometimes *fewer*): And,

2dly. The *Aſſumption*, or *Taking* of ONE or MORE of *thoſe* PARTS: Therefore,

A FRACTION is uſually *denoted* by TWO *Numbers*, the *one* ABOVE, and the *other* BELOW, a ſmall *Line*, thus, $\frac{2}{3}$, $\frac{3}{4}$, *&c.*

The

The *Lower* of which is uſually call'd the DENOMINATOR; the *Upper*, the NUMERATOR:

The DENOMINATOR is *ſo* call'd, becauſe it *denominates*, or *names* the PARTS into *which* the INTEGER is *divided*, or *ſhews* WHAT *Parts* they are.

The NUMERATOR is *ſo* call'd, becauſe it *numbers* or *tells* HOW MANY of the *Parts* (thus *denominated*) muſt be *taken*; thus, $\frac{2}{3}$ *Numbers*, *Two third Parts*, or *Two thirds*; $\frac{3}{4}$, *Three fourths*; $\frac{5}{6}$, *Five ſixths*, or *Sixth Parts*; &c.

Thus have I given the *Learner* the MOST SIMPLE *Expoſition* of FRACTIONS, and *Explanation* of their NATURE.

Now when the NUMERATOR of a FRACTION is *leſs* than the DENOMINATOR, it's call'd a PROPER *Fraction*; becauſe a FRACTION, *properly* ſo call'd, *denotes* ſomething *leſs* than UNITY, or ONE; as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, *&c.* But,

If it happens (as it often does) that the NUMERATOR be *equal to*, or *greater than*, the DENOMINATOR:

The FRACTION then is uſually call'd an IMPROPER *one*, as being ſomething either *equal to*, or *greater than* UNITY.

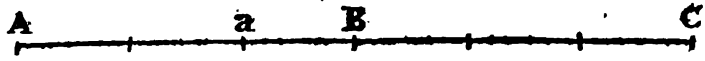
Such Sort of FRACTIONS are *theſe*, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, *&c. each* of *which* is *equal to* UNITY, or ONE; or *theſe*, $\frac{2}{1}$, $\frac{3}{2}$, $\frac{4}{3}$, *&c. each* of *which* is *greater than* UNITY: For if the NUMERATOR be *equal to* the DENOMINATOR, it ſhews, that *all the Parts* that the INTEGER is *divided* into, *i. e.* the INTEGER *itſelf* muſt be taken, and *not* any *particular Portion* of it (for *all* the PARTS together, are *equal to* the WHOLE, or the INTEGER).

And

And if the NUMERATOR be *greater than* the DENOMINATOR, it ſhews, that *more* of *ſuch homogeneal Parts* as the INTEGER is *divided* into, *than* (the DENOMINATOR *ſhews*) are *contain'd* in the INTEGER, muſt be *taken*; and, conſequently, the VALUE of *ſuch* an IMPROPER *Fraction*, is *greater than* UNITY, or the INTEGER.

But now, tho' MORE *Parts* of any ONE INTEGER *can't* be *taken*, than are *contain'd* in *that* INTEGER, as the NUMERATOR of an IMPROPER *Fraction* ſeems to require, yet *another* ſuch INTEGER (or *more* on Occaſion) *may* be *divided* into PARTS *homogeneal*, or *of the ſame Kind* with *thoſe which* the *firſt* INTEGER was *divided* into, and then *as many* of them may be *taken*, *as* the NUMERATOR (be what it will) of the IMPROPER *Fraction* requires: As for *Example*;

Imagine the *Line* A B to be ONE *Foot*, ONE *Yard*, ONE *Shilling*, or *any other* INTEGER whatſoever; and



Let it (for Inſtance) be *divided* into *three equal Parts*; then *one* of *thoſe Parts* is a FRACTION *properly* call'd *One* THIRD, or $\frac{1}{3}$ of the INTEGER; and *two* of *them* is a FRACTION *properly* call'd *Two* THIRDS, or $\frac{2}{3}$ of the INTEGER; but, if *three* of *them* be *taken*, the WHOLE is *taken*, and conſequently $\frac{3}{3}$ is *improperly* call'd a FRACTION: Now when the NUMERATOR is *greater than* the DENOMINATOR (as $\frac{4}{3}$, $\frac{5}{3}$, or $\frac{6}{3}$) and *denotes*, that *more* ſuch *Parts*, as the INTEGER A B is *divided* into, *than* are *therein contain'd*, muſt be *taken*;

Let, then, *another ſuch* INTEGER, B C, be *divided* into the *like Parts*; and then (ſince the *Parts* are *homogeneal*) if I *take* 4 of *them*, 'tis plain I take THREE *Thirds*, or the WHOLE, and 1 *Third* over, *viz. Four*

Thirds, which (*plac'd* or *denoted* as FRACTIONS uſually are, thus, $\frac{4}{3}$) is *improperly* call'd a FRACTION; but if I *take* 6 of *thoſe Parts*, *viz.* THIRDS, 'tis plain I take *all* the *Parts* that the *two* INTEGERS AB, and BC, are *divided* into; ſo that $\frac{6}{3}$ is *improperly* call'd a FRACTION, being *equal to* 2 WHOLES.

And, in like manner, by *taking another ſuch* INTEGER, and *dividing it* into *homogeneal Parts*, it may be ſhewn, that the *Improper* FRACTION $\frac{8}{3}$, is = 2 INTEGERS, and $\frac{2}{3}$ over; and that the IMPROPER *Fraction* $\frac{9}{3}$, is = *three* INTEGERS; and ſo on.

The *Nature* of FRACTIONS therefore may be more commodiouſly and generally *explain'd*, by the *Diviſion*, not only of *one*, but of *more* INTEGERS *taken together* (*viz.* of *as many as* the NUMERATOR *ſhews*) into *ſome equal Parts* (*viz. ſo many as* the DENOMINATOR *requires*) *one* of *which* PARTS the FRACTION *ſhews*, muſt be *taken*: For,

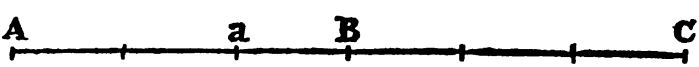
It's all one whether we ſay, the FRACTION $\frac{2}{3}$ *ſignifies* TWO *Thirds* of ONE *Integer*, or ONE *Third* of TWO *Integers*.

If we ſay $\frac{2}{3}$ *ſignifies* TWO *Thirds* of ONE *Integer*, we *then* ſuppoſe an INTEGER to be *divided* into 3 *equal Parts*, 2 of *which Parts*, the NUMERATOR *ſhews* muſt be *taken*: But,

If we ſay, $\frac{2}{3}$ *ſignifies* ONE *Third* of TWO *Integers*, we *then* ſuppoſe, *not one* INTEGER, *but two* INTEGERS (*ſhewn* by the NUMERATOR) to be *divided* into *three equal Parts* (*ſhewn* by the DENOMINATOR) and that *one* of *thoſe Parts* muſt be *taken*; by *either Method* the *ſame* thing is *denoted*: For,

'Tis

'Tis manifeſt, that A *a* is either $\frac{1}{3}$ of the



INTEGER AB, or $\frac{1}{3}$ of the 2 INTEGERS AB and BC.

Now THIS *Explanation* of FRACTIONS, is equally agreeable to *both* PROPER and IMPROPER *ones*:

For thus, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, *&c.* ſignify no more than THIRDS of the *Numbers* 2, 3, and 4, *&c. i. e.* They are no more than *ſo many* QUOTIENTS of the *Numbers* 2, 3, and 4, *&c. divided* by the *Number* 3.

Thus the PROPER *Fraction* $\frac{2}{3}$, *ſignifies* the QUOTIENT of 2 *divided* by 3.

Thus again, the IMPROPER *Fraction* $\frac{3}{3}$, *ſignifies* the QUOTIENT of 3 *divided* by 3, *i. e.* an INTEGER, or 1.

And thus, laſtly, the IMPROPER *Fraction* $\frac{4}{3}$, *ſignifies* the QUOTIENT of 4 *divided* by 3, *i. e. one* INTEGER, and *one Third.*

A SIMPLE or SINGLE *Fraction*, is *ſuch* a one *as* is expreſs'd by *one Numerator*, and *one Denominator*; ſo $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{7}$, $\frac{1}{1}$, $\frac{1}{2}$, *&c.* are *each* SINGLE *Fractions*: But,

A COMPOUND *Fraction* is a *Part* of a PART, or PARTS; or a *Fraction* of a FRACTION, or FRACTIONS; and is expreſs'd and diſtinguiſh'd by *two* or *more* SINGLE *Fractions*, *connected* uſually by the *Word* OF; ſo $\frac{1}{4}$ of $\frac{1}{7}$, and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, and the like, are COMPOUND *Fractions.*

A MIX'D *Number* is *ſo* call'd, *when* a FRACTION is *annex'd* to an INTEGER, or WHOLE *Number*; ſo 1 $\frac{1}{2}$, 7 $\frac{3}{5}$, 22 $\frac{1}{8}$, *&c.* are MIX'D *Numbers.*

AXIOM.

Any FRACTION *whatſoever*, is in Proportion to UNITY, or ONE, as the NUMERATOR of the *Fraction* is to its DENOMINATOR: Thus, for Inſtance, I ſay, 2 : 3 :: $\frac{2}{3}$: 1; for 1 is equal to $\frac{3}{3}$ (as was ſaid before); and it is manifeſt, that 2 : 3 :: (2 *Thirds*, or) $\frac{2}{3}$: (3 *Thirds*, or) $\frac{3}{3}$: Therefore we ſay,

UNIVERSALLY,

By a *Fraction* (either PROPER, or IMPROPER) *ſo much* of a *Homogeneal*, or *like* Quantity, is denoted, *as* has the *ſame* RATIO to *one* WHOLE, or *propos'd Quantity*, *as* has the NUMERATOR of the *Fraction*, to its *Denominator*: And hence, indeed, it follows;

1ſt. That the SAME *Fraction*, as to VALUE, may be denoted or expreſs'd *ſeveral Ways*; nay by *infinite ones*: And hence alſo it follows;

2dly. That ALL *thoſe Fractions* whoſe NUMERATORS have the *ſame* RATIO to their *reſpective* DENOMINATORS, are *equal to*, or of the *ſame* VALUE *with each other*, thus,

For *Example*'s Sake.

$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{10}{20}$, $\frac{100}{200}$, &c. becauſe in each, there is the *ſame* RATIO of the NUMERATOR to the DENOMINATOR (*viz. that* of 1 to 2):

And *this* now, I would have underſtood, *not only* of NUMBERS, *but alſo* of ALL QUANTITIES *homogeneal*, or *like* to *each other*, thus,

Suppoſe $\frac{A}{A+A}$, $\frac{A}{2A}$, $\frac{A}{B}$ (B being put the *double* of the *Quantity* A);

Whether

Whether the *Things* be *Numbers*, *Lines*, *Superficies*, *Bodies*, *Weights*, *Times*, &c. which are ſignify'd by the *Letters* A and B:

Thus alſo $\frac{a\ Pound}{two\ Pounds}$, $\frac{two\ Feet}{four\ Feet}$, denote the SAME *Fraction* as to VALUE *(viz. an Half)* becauſe in *each* there is the *ſame* RATIO of the NUMERATOR to the DENOMINATOR *(viz. that* of 1 to 2): Thus again,

$\frac{1}{2}$ D, $\frac{2}{4}$ D, $\frac{A}{A+A}$ D, $\frac{A}{B}$ D, $\frac{a\ Pound}{two\ Pounds}$ D, &c. ſignify the *ſame thing*, *viz. that* which has the *ſame* RATIO to the *propos'd Quantity* D, *(which* is accounted an *Integer)* as 1 has to 2, or as 2 to 4, or as A to A+A, or as A to B, or as a *Pound* to two *Pounds*; ALL *which* RATIOS, are the *ſame*, or *Equals*: Now,

From hence is manifeſt, either the *Identity*, or wonderful *Affinity* of RATIOS and FRACTIONS: For,

Since 'tis all one whether we ſay, the FRACTION $\frac{3}{4}$ (for *Example)* ſignifies *three fourths* of an INTEGER, or ſignifies *ſuch a Part* of an INTEGER, as has the *ſame* RATIO to the INTEGER, as 3 has to 4; I ſay, 'tis manifeſt, that FRACTIONS (whether PROPER, IMPROPER, or IRRATIONAL) are *nothing elſe but* the DENOMINATORS of RATIOS:

But thus much concerning the NATURE of FRACTIONS, concerning their *like* ESTIMATION as to VALUE, and concerning their AFFINITY with RATIOS.

Before I proceed to the *Arithmetical* OPERATIONS to be us'd in BROKEN *Numbers*; I mean, ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION, &c.;

I ſhall

I ſhall methodically *ſhew* how FRACTIONS muſt be neceſſarily prepar'd and fitted for *ſuch Operations*, which is done by *what* is commonly call'd REDUCTION: In order thereto, 'tis here very neceſſary that we firſt know,

How to find the greateſt COMMON MEASURE *of any two* GIVEN *Numbers.*

Now *this* is done by the following

RULE.

Divide the GREATER *Number* by the LESSER, and the DIVISOR, by the REMAINDER after this Diviſion, if there be any; and ſo on continually, until there be no *Remainder*; then the *laſt* DIVISOR (that *divides* without a *Remainder*) is the *greateſt* COMMON MEASURE, or DIVISOR of the *two given Numbers:*

As is ſhewn by EUCLID, *Lib.* 7. *Prop.* 2.

But if the REMAINDER at *laſt* be 1, 'tis an Argument, that the *propos'd Numbers* are *already* the *leaſt* in *that* RATIO; for *dividing* BOTH by 1, does not *leſſen* them: And *ſuch Numbers* are *what* EUCLID calls *Numbers* PRIME, the *one* to the *other.* EUCLID *Def.* 12. *Lib.* 7.

EX-

EXAMPLE I.

Find the *greateſt Common Meaſure* of the *two Numbers* 12 and 34; *which*, by the foregoing RULE, is thus done,

The *greateſt Common Meaſure* =

```
12)34(2
   24
   --
10)12(1
   10
   --
 2)10(5
   10
   --
    0
```

Thus, the *greateſt Common Meaſure* of the *Numbers* 12 and 34, is 2: For,

12 is contain'd *twice* in 34, and 10 *remains*: Again,

10 is contain'd *once* in 12, and 2 *remains*: Laſtly,

2 is contain'd exactly 5 *times* in 10:

Therefore this *laſt* DIVISOR 2, is the *greateſt Common Meaſure* of the *propos'd Numbers* 12 and 34.

EX-

EXAMPLE II.

Find the *greateſt Common Meaſure* of the *two Numbers* 13 and 34.

The OPERATION.

13)34(2
26
---
8)13(1
8
---
5)8(1
5
---
3)5(1
3
---
2)3(1
2
---
1)2(2
2

0

The *greateſt Common Meaſure* = 1

Here 1 is the *greateſt Common Meaſure* of the *propos'd Numbers* 13 and 34 (and thence *theſe Numbers* are already, *inter ſe primi*, the *leaſt* in the *ſame* RATIO), as is manifeſt from the OPERATION.

Having thus open'd the Way, by ſhewing how to find the *greateſt Common Meaſure* of *any two given Numbers*, I ſhall proceed directly to the

REDUCTION

REDUCTION *of* FRACTIONS.

PROP. I.

TO REDUCE *a Fraction to the* LEAST TERMS *poſſible, retaining the ſame* RATIO.

RULE.

Divide the NUMERATOR and DENOMINATOR *ſeverally*, by their *greateſt Common Meaſure*, and the *two* QUOTIENTS will *form* a FRACTION, and in *its* LEAST TERMS.

The *Truth* of this RULE, is evident from the *Corollary* of the 35th *Prop.* of the 7th *Book* of EUCLID; or, indeed, from *what* has been *herein* before ſaid of FRACTIONS and RATIOS.

EXAMPLE I.

Reduce $\frac{555}{629}$ to the *leaſt Terms* poſſible.

Seek firſt, the *greateſt Common Meaſure* of the NUMERATOR 555, and DENOMINATOR 629, after the Manner before taught, and you'll find it to be 37; with *which work* as the RULE directs, thus,

37)555(15 = *one Qu.*	37)629(17 = the *other Qu.*
37	37
185	259
185	259
0	0

Thus $\frac{15}{17}$ is the FRACTION *requir'd*, *that* is *equivalent* to the *given* FRACTION $\frac{555}{629}$, and expreſs'd in the *leaſt Terms* poſſible.

EXAMPLE II.

Reduce $\frac{4791}{9851}$, to the *leaſt Terms*.

Now here (in finding the *greateſt Common Meaſure*) the *Remainder* at laſt will be found to be 1; conſequently the *given Fraction* (by what hath been before obſerv'd) is in *its leaſt Terms* already.

But the *Learner* may obſerve, that, for the moſt Part, *ſome Common Meaſure or other* (if not the *greateſt*) may be deſcry'd at firſt Sight, by *which* the *given Fraction* may be *reduc'd* to *lower Terms*; and again, *thoſe lower Terms* may, perhaps, be ſtill *abbreviated* by *dividing* by the *ſame*, or *ſome other apparent Common Meaſure*; and ſo on, 'till the *given Fraction* be *reduc'd* to *its* LOWEST *Terms*, *which*, I ſay, may be moſt times ſooner done, than by firſt ſeeking the *greateſt Common Meaſure* for that Purpoſe.

As to the *Choice* of *theſe Common* MEASURES, it is, indeed, ſomewhat *uncertain* and *accidental*; for every one is at Liberty to *uſe ſuch* MEASURES, or DIVISORS, *as* firſt preſent themſelves unto his Mind, a *Fraction* being often capable of being *reduc'd* to the *leaſt Terms*, by *different* DIVISORS; *which*, as well as the *Manner* of performing theſe *Abbreviations*, may be ſeen by the ſubſequent OPERATIONS.

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EXAMPLE III.

Reduce $\frac{336}{560}$, to its *leaſt Terms* poſſible.

Which may be thus *abbreviated*,

2)	2)	2)	7)		
2)336	168	84	42	21	3
560	280	140	70	35	5

The *leaſt Terms* requir'd.

Or thus,

4)	7)		
4)336	84	21	3
560	140	35	5

As before.

Or thus,

8)	7)		
4)336	168	21	3
560	280	35	5

As before.

PROP. II.

To *Reduce* a COMPOUND *Fraction*, to a SIMPLE, or SINGLE *Fraction*.

RULE.

Multiply all the NUMERATORS *continually, for the* NUMERATOR *of the* SINGLE *Fraction; and multiply alſo, all the* DENOMINATORS *continually, for the* DENOMINATOR *thereof.*

EXAMPLE I

Reduce the COMPOUND *Fraction* $\frac{1}{2}$ of $\frac{1}{3}$, to a SINGLE *Fraction.*

Thus, the *Numerators* $1 \times 1 = 1$, and the *Denominators* $2 \times 3 = 6$, whence $\frac{1}{6}$ is the SINGLE *Fraction* requir'd, *equivalent* to the propos'd COMPOUND *one*, $\frac{1}{2}$ of $\frac{1}{3}$.

EXAMPLE II.

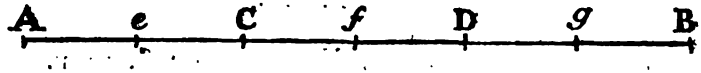
Reduce $\frac{2}{3}$ of $\frac{3}{4}$, to a SINGLE *Fraction.*

Thus $2 \times 3 = 6$, and $3 \times 4 = 12$;

Wherefore $\frac{6}{12}$ is the SINGLE *Fraction* ſought by the RULE, which being *abbreviated* (by *Prop.* I.) becomes $\frac{1}{2}$; and thus the COMPOUND *Fraction*, $\frac{2}{3}$ of $\frac{3}{4}$, is = the SINGLE *Fraction* $\frac{1}{2}$.

The TRUTH of *this* RULE may be thus *illuſtrated*;

Let the *Line* AB repreſent 1 INTEGER of *any Denomination whatſoever.*



Suppoſe *this* INTEGER (to make it applicable to the 1ſt *Example*) be *divided* into THREE *equal* PARTS, as AC, CD, DB; then EACH of *thoſe* PARTS, is $\frac{1}{3}$ of the WHOLE AB: Again,

Suppoſe EACH of *thoſe* THIRD PARTS, AC, CD, DB, be *biſected* in the *Points*, *e*, *f*, *g*; 'tis very plain, that the INTEGER AB, will then be *divided* into SIX *equal* PARTS, *viz.* A*e*, *e*C, C*f*, *f*D, D*g*, *g*B; ſo that EACH of *theſe* PARTS (A*e* for Inſtance) is then $\frac{1}{6}$ of the

the WHOLE AB: Now, whereas AC is $= \frac{1}{3}$ of the INTEGER AB, by *Conſtruction*; and whereas A*e* is $= \frac{1}{2}$ of AC, *i. e.* $= \frac{1}{2}$ of $\frac{1}{3}$ of the INTEGER AB; it evidently appears, that $\frac{1}{2}$ of $\frac{1}{3}$ of the INTEGER AB, and $\frac{1}{6}$ of the INTEGER AB, are of *equal* VALUE.

The *like* might be *ſhewn*, if the FRACTION was *more* COMPOUNDED, as in

EXAMPLE III.

Reduce the COMPOUND *Fraction* $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, to an *equivalent* SIMPLE *Fraction*.

Thus, $1 \times 2 \times 3$, produces 6 for the NUMERATOR of the SIMPLE *Fraction*; and $2 \times 3 \times 4$, produces 24 for the DENOMINATOR *thereof*: Thus, $\frac{6}{24}$, or (by *Abbreviation*) $\frac{1}{4}$ is the SIMPLE *Fraction* ſought: For,

If the INTEGER AB be *divided* into 4 *equal* PARTS, *viz.* AC, CD, DE, EB, it is *ſelf-evident*,

That AC is $\frac{1}{2}$ of AD; that AD is $\frac{2}{3}$ of AE; and that AE is $\frac{3}{4}$ of the INTEGER AB; *i. e.* it is evident, that AC is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of the INTEGER AB: but AC is alſo $\frac{1}{4}$ of the *ſame* INTEGER AB, by *Conſtruction*; whence 'tis very plain, that the COMPOUND *Fraction* ($\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$) and the SIMPLE *Fraction* ($\frac{1}{4}$) are *Fractions* EQUIVALENT to *each other*.

And here it may be properly enough obſerv'd, that altho' (in the DEFINITION of a COMPOUND *Fraction*) we have ſaid, that FRACTIONS of FRACTIONS are uſually *denoted* by the *Word* OF, thus, $\frac{1}{2}$ of $\frac{1}{3}$; and $\frac{1}{4}$ of $\frac{2}{3}$, *&c.* yet they may be every Whit as well *denoted* thus, $\frac{1}{2} \times \frac{1}{3}$

$\frac{1}{2} \times \frac{1}{3}$; and $\frac{3}{4} \times \frac{2}{3}$, *&c.* for a FRACTION of a FRACTION, is *nothing elſe but* a FRACTION *compounded* of *two Fractions multiply'd together*, as ſhall be *ſhewn* under the RULE of MULTIPLICATION, in its *proper Place.*

PROP. III.

To *Reduce* FRACTIONS of *different* DENOMINATONS, to other *equivalent* FRACTIONS, having the *ſame*, or a *common* DENOMINATOR.

RULE.

Multiply each NUMERATOR *into all the* DENOMINATORS *continually, except its own* DENOMINATOR, *for new* NUMERATORS; *and multiply all the* DENOMINATORS *continually into one another, for a common* DENOMINATOR *to theſe new* NUMERATORS.

The TRUTH of *this* RULE, is evident from the SCHOLIUM of *Prop.* 18. of the 7th *Book* of EUCLID: Or it may be deduc'd from the 17th *Prop.* of the *ſame Book* of EUCLID; which I ſhall make plain immediately.

EXAMPLE I.

Reduce $\frac{1}{3}$ and $\frac{1}{4}$, to *other* equivalent FRACTIONS, that ſhall have the *ſame*, or a *common* DENOMINATOR.

To do *this* then by the RULE, I muſt *multiply* the NUMERATOR (1) of the *former* FRACTION, into the DENOMINATOR (4) of the *latter*, for *one* of the *new* NUMERATORS; I muſt alſo *multiply* the NUMERATOR (1) of the *latter* FRACTION, by the DENOMINATOR (3) of the *former*, for the *other new* NUMERATOR; whereby the *new* NUMERATORS will be found to be 4 and 3; and, laſtly, I muſt *multiply* the DENOMINATORS

(3

(3 and 4) together for a *common* DENOMINATOR to theſe *new* NUMERATORS, which is 12; for $3 \times 4 = 12$: And thus do I gain two *new* FRACTIONS, *viz.* $\frac{4}{12}$ and $\frac{3}{12}$, *which* have one *common* DENOMINATOR, *viz.* 12; and *which*, I ſay, are of *equal* VALUE with the *two given* ONES, $\frac{1}{3}$ and $\frac{1}{4}$.

For, by *ſo* multiplying the NUMERATOR (1) of the *former* FRACTION, by the DENOMINATOR (4) of the *latter*; and by *ſo* multiplying the DENOMINATORS (3 and 4) together, for a *common* DENOMINATOR, I do, in Reality, *multiply ſeverally* the *two* TERMS (*viz* the NUMERATOR 1, and DENOMINATOR 3) of the *former* FRACTION, by *one* and the *ſame* NUMBER (4):

But the NUMBERS *produc'd* by *ſuch Multiplication*, (*viz.* 4 and 12) are *in the ſame* PROPORTION, *that* the NUMBERS *multiply'd* (*viz.* 1 and 3) are, by the 17th *Prop.* of the 7th *Book* of EUCLID, *i. e.* 1 : 3 : : 4 : 12: Therefore the *new* FRACTION $\frac{4}{12}$, is = the *given* FRACTION $\frac{1}{3}$. Again,

In the *like Manner*, by the OPERATION *above*, according to the *foregoing* RULE, I do, in Reality, *multiply ſeverally*, the *two* TERMS (1 and 4) of the *latter* FRACTION, by *one* and the *ſame* NUMBER (3); and therefore, as before, the NUMBERS *produc'd* thereby (3 and 12) are *in the ſame* PROPORTION *that* the NUMBERS *multiply'd* (1 and 4) are (by *Euclid* 17. 7.) *i. e.* 1 : 4 : : 3 : 12; and therefore the *new* FRACTION $\frac{3}{12}$, is = the *given* FRACTION $\frac{1}{4}$.

Thus have I clearly PROV'D, that by *purſuing* the *foregoing* RULE, may be *gain'd new* FRACTIONS, *which* ſhall have *one common* DENOMINATOR, and be of *equal* VALUE with the FRACTIONS *propos'd*, having *different* DENOMINATORS.

And

And now, *what Number* of FRACTIONS *ſoever occur* to be *reduc'd* to a *common* DENOMINATOR, the very *ſame Method* muſt be *us'd*.

EXAMPLE II.

Let the FRACTIONS $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$, be to be *reduc'd* to *other equivalent* FRACTIONS, *which* ſhall have a *common* DENOMINATOR.

Now theſe FRACTIONS $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$, will, by the RULE, be *reduc'd* to *theſe equivalent* ONES, $\frac{15}{30}$, $\frac{20}{30}$, and $\frac{12}{30}$; for $1 \times 3 \times 5$, are $= 15$, for a *firſt new* NUMERATOR; and $2 \times 3 \times 5$, are $= 30$, for a *common* DENOMINATOR; and thus the *firſt given* FRACTION $\frac{1}{2}$, is $= \frac{15}{30}$: Again, $2 \times 2 \times 5$, is $= 20$, and $3 \times 2 \times 5$, are, as before, $= 30$; and thus the *ſecond given* FRACTION $\frac{2}{3}$, is $= \frac{20}{30}$: Laſtly, $2 \times 2 \times 3$, are $=$ to 12, and $5 \times 2 \times 3$, are $= 30$, as before; and thus $\frac{2}{5}$, are $= \frac{12}{30}$; and, therefore, $\frac{15}{30}$, $\frac{20}{30}$, and $\frac{12}{30}$, are FRACTIONS of the *ſame* VALUE with the *given* ONES, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{2}{5}$; and which have *one common* DENOMINATOR.

But now,

If *ſuch* a *common* DENOMINATOR offers itſelf to us, as is *leſs* than *that* gain'd by the *continual Multiplication* of the *given* DENOMINATORS, we may ſometimes make uſe of it with Advantage, with this Conſideration, that the *ſame* RATIO as *that common* DENOMINATOR has to the DENOMINATOR of the *given* FRACTION, the *very ſame* muſt the *aſſum'd* NUMERATOR have to the *given* NUMERATOR, *leſt* the VALUE of the *given* FRACTION be *alter'd*. Thus in

EX-

EXAMPLE III.

Suppoſe the FRACTIONS $\frac{7}{12}$, and $\frac{1}{3}$, are to be *reduc'd* to the *ſame* DENOMINATOR.

If now, inſtead of 3, the DENOMINATOR of the *latter* FRACTION, I ſubſtitute 12, *which* is *quadruple* of, or 4 times 3; then Care muſt be taken, in the Room of 1, the NUMERATOR of the *ſame* FRACTION, to put 4, *its Quadruple*, that $\frac{4}{12}$, and $\frac{1}{3}$, may retain the *ſame* RATIO of TERMS to *each other*.

PROP. IV.

To *reduce* an IMPROPER *Fraction* to a WHOLE, or to a MIX'D *Number*.

RULE.

Divide the NUMERATOR *by the* DENOMINATOR, *and the* QUOTIENT *is the* WHOLE or MIX'D *Number ſought, viz.* a WHOLE *Number* (if there be *no Remainder*) *equivalent* to the IMPROPER *Fraction* given; But,

If there be a *Remainder* (after ſuch *Diviſion*) it muſt be made the NUMERATOR of the (*Diviſor*, or) given DENOMINATOR, and annex'd to the *Integral* Part of the QUOTIENT; and thus the *given* IMPROPER *Fraction* will be *reduc'd* to an *equivalent* MIX'D *Number*.

The RATIONALE of *this* RULE may, I think, be manifeſtly enough deduc'd from *what* has been already ſaid in the *Explanation* of FRACTIONS, particularly under *that* of an IMPROPER *Fraction*.

For, ſince (as has been ſaid) a FRACTION (whether PROPER or IMPROPER) is *nothing but* the INDEX of an *imperfect* DIVISION:

That DIVISION, actually *perform'd*, *does* that very Thing the FRACTION ſhews *ſhould* be done.

EXAMPLE I.

Reduce the IMPROPER *Fraction* $\frac{24}{6}$, to an *equivalent* WHOLE or MIX'D *Number*.

The OPERATION according to the RULE.

6)24(4
0

Thus, the IMPROPER *Fraction* $\frac{24}{6}$, is = the WHOLE *Number* 4, becauſe 24 *divided* by 6, *quotes* 4.

EXAMPLE II.

Reduce the IMPROPER *Fraction* $\frac{24}{7}$, to a WHOLE or a MIX'D *Number*.

7)24(3 $\frac{3}{7}$
21
—
3

And thus the IMPROPER *Fraction* $\frac{24}{7}$, *is* = the MIX'D *Number* 3 $\frac{3}{7}$, becauſe 24, *divided* by 7, *quotes* 3 $\frac{3}{7}$.

PROP. V.

To *reduce* any *Number*, whether an INTEGER, or a MIX'D *Number*, to an IMPROPER *Fraction*.

This PROPOSITION being the *Reverſe* of the *Laſt*, muſt neceſſarily be *ſolv'd* by a RULE directly *contrary* to the RULE *whereby* THAT is *ſolv'd*: 'Tis *done* therefore by *this*

RULE.

Multiply the INTEGER *into the given or aſſign'd* DENOMINATOR (*when the Number to be reduc'd is a* WHOLE *Number*) *and then the* PRODUCT *itſelf is the* NUMERATOR *of the* IMPROPER *Fraction: But, (when the Number to be reduc'd is a* MIX'D *one) multiply the* INTEGRAL *Part (of ſuch* MIX'D *Number) by the* DENOMINATOR *of the* FRACTIONAL *Part, and to the* PRODUCT, *add the* NUMERATOR *of the ſaid* FRACTIONAL *Part; and then the* SUM *is the* NUMERATOR, *which, with the ſame* DENOMINATOR, *forms the* IMPROPER *Fraction requir'd.*

This RULE (being plainly the *Reverſe* of the LAST) needs *no* DEMONSTRATION: That is,

Since an IMPROPER *Fraction* is *reduc'd* to a WHOLE, or a MIX'D *Number*, by DIVISION, 'tis very evident, that, on the *contrary*, a WHOLE, or a MIX'D *Number*, muſt be *reduc'd* to an IMPROPER *Fraction* by MULTIPLICATION.

THIS *Propoſition*, therefore, and the LAST, *mutually* PROVE *one another.*

EXAMPLE I.

Reduce the WHOLE *Number* 4, to an IMPROPER *Fraction*, whoſe DENOMINATOR ſhall be 6.

Here (the INTEGER) 4 × (the *aſſign'd* DENOMINATOR) 6, makes 24, = the NUMERATOR of the IMPROPER *Fraction* requir'd.

Thus the INTEGER 4 is *reduc'd* to the IMPROPER *Fraction* $\frac{24}{6}$, by MULTIPLICATION, as the *ſame* IMPROPER *Fraction* $\frac{24}{6}$, was before *reduc'd* to the WHOLE *Number* 4 by DIVISION.

EXAMPLE II.

Reduce the MIX'D *Number* $3\frac{3}{7}$, to an IMPROPER *Fraction.*

Here the INTEGRAL *Part* 3 × the DENOMINATOR 7, *produces* 21; and *this* PRODUCT 21 + the NUMERATOR 3, makes 24; *which*, according to the RULE, is the NUMERATOR of the IMPROPER *Fraction* requir'd, *whoſe* DENOMINATOR, is the *given* DENOMINATOR 7; and, thus $\frac{24}{7}$, is the IMPROPER *Fraction* requir'd, *equivalent* to the *given* MIX'D *Number* $3\frac{3}{7}$.

Thus again, it may be obſerv'd, that as the IMPROPER *Fraction* $\frac{24}{7}$, was (under the 4th *Prop.*) *reduc'd* to the MIX'D *Number* $3\frac{3}{7}$ by DIVISION, ſo *here*, by a *contrary* OPERATION (*viz.* MULTIPLICATION) is the MIX'D *Number* $3\frac{3}{7}$, turn'd back again into the IMPROPER *Fraction* $\frac{24}{7}$.

Moreover, the *Learner* may take Notice, that,

When

When the *Number* to be *reduc'd* to an IMPROPER *Fraction*, is a WHOLE, and *not* a MIX'D, *Number*, he has nothing more to do, than to *place* the *given* WHOLE *Number* as a NUMERATOR, and 1 underneath it as a DENOMINATOR, provided there be *no particular* DENOMINATOR *aſſign'd*.

Thus, the WHOLE *Numbers*, 1, 2, 4, 49, *&c.* are turn'd into *equivalent* IMPROPER *Fractions*, by *placing* 1 under *them* reſpectively, as a DENOMINATOR; for $\frac{1}{1}$ is $= 1$, $\frac{2}{1} = 2$, $\frac{4}{1} = 4$, $\frac{49}{1} = 49$, *&c.* as is ſufficiently manifeſt from *what* has been *before ſaid*, of the NATURE of *Fractions*.

PROP. VI.

TO *reduce* a FRACTION to *another* EQUIVALENT *Fraction*, that ſhall have any *aſſign'd* DENOMINATOR:

In the firſt Place, 'tis plain, that here's only a NUMERATOR wanting, to compleat the FRACTION *requir'd*, the DENOMINATOR being *always* ASSIGN'D.

If *ſuch* a NUMERATOR then be found, *as* has the *ſame* RATIO to the *aſſign'd* DENOMINATOR, as the NUMERATOR of the *given* FRACTION, has to *its* DENOMINATOR, the thing is done: And the REASON is *manifeſt*, *viz.* becauſe whilſt the *ſame* RATIO is preſerv'd between the NUMERATOR, and the DENOMINATOR, the *ſame* VALUE of the FRACTION *remains*.

Wherefore, it will always hold,

As the DENOMINATOR of the *given* FRACTION,

To *its* NUMERATOR;

So

So *is* the *aſſign'd* DENOMINATOR,

To *its* NUMERATOR:

From *whence* is plainly deduc'd the *following common*

RULE.

Multiply the NUMERATOR *of the given* FRACTION, *by the aſſign'd* DENOMINATOR, *and divide the* PRODUCT *by the* DENOMINATOR *of the given* FRACTION; *and the* QUOTIENT *ſhall be the* NUMERATOR *of the aſſign'd* DENOMINATOR.

EXAMPLE I.

Reduce the FRACTION $\frac{1}{2}$, to *another* of the *ſame* VALUE, *which* may have the *aſſign'd* DENOMINATOR 12.

Thus, by the RULE, $1 \times 12 = 12$, and $12 \div 2$ is $= 6$; ſo that $\frac{6}{12}$ is the FRACTION found $=$ the *given* FRACTION $\frac{1}{2}$.

EXAMPLE II.

Reduce $\frac{84}{126}$, to *another* EQUIVALENT *Fraction, whoſe* DENOMINATOR ſhall be 3.

By the *preceding common* RULE, the OPERATION is *this*,

84
3

126)252(2 = the NUMERATOR ſought.

Thus

Thus $\frac{2}{3}$ is the FRACTION found to be *equivalent* to the *given* FRACTION $\frac{84}{120}$.

And now, from *this* PROPOSITION, may be clearly drawn, ſeveral uſeful

COROLLARIES.

1. From *hence* may be gain'd a *Method* of *finding* the VALUE of a FRACTION, or *propos'd Parts* of *any* INTEGER, in *other Parts* more *known* and *familiar* to us:

For *Example*,

Suppoſe $\frac{2}{3}$ of an *Engliſh* SHILLING be propos'd, and I would know the VALUE of *that* FRACTION in PENCE, or 12th *Parts* of a SHILLING;

I *reduce* the *propos'd* FRACTION $\frac{2}{3}$, to *another* = it, *whoſe* DENOMINATOR ſhall be 12 (the *Number* of *Parts*, call'd PENCE, the *Engliſh* SHILLING is *divided* into):

And thus, becauſe the DENOMINATOR 3, of the FRACTION *propos'd*, is to *its* NUMERATOR 2, as the *new* DENOMINATOR 12, is to 8: I find that $\frac{2}{3}$ (or *two Thirds*) of an *Engliſh* SHILLING, is of the *ſame* VALUE with $\frac{8}{12}$ (*eight Twelfths*) *i.e.* with *eight* PENCE. Whence,

To find the VALUE of a *Fraction* in ſome *known Parts* of the INTEGER, we get *this general*

RULE.

Multiply the NUMERATOR *of the given* FRACTION, *by the known Parts of the next inferior* DENOMINATION, *and divide that* PRODUCT *by the* DENOMINATOR, *and the*

the QUOTIENT *will be the* VALUE *of the propos'd* FRACTION, *in that inferior* DENOMINATION.

But now we muſt underſtand, that a *given* FRACTION *can't* thus be *always reduc'd* to a *given* DENOMINATOR, but ſometimes for the *ſought* NUMERATOR, will come forth, *either* a FRACTION, *or* a MIX'D *Number*; and in *ſuch Caſe*, the *given Fraction can't* be deſign'd by a SIMPLE *Fraction*, *that* ſhall have a *given* DENOMINATOR; but there will be need of a COMPOUND *Fraction*.

Thus, ſuppoſe I would *reduce* the FRACTION $\frac{2}{5}$, to *twelfth Parts* (*i. e.* to a FRACTION = it, *whoſe* DENOMINATOR may be 12): Now,

Becauſe 'tis $5 : 2 :: 12 : \frac{24}{5} = 4\frac{4}{5}$, the FRACTION *ſought* would be $\frac{4\frac{4}{5}}{12}$, *i. e. four Twelfths*, together with $\frac{4}{5}$ of *one Twelfth* more; and thus, for Inſtance, $\frac{2}{5}$ of an *Engliſh* SHILLING, will be in VALUE $4\frac{4}{5}$ of *Engliſh* PENCE, and $\frac{4}{5}$ of *another* PENNY:

But then, by the ſame *general* RULE, the VALUE of *this* FRACTION, $\frac{4}{5}$, of a PENNY, may be found in the *next inferior* DENOMINATION, *viz.* FARTHINGS; thus,

If I *multiply* the NUMERATOR 4, by 4 (being the *Number* of FARTHINGS, or PARTS, the *Engliſh* PENNY is *divided* into) and *divide* the PRODUCT 16, by the DENOMINATOR 5, the QUOTIENT $3\frac{1}{5}$, will be the VALUE of the FRACTION ($\frac{4}{5}$ of a PENNY) in FARTHINGS, *i.e.* 3 FARTHINGS, and $\frac{1}{5}$ of *another* FARTHING: thus, having deſcended to the *leaſt* KNOWN *Parts*, I find the VALUE of the *propos'd Fraction* (*viz.* $\frac{2}{5}$ of a SHILLING) to be 4 PENCE 3 FARTHINGS, and $\frac{1}{5}$ of a FARTHING more.

2

And

And in like manner, the VALUE of *any other* FRACTION of MONEY, WEIGHT, MEASURE, TIME, *&c.* may be found in the *Common* and KNOWN *Parts* of the INTEGER.

2. From *this* PROPOSITION, we plainly perceive *how* to *convert* VULGAR *Fractions* into DECIMALS, SEXAGESIMALS, *&c.* and the *contrary*; ſince 'tis,

As the DENOMINATOR of the VULGAR *Fraction*,

To *its* NUMERATOR;

So is (10, or 100, or 1000, *&c.*) the *aſſign'd* DENOMINATOR of the DECIMAL *Fraction*,

To *its* NUMERATOR *ſought*:

And *ſo is* 60 the *aſſign'd* DENOMINATOR of the SEXAGESIMAL,

To *its* NUMERATOR *ſought*; and the *contrary*.

3. Hence alſo we learn, *how* to *reduce* a VULGAR *Fraction* into *another* EQUIVALENT *Fraction that* ſhall have *any aſſign'd* NUMERATOR: For 'tis, by the *Inverſe* RATIO,

As the NUMERATOR of the *given Fraction*,

To *its* DENOMINATOR;

So is the *aſſign'd* NUMERATOR,

To *its* DENOMINATOR *requir'd*.

Thus, for Inſtance, the *Fraction* $\frac{11}{120}$, *reduc'd* to *another Fraction* of the *ſame* VALUE with *it*, and *whoſe*

NUMERATOR you would have to be 2, will become $\frac{2}{3}$: For,

84 : 126 :: 2 : Q = 3, The DENOMINATOR *ſought.*

2

84)252(3

0

And now, having *explain'd* what ſeem'd *neceſſary* to the *true* UNDERSTANDING the NATURE of FRACTIONS, or BROKEN *Numbers*, and *their various* REDUCTIONS, I ſhall proceed to the *Explanation* of the *Arithmetical* OPERATIONS to be *us'd* in FRACTIONS, *which* are, indeed, the *ſame as* are *us'd* in WHOLE *Numbers*, or INTEGERS: I mean ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION.

ADDITION *and* SUBTRACTION *of* VULGAR *FRACTIONS.*

IF the *Fractions* to be *added*, or *ſubtracted from*, each other, are of the *ſame* NAME (*i. e.* if they have the *ſame* DENOMINATOR) there is no Difficulty in *theſe* OPERATIONS; For,

By ſubſtituting for the *given* NUMERATORS, the SUM or DIFFERENCE of thoſe NUMERATORS, *retaining* the *common* DENOMINATOR they *firſt* had, you have the SUM or DIFFERENCE of the FRACTIONS *propos'd*; thus,

If the *Fractions* $\frac{2}{6}$ and $\frac{3}{6}$, are to be *added together*, the SUM will be $\frac{5}{6}$; for 2 and 3 make 5, be the *Things* that are number'd *what they will* (ſuppoſe) whether INTEGERS or PARTS.

For

For the ſame Reaſon, *as* 2 and 3 *Men*, are 5 *Men*; or 2 and 3 *Months*, 5 *Months*; or 2 and 3 *Integers*, 5 *Integers*:

For the very ſame Reaſon, 2 and 3 *Parts*, are 5 *Parts*; 2 and 3 *Halfs*, are 5 *Halfs*; 2 and 3 *Sixths*, 5 *Sixths*, *i. e.* $\frac{2}{6} + \frac{3}{6} = \frac{5}{6} = \frac{2+3}{6}$.

Thus, again,

If from $\frac{5}{6}$ be taken $\frac{3}{6}$, there remain $\frac{2}{6}$: For, 'tis *univerſally* TRUE, that $5 - 3$, is $= 2$, be the *Things* number'd, INTEGERS, or PARTS: For,

As $5 - 3$ *Men*, are 2 *Men*; and $5 - 3$ *Months*, are 2 *Months*; or $5 - 3$ *Integers*, are 2 *Integers*; *ſo* $5 - 3$ *Sixths*, are 2 *Sixths*, *i. e.* $\frac{5}{6} - \frac{3}{6} = \frac{2}{6} = \frac{5-3}{6}$.

But now,

The *ſame Method* muſt be taken, where *more Fractions than two*, are to be continually *added to*, or *ſubtracted from one another*, thus,

$\frac{2}{5} + \frac{3}{5} - \frac{4}{5} = \frac{1}{5}$, becauſe $2 + 3 - 4$, are $= 1$; and the ſame in others.

But, if the *Fractions* to be *added to*, or *ſubtracted from* one another, are *not* of the *ſame* DENOMINATION, then there muſt be a *Preparation* made, *i. e. they* muſt be *reduc'd* to the *ſame* NAME, before *Addition* and *Subtraction* can be *made*; and, when *they* are *reduc'd* to the *ſame* NAME, DENOMINATION, or DENOMINATOR, they are *manag'd* as before; thus,

Suppoſe the *Fractions* $\frac{1}{2}$ and $\frac{1}{4}$, are to be *added together*, or that the *latter* be to be *taken from* the *former*;

Inſtead of $\frac{1}{2}$, I ſubſtitute *its* EQUAL $\frac{2}{4}$ (for there's the *ſame* RATIO of 1 to 2, as of 2 to 4, therefore the *Fractions* $\frac{1}{2}$ and $\frac{2}{4}$, are of the *ſame* VALUE with *each other*, as was ſhewn in the EXPLANATION of *Fractions*) and thus, $\frac{1}{2} + \frac{1}{4}$ are $= \frac{2}{4} + \frac{1}{4}$, *i. e.* $\frac{3}{4}$: And, $\frac{1}{2} - \frac{1}{4}$ is $= \frac{2}{4} - \frac{1}{4}$, *i. e.* $\frac{1}{4}$: Thus, again, $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$: And, $\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$.

After the *ſame* manner, if to the *Number* 3, or $\frac{3}{1}$ (*which* are *alike*, or of the *ſame* VALUE; for, the *Diviſion* by 1, alters *not* the Quantity of the *Thing divided*) I ſay, If to the *Number* 3, or $\frac{3}{1}$, be to be *added* the *Fraction* $\frac{2}{5}$; *this* may be *done* by *placing* them thus, not changing the *Terms*, $3 + \frac{2}{5}$, or thus, $3\frac{2}{5}$; But now, (as ſometimes it proves neceſſary) the SUM may be *given* in an IMPROPER *Fraction*; as thus, in the foregoing *Example*, $3 + \frac{2}{5}$, *i. e.* $\frac{3}{1} + \frac{2}{5}$, becomes $= \frac{15}{5} + \frac{2}{5}$, *i. e.* $= \frac{17}{5}$; by ſubſtituting in the Room of 3, or $\frac{3}{1}$ (3 *Integers*) $\frac{15}{5}$; *fifteen* FIFTHS being of the *ſame* VALUE with *three* INTEGERS; for, ſince *five* FIFTHS conſtitute *one* INTEGER, *fifteen* FIFTHS, *i. e.* 3 times *five* FIFTHS, will conſtitute 3 *Integers*. Again,

If from 3 you would take $\frac{2}{5}$, in the Room of 3, ſubſtitute either $\frac{15}{5}$, or $2\frac{5}{5}$, or $2 + \frac{5}{5}$; and *Subſtraction* being made, $3 - \frac{2}{5}$, will be $= \frac{15}{5} - \frac{2}{5}$, *i. e.* $= \frac{13}{5}$: Or, $3 - \frac{2}{5}$, will be $= 2 + \frac{5}{5} - \frac{2}{5} = 2 + \frac{3}{5}$; (for $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$, *before* which *Fraction*, if we *place* the INTEGER 2, we ſhall have $2 + \frac{3}{5}$, or $2\frac{3}{5}$.)

Thus, then, from what has been ſaid, we may fairly and naturally collect a *general* RULE for *adding* and *ſubtracting* of FRACTIONS; *which*, in ſhort, is *this*

RULE.

RULE.

If the FRACTIONS *to be added to, or ſubtracted from each other, are ſingle, and have the* SAME, *or a* COMMON DENOMINATOR, *add or ſubtract the* NUMERATORS *to, or from each other, and their* SUM *or* DIFFERENCE *plac'd as a* NUMERATOR *to ſuch common* DENOMINATOR, *will be the* SUM, *or the* DIFFERENCE *of the Fractions propos'd; but if the* FRACTIONS *have* DIFFERENT DENOMINATORS, *they muſt, in the firſt Place, be reduc'd to a* COMMON DENOMINATOR, *and then manag'd after the ſame Manner.*

Here we ſee 'tis abſolutely neceſſary, that we know how to *reduce* FRACTIONS of *different* DENOMINATORS, to *equivalent* FRACTIONS, having a *common* DENOMINATOR, which is done by *Prop.* III. of REDUCTION of *Fractions.*

'Tis alſo requiſite to know how to *reduce* COMPOUND *Fractions* to SINGLE *ones*, which is done by *Prop.* II.

Though the TRUTH of *this* RULE be very evident, from *what* has been purpoſely *premis'd*, yet that the USE of *it* may be made eaſy and familiar to the *Learner*, I ſhall, as a farther *Illuſtration* of *it*, give him the SOLUTIONS of the *Examples* following.

EXAMPLE I.

Add the SIMPLE *Fractions* $\frac{1}{8}$, $\frac{2}{8}$, and $\frac{3}{8}$ together.

Here the *Fractions* propos'd to be *added* into *one* SUM, having ALL the *ſame* DENOMINATOR, without any *Preparation*, I *add* their NUMERATORS together, and find, that they, *viz.* $1 + 2 + 3$, make 6, and thus,

thus, $\frac{6}{8}$, or (by REDUCTION by *Prop.* I.) $\frac{3}{4}$ is the SUM of the *Fractions* propos'd to be *added* together.

EXAMPLE II.

Let the SIMPLE *Fractions* $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{2}{7}$, be *added* into *one* SUM.

Here the *Fractions* propos'd, having ALL *different* DENOMINATORS, I first *reduce* them to *other Fractions* of the *same* VALUE, and *which* shall have a *common* DENOMINATOR, by *Prop.* III. thus,

$$\left.\begin{array}{l} 1 \times 5 \times 7 = 35 \\ 1 \times 3 \times 7 = 21 \\ 2 \times 5 \times 3 = 30 \end{array}\right\} = \text{the } \textit{new} \text{ NUMERATORS.}$$

$3 \times 5 \times 7 = 105 =$ the *common* DENOMINATOR.

Whereby the *given Fractions* $\frac{1}{3}$, $\frac{1}{7}$, and $\frac{2}{7}$, are *reduc'd* to the *equivalent ones* $\frac{35}{105}$, $\frac{21}{105}$, and $\frac{30}{105}$; *which Fractions added* into *one* SUM, as before (*viz.* by *adding* the NUMERATORS 35, 21, and 30 together) make $\frac{86}{105}$ = the SUM of the *Fractions* propos'd.

EXAMPLE III.

Let the *Fractions* $\frac{1}{3}$ and $\frac{5}{6}$ of $\frac{1}{4}$, be to be *added* together.

In *this Example*, there being a COMPOUND *Fraction* (*viz.* $\frac{5}{6}$ of $\frac{1}{4}$) it must be *reduc'd* to a SINGLE *one* by *Prop.* II. thus, $5 \times 1 = 5$, and $6 \times 4 = 24$; whence the COMPOUND *Fraction* becomes $\frac{5}{24}$; and then, if the SIMPLE *Fractions* $\frac{1}{3}$ and $\frac{5}{24}$, be *reduc'd* and *added* together, according to the *foregoing* RULE; and, as hath been

been ſhewn by *Example* II. the SUM of the *Fractions* propos'd to be *added* together, will be found to be $\frac{15}{24}$, thus,

$$\left.\begin{array}{l} 1 \times 24 = 24 \\ 5 \times\ 3 = 15 \end{array}\right\} = \text{the } \textit{new} \text{ NUMERATORS.}$$

$3 \times 24 = 72$ = the *common* DENOMINATOR.

But $24 + 15 = 39$, thus $\frac{39}{72}$ (by *Prop.* I.) $\frac{15}{24}$ is the SUM requir'd.

EXAMPLE IV.

Let $5\frac{3}{4}$ be to be *added to* $6\frac{2}{4}$.

'Tis very plain, that, when MIX'D *Numbers* are given to be *added* together, the SUM of the INTEGERS, *added* to the SUM of the *Fractional* PARTS, is the SUM requir'd.

Firſt, then, the *Fractions* $\frac{3}{4}$ and $\frac{1}{4}$, being *added* according to the foregoing RULE, make $\frac{4}{4}$, or (by *Prop.* IV.) $1\frac{1}{4}$, which *added to* 11, the SUM of the INTEGERS (*viz.* $5 + 6$) makes $12\frac{1}{4}$, the SUM of the MIX'D *Numbers* given to be *added*.

Thus, $5\frac{3}{4}$ / $6\frac{2}{4}$ / $12\frac{1}{4}$

Or thus, 5 / 6 / $1\frac{1}{4}$ / $12\frac{1}{4}$ = the SUM requir'd.

Or, the MIX'D *Numbers* propos'd to be *added*, $5\frac{3}{4}$, and $6\frac{2}{4}$, may be firſt *reduc'd* to IMPROPER *Fractions* (by *Prop.* V.) and then *thoſe* IMPROPER *Fractions* may be *reduc'd* (if need be) and *added* together, as the *general* RULE

RULE directs; thus, the MIX'D *Number* $5\frac{3}{4}$, becomes the IMPROPER *Fraction* $\frac{23}{4}$; and the MIX'D *Number* $6\frac{2}{4}$, becomes $\frac{26}{4}$ (by *Prop.* V.); Now *theſe* IMPROPER *Fractions* $\frac{23}{4}$ and $\frac{26}{4}$, being *added* together, *they* having a *common* DENOMINATOR, make $\frac{49}{4}$, *which* (in an IMPROPER *Fraction*) is the SUM requir'd; for 23 + 26 *(Fourths)* are = 49 *(Fourths) which* IMPROPER *Fraction*, $\frac{49}{4}$, may (by *Prop.* IV.) be found to be = $12\frac{1}{4}$, = the SUM requir'd, as above.

Theſe *Examples*, I think, are *ſufficient*, in ADDITION of *Fractions*: I ſhall, therefore, now give *ſome* in SUBTRACTION.

EXAMPLE I.

Suppoſe $\frac{5}{9}$ be to be *ſubtracted* from $\frac{7}{9}$.

Here the propos'd *Fractions* being of the *ſame* NAME, *no Preparation* is wanting, but at once,

From the NUMERATOR 7
Subtract the NUMERATOR 5

And 2 Remain.

Thus, $\frac{2}{9}$ is the DIFFERENCE between $\frac{7}{9}$ and $\frac{5}{9}$.

EXAMPLE II.

Suppoſe $\frac{5}{9}$ be to be *ſubtracted* from $\frac{7}{8}$.

Here, the propos'd *Fractions*, having *different* DENOMINATORS, muſt be firſt *reduc'd* (by *Prop.* III.) to *other Fractions* having a *common* DENOMINATOR, and then *one* NUMERATOR muſt be *ſubtracted* from the *other*, as in the firſt *Example*, thus,

By

By *Prop.* III. $\left\{\begin{array}{l} 5\times 8 = 40 \\ 7\times 9 = 63 \end{array}\right\}$ The *new* NUMERATORS.

$9\times 8 = 72$ The *common* DENOMINATOR.

And then, if from $6\frac{3}{72}$, you *ſubtract* $4\frac{0}{72}$, there will *remain* $2\frac{3}{72}$: For,

From the NUMERATOR	63
Subtract the NUMERATOR	40
And	23 Remain.

And thus $\frac{23}{72}$ is found to be the *true* DIFFERENCE between the *propos'd* FRACTIONS $\frac{5}{9}$ and $\frac{7}{8}$.

EXAMPLE III.

From $\frac{2}{3}$ of $\frac{3}{4}$, *ſubtract* $\frac{1}{2}$ of $\frac{1}{4}$.

Firſt *reduce* the COMPOUND *Fractions* into SINGLE *ones* (by *Prop.* II.) and then with them proceed to SUBTRACTION, as in the 1ſt or 2d *Example*; thus,

The COMPOUND *Fraction* $\frac{2}{3}$ of $\frac{3}{4}$, becomes $= \frac{6}{12} = \frac{4}{8}$, by *Reduction*.

And the COMPOUND *Fraction* $\frac{1}{2}$ of $\frac{1}{4}$, becomes $= \frac{1}{8}$: Then,

From the NUMERATOR	4
Subtract the NUMERATOR	1
And	3 remain, *viz.* $\frac{3}{8}$ remain.

Which $\frac{3}{8}$ is the *true* DIFFERENCE requir'd, between the COMPOUND *Fractions* propos'd.

EXAMPLE IV.

Let $5\frac{1}{3}$ be *ſubtracted* from $7\frac{2}{3}$.

First *reduce theſe* propos'd MIX'D *Numbers* into IMPROPER *Fractions* (by *Prop.* V.) and then proceed to *ſubtract* thoſe IMPROPER *Fractions* from one another, by the foregoing *general* RULE; thus,

By *Prop.* V. the MIX'D *Number* $\left\{\begin{matrix}7\frac{2}{3}\\5\frac{1}{3}\end{matrix}\right\}$ is = the IMPROPER *Fraction* $\left\{\begin{matrix}\frac{23}{3}\\\frac{16}{3}\end{matrix}\right\}$ Now,

Theſe IMPROPER *Fractions* having a *common* DENOMINATOR, 3, require *no farther* REDUCTION before SUBTRACTION is made; therefore,

If from the NUMERATOR 23
You *ſubtract* the NUMERATOR 16

There will *remain* 7, *i.e.* 7 *Thirds*, or $\frac{7}{3}$.

Which $\frac{7}{3}$ is the DIFFERENCE between the MIX'D *Numbers* propos'd to be *ſubtracted* from one another.

But this IMPROPER *Fraction* $\frac{7}{3}$, will be found (by *Prop.* IV.) $= 2\frac{1}{3}$; and thus, laſtly, $2\frac{1}{3}$ is the DIFFERENCE between the *propos'd* MIX'D *Numbers*.

But here the *Learner* may obſerve, that *whenſoever*, in SUBTRACTION, we have to do with a WHOLE or MIX'D

MIX'D *Number* or MIX'D *Numbers*, it is not always neceſſary that we firſt *reduce* them to IMPROPER *Fractions*; but that after *reducing* the *fractional* PARTS of the *propos'd* MIX'D *Numbers* (if need be) to a *common* DENOMINATOR, we may *place* the *integral* and *fractional* PARTS, properly and diſtinctly under one another, and then *ſubtract* them (as we *do* WHOLE *Numbers*) with greater Eaſe and Expedition, than by firſt *reducing* them into IMPROPER *Fractions*, and then proceeding as is, in the 4th *Example*, above taught: And *this manner* of SUBTRACTING, muſt appear very plain from the following *Examples* and *Operations*.

And, in the firſt Place, take the ſame

Example IV.

From $7\frac{2}{3}$
Subtract $5\frac{1}{3}$

Remainder $2\frac{1}{3}$ as above.

Example V.

From $7\frac{2}{3}$
Subtract 5

Remainder $2\frac{2}{3}$

Example VI.

From 7
Subtract $5\frac{1}{3}$

Remainder $1\frac{2}{3}$

Example VII.

From 7
Subtract $0\frac{1}{3}$

Remainder $6\frac{2}{3}$

Example VIII.

From $7\frac{2}{3}$
Take $\frac{1}{3}$

Remainder $7\frac{1}{3}$

Example IX.

From $7\frac{1}{3}$
Take $\frac{2}{3}$

Remainder $6\frac{2}{3}$

Example X.

From	1
Take	$0\frac{1}{3}$
Remainder	$\frac{2}{3}$

Example XI.

From	$7\frac{1}{5} =$	$7\frac{3}{15}$
Take	$5\frac{2}{3} =$	$5\frac{10}{15}$
Remainder		$1\frac{8}{15}$

In *this* laſt *Example*, the *fractional* PARTS ($\frac{1}{5}$ and $\frac{2}{3}$) of the *propos'd* MIX'D *Numbers* ($7\frac{1}{5}$ and $5\frac{2}{3}$) having DIFFERENT *Denominators*, are, in the firſt Place, *reduc'd* (by *Prop.* III.) into OTHER *Fractions*, that have a COMMON *Denominator* (*viz.* $\frac{3}{15}$ and $\frac{10}{15}$) and then the SUBTRACTION is *thus* perform'd;

Since I *can't* take 10 *Fifteenths* from 3 *Fifteenths*, I *borrow* (according to the *common Method* us'd in *Subtraction* of WHOLE *Numbers*) 1 INTEGER, or $\frac{15}{15}$ (*i. e.* always *ſo many* PARTS as the *common* DENOMINATOR *ſhews* the INTEGER to be *divided* into) and *add* them to $\frac{3}{15}$; and from *that* SUM (*viz.* $\frac{18}{15}$) I take $\frac{10}{15}$, and ſubſcribe the $\frac{8}{15}$ that *remain*; and then *carrying* the 1 which I *borrow'd*, to the INTEGER 5, I *ſubtract* 6 from 7, and ſubſcribe the 1 that *remains*; and thus $1\frac{8}{15}$ is the *true* DIFFERENCE or REMAINDER *ſought*. The *ſame* in *others*.

Thus much concerning ADDITION and SUBTRACTION of *Fractions*.

I ſhall now propoſe a few more *Queſtions* for the *Learner's* Exerciſe therein, and then proceed to MULTIPLICATION of *Fractions*.

Queſtions

Queſtions to exerciſe ADDITION *and* SUBTRACTION *of* VULGAR *Fractions.*

I. *What's* the SUM of $\frac{3}{7}$ of a *Pound*, and $\frac{3}{7}$ of a *Shilling*, in a *Fraction* of a *Pound Sterling*?

Anſwer. $\frac{87}{140}$ of a *Pound Sterling.*

II. The *Difference* of *two Numbers* is 21 $\frac{75}{97}$, the *leſſer* of the *two* is 17 $\frac{3}{7}$; *what's* the *greater*?

Anſwer. 39 $\frac{137}{679}$.

III. A *Merchant* has $\frac{3}{16}$, and $\frac{1}{2}$ of $\frac{1}{4}$ of a *Ship*; *what Part* then hath *he* of the WHOLE?

Anſwer. $\frac{5}{16}$ of the *Ship.*

IV. *What* Number added to 9 $\frac{3}{8}$, ſhall make the *Sum* 16 $\frac{1}{2}$?

Anſwer. 7 $\frac{1}{8}$.

V. Two *Men* have *Shares* in a *Ship*, A hath $\frac{3}{7}$, and B $\frac{4}{9}$ of the *Ship*: I demand *whoſe* is the *greater* Share, and by *how much*?

Anſwer. B's is the *greater* Share by $\frac{1}{63}$.

VI. In 3 *Boxes* are 67 $\frac{3}{8}$ Yards of *Cambrick*, *viz.* in N° 1. are 21 $\frac{1}{2}$ *Yards*, in N° 2. 22 $\frac{1}{4}$ *Yards*: *what's* in N° 3?

Anſwer. 23 $\frac{5}{8}$ *Yards.*

MULTI-

MULTIPLICATION *of* VULGAR *Fractions.*

ALL the *Preparation* that is occaſionally *neceſſary* to be made for the MULTIPLICATION (or, indeed, for the DIVISION) of the TERMS *propos'd* to be *multiply'd* (or *divided*) is *this*, *viz.* COMPOUND *Fractions* muſt be *reduc'd* to SINGLE *ones*, by *Prop.* II. And WHOLE or MIX'D *Numbers*, muſt be *reduc'd* to IMPROPER *Fractions*, by *Prop.* V. And then MULTIPLICATION may be *perform'd* by this *general*

RULE.

Multiply the { NUMERATORS / DENOMINATORS } *together for the* { NUMERATOR / DENOMINATOR } *of the* PRODUCT *requir'd.*

The REASON of *this* RULE will be made *manifeſt* by *what* follows.

EXAMPLE I.

Let the *Fraction* $\frac{4}{7}$ be *multiply'd* by the *Fraction* $\frac{2}{3}$.

Here *no Preparation* is requir'd, the TERMS propos'd to be *multiply'd* together, being SINGLE *Fractions*; but, at once, by the *general* RULE, $\frac{4}{7}$ *multiply'd* by $\frac{2}{3}$, produces $\frac{8}{21}$: For,

The

The { NUMERATOR 4 / DENOMINATOR 5 } *multiply'd* by the { NUMERATOR 2 / DENOMINATOR 3 } produces { 8 / 15

EXAMPLE II.

Let the *Fraction* $\frac{4}{5}$ be *multiply'd* by the *Number* 2.

Here the WHOLE *Number* 2 muſt be made an IMPROPER *Fraction*, *which* is done by ſubſcribing 1 under it, for a DENOMINATOR; and then we muſt *multiply* as the *general* RULE directs; thus,

$\frac{4}{5} \times \frac{2}{1}$, produces $\frac{8}{5} = 1\frac{3}{5}$, by *Prop.* IV.

But now, when a *Fraction* is to be *multiply'd* by an INTEGER, it may be done by only *multiplying* the NUMERATOR of the *Fraction*, by the INTEGER, without ſubſcribing the DENOMINATOR 1, ſince 1 doth not *multiply*; thus,

$\frac{4}{5} \times 2 = \frac{8}{5}$, as before:

And that *this* is the *true* PRODUCT is plain; for, *twice* four is 8, be the *Things* to be *number'd what* they will, whether INTEGERS or PARTS; for, for the ſame Reaſon as *twice* four MEN are *eight* MEN; *twice* four BODIES, *eight* BODIES; *twice* four INTEGERS, *eight* INTEGERS; *twice* four THOUSANDS, *eight* THOUSANDS, *&c.* for the very *ſame Reaſon* (*viz.* becauſe *twice* four *ſimply* are *eight*) *twice* four PARTS are *eight* PARTS; *twice* four FOURTHS, *eight* FOURTHS; *twice* four FIFTHS, *eight* FIFTHS (or 5th *Parts*) *&c.*; thus,

$\frac{2}{3} \times A$,

$\frac{1}{3} \times A$, or $\frac{1}{3} A$, will be $\frac{2\,A}{3}$: Thus,

$D \times \frac{A}{B}$, or $\frac{A}{B} \times D$, or $\frac{A}{B} D$, will be $\frac{A\,D}{B}$; and the *ſame* in *others*.

But as the *Fraction* $\frac{4}{7}$, *multiply'd* by the INTEGER 2, produces $\frac{8}{7}$; ſo if the *ſame Fraction* $\frac{4}{7}$, be *multiply'd* by the *Fraction* $\frac{2}{3}$ (*which* is the 3d Part of the former *Multiplier*) there will be produc'd $\frac{8}{21}$ (as in *Example* I.) *i. e.* the third of the former *Product*;

For, as 1 *fifteenth* Part of any INTEGER, is 1 *third* Part of 1 *fifth* Part of the *ſame* INTEGER, ſo 8 *Fifteenths* is 1 *Third* of 8 *Fifths*; $\frac{8}{21}$ therefore, I ſay, (being $\frac{1}{3}$ of $\frac{8}{7}$) is the *true* PRODUCT ariſing from the *Multiplication* of the two *Fractions* $\frac{4}{7}$ and $\frac{2}{3}$, into *one another*.

The REASON is manifeſt enough; for whoſo *multiplies* by 2, ſeeks a DOUBLE; but whoſo *multiplies* by $\frac{2}{3}$, ſeeks a *Third* of a DOUBLE (or the *Double* of a THIRD); and thus, *what* is produc'd by *doubling*, or *multiplying* by 2, is to be *triſected*, that a *Third* of *that* DOUBLE may be gain'd, *which*, by what hath been ſaid, is plainly done by *tripling* the DENOMINATOR, or *multiplying* it by 3: From all *which*, the *Truth* of the RULE muſt, I think, moſt manifeſtly appear: It is alſo plain, from *what* has been ſaid,

1ſt. That the *Multiplication* of the NUMERATOR of a *Fraction*, MULTIPLIES the *Fraction*.

2dly. That the PRODUCT, as $\frac{8}{21}$, ariſing from the MULTIPLICATION of *any* two FRACTIONS, $\frac{4}{7}$ and $\frac{2}{3}$, into *one another*, is *but* the VALUE of a *Fraction* of a FRACTION; ſo that a COMPOUND *Fraction* may be *expreſs'd* as well *thus*, *viz.*

$\frac{4}{7} \times \frac{2}{3}$

†

$\frac{4}{7} \times \frac{2}{3}$, or $\frac{2}{3} \times \frac{4}{7}$, as *thus*, *viz.*

$\frac{4}{7}$ of $\frac{2}{3}$, or $\frac{2}{3}$ of $\frac{4}{7}$, the *Expreſſions* ſignifying the *ſame* VALUE, as has been before hinted.

But, ſince it appears that *multiplying* the DENOMINATOR of a *Fraction*, really *divides* it; it *muſt*, on the contrary, be as true, that the *Diviſion* of the DENOMINATOR of a *Fraction*, really *multiplies* it: Wherefore,

If a FRACTION be *multiply'd* by an INTEGER, it may (whene'er the INTEGER is an ALIQUOT *Part* of the DENOMINATOR of the *given Fraction*) be done by *dividing* the DENOMINATOR by the *given* INTEGER, inſtead of *multiplying* the NUMERATOR; thus,

If, for Inſtance, the *Fraction* $\frac{3}{4}$ be to be *doubl'd*, 'tis all one *whether* the NUMERATOR be *doubl'd*, by *which* we get $\frac{6}{4}$; *or whether* the DENOMINATOR be *halv'd*, by *which* we get $\frac{3}{2}$; For,

The *Fractions* gain'd $\frac{6}{4}$ and $\frac{3}{2}$, are *both* of the *ſame* VALUE; becauſe in *both* there is the *ſame* RATIO of the NUMERATOR to the DENOMINATOR; for 6 : 4 :: 3 : 2.

The like in SYMBOLS,

$\frac{A}{B\,C} \times B$ is $= \frac{B\,A}{B\,C}$, if the NUMERATOR A be *multiply'd* into the INTEGER B; or 'tis $= \frac{A}{C}$, if the DENOMINATOR BC be *divided* by the INTEGER B: Now, the *Fractions* $\frac{B\,A}{B\,C}$ and $\frac{A}{C}$, have the *ſame* VALUE; for, BA : BC :: A : C. Therefore,

It is altogether the *ſame thing*, as to the VALUE of the *Fraction* or PRODUCT *gain'd*, *whether* we *multiply* the NUMERATOR, or *divide* the DENOMINATOR.

EXAMPLE III.

Multiply the SINGLE *Fraction* $\frac{2}{3}$, by the COMPOUND *one* $\frac{1}{2}$ of $\frac{1}{3}$.

1ſt. The COMPOUND *Fraction* $\frac{1}{2}$ of $\frac{1}{3}$, is (by *Prop.* II.) $= \frac{1}{6} = \frac{1}{3}$, by *Abbreviation*.

Then, by the *general* RULE, $\frac{2}{3} \times \frac{1}{3}$, produces $\frac{2}{9}$, *which* is the *true* PRODUCT ſought.

EXAMPLE IV.

Multiply the MIX'D *Number* $24\frac{3}{4}$ by the *Fraction* $\frac{1}{8}$.

1ſt. *Reduce* the MIX'D *Number* $24\frac{3}{4}$, into an IMPROPER *Fraction* (*viz.* $\frac{99}{4}$) by *Prop.* V.

Then, by the *general* RULE, $\frac{99}{4} \times \frac{1}{8}$, produces $\frac{99}{32}$ for the *Anſwer* ſought; but *this Anſwer*, *reduc'd* by *Prop.* IV. is $= 3\frac{3}{32}$; and thus we find, that $24\frac{3}{4} \times \frac{1}{8}$, produces truly $3\frac{3}{32}$.

EXAMPLE V.

Multiply the MIX'D *Number* $8\frac{1}{2}$, by the MIX'D *Number* $6\frac{1}{4}$.

Theſe MIX'D *Numbers*, *reduc'd* (by *Prop.* V.) into IMPROPER *Fractions*, become, the *one* $\frac{17}{2}$, and the *other* $\frac{25}{4}$: *Which* IMPROPER *Fractions*, *multiply'd* into one another

†

another by the foregoing RULE, produce $\frac{425}{8}$, $= 53\frac{1}{8}$, by *Prop.* IV.

What has been ſaid, I think, is *plain*, and *ſufficient* to *inſtruct* any one in MULTIPLICATION of *Fractions*; I ſhall therefore *conclude* this RULE with a *Remark* or two, and proceed to DIVISION.

SCHOLIA.

1. We may obſerve that, when *either* a WHOLE or a MIX'D *Number* is *multiply'd* by a PROPER *Fraction*, the PRODUCT is always LESS than the MULTIPLICAND: And

2. That, when a PROPER *Fraction* is *multiply'd* by *either* a WHOLE or a MIX'D *Number*, the PRODUCT is always LESS than the MULTIPLIER: And

3. That, when the *two* FACTORS are *both* PROPER *Fractions*, the PRODUCT is always LESS than *either* of *them*.

The REASON is *evident* from the *known conſtant* PROPORTION, deduc'd from the 15th *Def.* of the 7th *Book* of *Euclid*, *viz.*

That in *every* MULTIPLICATION a UNIT is to the MULTIPLIER, as the MULTIPLICAND is to the PRODUCT; thus,

1ſt 1 : $\frac{1}{2}$
2dly 1 : 4 } (the MULTIPLIER) : : $\left\{\begin{matrix}4\\ \frac{1}{2}\\ \frac{1}{2}\end{matrix}\right\}$ (the MULTIPLICAND) : $\left\{\begin{matrix}2\\ 2\\ \frac{1}{4}\end{matrix}\right\}$ the PRODUCT.
3dly 1 : $\frac{1}{2}$

DIVISION *of* VULGAR *FRACTIONS.*

AFTER the *ſame Preparation* is made (as often as Occaſion requires) as is mention'd in MULTIPLICATION of *Fractions*, DIVISION may be *perform'd* by the following

RULE.

Multiply the NUMERATOR *of the* DIVISOR *into the* DENOMINATOR *of the* DIVIDEND, *and the* PRODUCT *will be the* DENOMINATOR *of the* QUOTIENT; *then multiply the* DENOMINATOR *of the* DIVISOR *into the* NUMERATOR *of the* DIVIDEND, *and the* PRODUCT *will be the* NUMERATOR *of the* QUOTIENT. Or,

Which is doing the ſame thing,

Invert the DIVISOR, *and then multiply the* DIVIDEND *and the inverted* DIVISOR *together, by the* RULE *of* MULTIPLICATION, *and the* PRODUCT *will be the* QUOTIENT *requir'd.*

This RULE (as it ought to be) is the *Converſe* of *that* of MULTIPLICATION, as ſhall be *ſhewn*, and the REASON of it made *manifeſt*, from the *Examples* and *Arguments* following.

EXAMPLE I.

Suppoſe the *Fraction* $\frac{4}{7}$ be to be *divided* by $\frac{2}{3}$.

Here, being *no* Occaſion for *any Preparation* to be *made*, the OPERATION is, by the RULE, directly *perform'd*; thus,

$\frac{2}{3})\frac{4}{7}(\frac{12}{10}$, The QUOTIENT. Or,

By

By *inverting* the DIVISOR, and *multiplying* thus, $\frac{3}{2} \times \frac{4}{5} = \frac{12}{10}$ = the QUOTIENT.

And thus $\frac{12}{10}$ ($= \frac{6}{5} = 1\frac{1}{5}$, by *Reduction*) is the QUOTIENT, *either Way*, gain'd.

And that it muſt always be *ſo*, is ſelf-evident; For,

Whether we *divide* according to the above *given* RULE, or *invert* the DIVISOR, and proceed according to the RULE of MULTIPLICATION; we *either Way multiply* the SAME *Numbers* together, and conſequently *muſt* either Way *gain* the *ſame* PRODUCTS, *which* conſtitute the QUOTIENT requir'd.

EXAMPLE II.

Suppoſe the *Fraction* $\frac{4}{5}$ be to be *divided* by 2.

Here the DIVISOR 2, being an INTEGER, muſt be firſt *reduc'd* to an IMPROPER *Fraction*, by ſubſcribing 1 for its DENOMINATOR, thus, $\frac{2}{1}$; then we may work thus,

$\frac{2}{1}) \frac{4}{5} (\frac{4}{10}$, the *Quotient*.

Or, *thus*,

$\frac{1}{2} \times \frac{4}{5} = \frac{4}{10}$, the *Quotient*.

And thus $\frac{4}{5}$ *divided* by 2, *quotes* $\frac{4}{10}$, which is $= \frac{2}{5}$, by *Reduction*.

But now, when a FRACTION is to be *divided* by an INTEGER, it may (notwithſtanding the RULE *above*) be *done* by only *dividing* the NUMERATOR of the *Fraction* by the INTEGER (provided the INTEGER be an

ALIQUOT

ALIQUOT *Part* of the NUMERATOR) retaining the *ſame* DENOMINATOR; thus,

$\frac{4}{7}$ *divided* by 2, *quotes* $\frac{2}{7}$, as before, becauſe 4 *divided* by 2, *quotes* 2: For,

Since *univerſally* the *Half* of the *Number* 4 is 2, be *they* what they will *which* are *number'd*; Therefore *Half* of *four* INTEGERS, are *two* INTEGERS; and therefore *half* of *four* PARTS, are *two* PARTS; *half* of *four* FIFTHS, *two* FIFTHS, &c.

But, as often as *ſuch Diviſion* can't conveniently be *made* (*i. e.* when the INTEGER, or *given* DIVISOR, is *not* an ALIQUOT *Part* of the NUMERATOR, and therefore *can't meaſure* it without a *Remainder*) then, inſtead of *dividing* the NUMERATOR, we may *multiply* the DENOMINATOR by the *given* INTEGER, and the *Fraction* is *divided*, as has been mention'd under the RULE of MULTIPLICATION; for it all comes to one as to *Value*, whether we *divide* the NUMERATOR, or *multiply* the DENOMINATOR: For,

By *how much* the *bigger* the DENOMINATOR is, *ſo much* are the *Parts* of the INTEGER ſuppos'd to be *leſs*; and therefore, the *ſame* NUMERATOR remaining, the *Fraction* is *diminiſh'd*.

Thus, ſuppoſe as before, the *Fraction* $\frac{4}{7}$ be to be *divided* by the INTEGER 2, the QUOTIENT will be $\frac{2}{7}$, if the NUMERATOR 4 be *halv'd*, or *divided* by the INTEGER 2: Or,

If the DENOMINATOR 5 be *doubl'd*, or *multiply'd* by the INTEGER 2, the QUOTIENT will be $\frac{4}{10}$:

Now theſe *Quotients* $\frac{2}{7}$ and $\frac{4}{10}$, are *both* of the *ſame* VALUE, becauſe in *both* there is the *ſame* RATIO of the NUMERATOR to the DENOMINATOR; thus,

2 : 5

2 : 5 :: 4 : 10. And again,

The *Fraction* $\frac{3}{5}$, *divided* by the INTEGER 2, *quotes* $\frac{3}{10}$, *which* QUOTIENT is gain'd by *multiplying* the DENOMINATOR 5, by the INTEGER 2; the NUMERATOR 3 *not* being conveniently to be *divided* by the INTEGER 2; the *Number* 2 *not* being an ALIQUOT *Part* of the *Number* 3.

Now, from *what* has been ſaid, and from the *Conſideration*, that if the *ſame* DIVIDEND be *divided* by *different* DIVISORS, *different* QUOTIENTS will be *gain'd*, *viz.* ONE *Quotient* will be *ſo many* times *leſs* than the OTHER, as ONE *Diviſor* is *greater* than the OTHER; the GREATER *Diviſor* always giving the LESS *Quotient*.

From *this Conſideration*, I ſay, and from *what* has been *premis'd*, the REASON of the *Rule* muſt be very *manifeſt*: For,

When $\frac{4}{7}$ is to be *divided* by $\frac{2}{3}$, as in *Example* I. *what* does the RULE direct us *firſt* to *do*? Why, it directs us *firſt* to *multiply* the NUMERATOR of the DIVISOR, into the DENOMINATOR of the DIVIDEND; by which means we firſt *divide* $\frac{4}{7}$ by the INTEGER 2, and *gain* the QUOTIENT $\frac{4}{10}$, as has been before ſhewn.

Now, ſince $\frac{4}{7}$ *divided* by the INTEGER 2, truly *quotes* $\frac{4}{10}$, the *ſame* DIVIDEND $\frac{4}{7}$, *divided* by the *given* DIVISOR $\frac{2}{3}$, *i. e.* a third of 2, muſt *quote* 3 times the QUOTIENT $\frac{4}{10}$, *gain'd* by the DIVISOR 2; accordingly, the RULE, after we have thus *divided* by 2, directs us to *multiply* the DENOMINATOR 3, of the DIVISOR, into the NUMERATOR 4, of the DIVIDEND; thus, $\frac{4}{10} \times 3$ gives $\frac{12}{10}$, *which* is the *true* QUOTIENT requir'd from the *Diviſion* of $\frac{4}{7}$ by $\frac{2}{3}$: whence the *Truth* of the RULE is *manifeſt*.

EX-

EXAMPLE III.

Suppose $\frac{1}{2}$ of $\frac{1}{20}$ to be *divided* by $\frac{3}{4}$ of $\frac{1}{12}$.

First, *reduce* the COMPOUND *Fractions* to SIMPLE *ones*; thus,

$\frac{1}{2}$ of $\frac{1}{20}$ is $= \frac{1}{40}$, and $\frac{3}{4}$ of $\frac{1}{12} = \frac{3}{48} = \frac{1}{16}$.

Then *divide* thus,

$\frac{1}{16}) \frac{1}{40} (\frac{16}{40} = \frac{2}{5}$.

Or, *thus*,

$\frac{16}{1} \times \frac{1}{40} = \frac{16}{40} = \frac{2}{5}$ = the QUOTIENT.

EXAMPLE IV.

Let 48 $\frac{1}{2}$ be to be *divided* by 3 $\frac{2}{3}$.

Here *reduce* the MIX'D *Numbers* to IMPROPER *Fractions*, and 48 $\frac{1}{2}$ will become $= \frac{97}{2}$, and 3 $\frac{2}{3} = \frac{11}{3}$, by *Prop.* V. and then *gain* your *Quotient*; thus,

$\frac{11}{3}) \frac{97}{2} (\frac{291}{22} = 13 \frac{5}{22}$, the *Quotient*.

Or, *thus*,

$\frac{3}{11} \times \frac{97}{2} = \frac{291}{22} = 13 \frac{5}{22}$, the *Quotient* requir'd.

Thus in SYMBOLS,

Suppose the *Fraction* $\frac{A}{B}$ be to be *divided* by the *Fraction* $\frac{D}{C}$.

$\frac{D}{C}$

t

Quotient.

$$\frac{D}{C}\Big)\frac{A}{B}\Big(\frac{AC}{BD} = \frac{A}{B} \times \frac{C}{D}.$$

And hence it is manifest (which should be, and as I have before, under *Example* I. particularly *noted*) that the *same thing*, in every Respect, is *gain'd*, whether we *divide* by $\frac{D}{C}$, or *multiply* by $\frac{C}{D}$; for, the *same Quantity* will *either Way* come forth:

Also, *whether* we *divide* by $\frac{2}{3}$, or *multiply* by $\frac{3}{2}$; and *so* in *all others*, *be* the *Quantity* (thus to be *divided* or *multiply'd*) an INTEGER, or a FRACTION.

From the WHOLE then, it is *manifest*, that the *Doctrine* of MULTIPLICATION and DIVISION (in FRACTIONS) is, in short, *this*, *viz.*

The *Multiplication* of the NUMERATOR, *multiplies* the *Fraction*: And,

The *Division* of the NUMERATOR, *divides* the *Fraction*: Or,

The *Multiplication* of the DENOMINATOR, *divides* the *Fraction*: And,

The *Division* of the DENOMINATOR, *multiplies* the *Fraction*.

Whoso *multiplies* by $\frac{2}{3}$, he, in Reality, *multiplies* by 2, and *divides* by 3: for,

He *multiplies* the NUMERATOR of the *propos'd Quantity* (which is *multiplying*) by 2, and *multiplies* the DENOMINATOR thereof (which is *dividing*) by 3:

And whoſo *divides* by $\frac{2}{3}$, he, in Reality, *divides* by 2, and *multiplies* by 3: for,

He *multiplies* the DENOMINATOR of the *propos'd Quantity* (which is *dividing*) by 2, and *multiplies* the NUMERATOR thereof (which is *multiplying*) by 3.

SCHOLIUM.

It is to be obſerv'd, that the QUOTIENT *gain'd* by *dividing any propos'd Quantity* or DIVIDEND, by a PROPER *Fraction*, is always *greater* than the DIVIDEND: For,

In DIVISION, a UNIT is to the QUOTIENT, as the DIVISOR is to the DIVIDEND; from the 23d *Defin.* of the 7th *Book* of *Euclid's Elements.*

Thus, 1 : $\left\{\begin{matrix}\frac{3}{8}\\16\end{matrix}\right\}$ (The *Quotient*) : : $\left\{\begin{matrix}\frac{1}{3}\\\frac{1}{2}\end{matrix}\right\}$ (The *Diviſor*) : $\left\{\begin{matrix}\frac{1}{8}\\8\end{matrix}\right\}$ The *Dividend.*

I ſhall now propoſe a few *Queſtions* for the *Learner's* Exerciſe in *Multiplication* and *Diviſion* of *Vulgar Fractions*, and then proceed to *Treat of* DECIMAL and SEXAGESIMAL *Fractions.*

QUESTIONS.

I. *What* Number is that which if *divided* by $8\frac{3}{8}$, will *quote* $7\frac{1}{2}$?

Anſwer. $62\frac{1}{16}$.

II. *What*

II. *What* Number is $6\frac{1}{9}$ the *third Part* of?

Answer. $8\frac{3}{9}$.

III. In $478\frac{3}{4}$ *Rods* (*each* $5\frac{1}{2}$ *Yards*) how many *Yards*?

Answer. $2633\frac{1}{8}$.

IV. *What* Number *multiply'd* by $7\frac{1}{2}$ will *produce* 64?

Answer. $8\frac{8}{17}$.

V. *What* Number is $18\frac{3}{9}$ the *Triple* of?

Answer. $6\frac{1}{9}$.

VI. In $2633\frac{1}{8}$ *Yards*, how many *Rods* (*each* $5\frac{1}{2}$ *Yards*)?

Answer. $478\frac{3}{4}$.

Of DECIMAL *FRACTIONS.*

HAVING before given the *Learner* ſeveral EXPLANATIONS of the NATURE of *Fractions* in *general*; and having alſo *explain'd* the *various* REDUCTIONS and the *Arithmetical* OPERATIONS to be *us'd* in COMMON or VULGAR *Fractions*.

I come now to ſay ſomething as well of SEXAGESIMAL *Fractions*, which were moſt in uſe with the ANCIENTS, as of DECIMAL *Fractions*, which are moſt us'd by the MODERNS.

Firſt, then, I ſhall treat of DECIMAL *Fractions.*

In the DECIMAL *Parts* of the MODERNS, every INTEGER is ſuppos'd to be *divided* into 10 *(Decimal) Parts*; and *every* TENTH into 10 *Centeſimal*, or *Hundredth Parts*; *every* CENTESIMAL into 10 *Milleſimal*, or *Thouſandth Parts*, and ſo on; by *deſcending* in a continual *Subdecuple* RATIO below the *Place* of UNITY; as we *aſcend* above the *Place* of UNITY in a continual *decuple* RATIO to TENS, HUNDREDS, THOUSANDS, *&c.*

Which RATIO, indeed, of *dividing* an INTEGER, is much more commodious than the SEXAGENARY *one* of the ANCIENTS; becauſe DECIMAL *Fractions* are *manag'd* in the very *ſame Manner as* WHOLE *Numbers* are; and *ſuffer no Difficulty* in CALCULATION; and, indeed, for that very Reaſon, *viz.* becauſe they *encreaſe* and *decreaſe* orderly and continually, in the very *ſame* RATIO *as* WHOLE *Numbers do*:

The *Manner* of *which* continu'd *Decimal Encreaſe* and *Decreaſe* (from whence *Fractions* of *this Sort*, take their *Name*) muſt, by *Inſpection*, appear plain from the following

TABLE

TABLE *of* NOTATION.

Integers.							*Decimal Parts.*						
6	5	4	3	2	1	0,	1	2	3	4	5	6, &c. *ad Infinitum.*	
C's of Thousands.	*X's of Thousands.*	*Thousands.*	*Hundreds.*	*Tens.*	*Units.*		*Tenth*, or 10th *Part of an Unit.*	*Hundredths*, or 100th	*Thousandths*, or 1000th	*X. Thousandths*, or 10000th	*C. Thousandths*, or 100000th	*Millionths*, or 1000000th	*Parts of an Unit.*
Millions.													

Upon due Conſideration, the *Learner* may, from *this* TABLE, eaſily underſtand *how* to *write down* and *expreſs* truly *any decimal* PARTS: For,

Since *each* of the *Figures* above mention'd, in *what Place ſoever* (*whether* among the INTEGERS, or the *decimal* PARTS) *ſignifies* juſt *ten times* as much as the *ſame Figure* would *do*, in the *Place* next *below* it towards the

the *right Hand*; and but a *tenth* Part of *what* it would *ſignify* in the *Place* next *above* it towards the *left Hand*: It plainly follows, that,

As the *Firſt*, *Second*, *Third*, &c. *Places* ABOVE *that* of UNITS, ſignify *Tens*, *Hundreds*, *Thouſands*, &c.

So the *Firſt*, *Second*, *Third*, &c. *Places*, BELOW *that* of UNITS, will ſignify *Tenths*, *Hundredths*, *Thouſandths*, &c. 10th *Parts*, 100th *Parts*, 1000th *Parts*, &c. of an UNIT. Hence,

The REASON why *theſe* DENOMINATORS of DECIMAL *Parts* are *very ſeldom*, or *never*, SET DOWN, *is*, becauſe the *peculiar* VALUE or DENOMINATION of *each* PART or PLACE, is *known* BY ITS DISTANCE FROM UNITY: But,

DECIMAL *Parts* being written in the ſame *Line* with the INTEGERS, and uſually *diſtinguiſh'd* from *them* by a COMMA, as in the *foregoing* TABLE (though ſometimes by a POINT, thus, 0.5; and ſometimes thus, 0⌊5;) *be the Number of them what it will*, take their DENOMINATION from the PLACE of their *laſt Figure* on the *right Hand*;

Thus { 0,5 / 0,50 / 0,500 } are { 5 *Tenths.* / 50 *Hundredths.* / 500 *Thouſandths.* }

And,

Thus { 0,5 / 0,05 / 0,005 } are { 5 *Tenths.* / 5 *Hundredths.* / 5 *Thouſandths.* }

And thus the *Number* 234,567, *ſignifies* Two hundred thirty-four UNITS, together with Five hundred ſixty-ſeven *Milleſimal*, or *Thouſandth* PARTS of an *Unit*; the

LAST

LAST *Figure* of the *Decimal* PARTS being in the *Place* of THOUSANDTHS:

For 500 THOUSANDTHS, are = 5 TENTHS; and 60 THOUSANDTHS = 6 HUNDREDTHS; therefore,

The *Decimal* PARTS (,567) may *as well* be collectively call'd 567 THOUSANDTHS (*Thouſandths* being the DENOMINATOR of the LAST *Figure*) *as* diſtinctly call'd 5 *Tenths*, 6 *Hundredths*, and 7 *Thouſandths*: The *ſame* in *Others*.

Here it may be noted, that *Cyphers* plac'd after, or on the *right* Hand of *Decimal* Parts, do neither *encreaſe* nor *decreaſe* their VALUE: So that 0,5 and 0,50 and 0,500 and 0,5000 are *Decimals*, all of *one* and the *ſame* VALUE; for,

5 : 10 :: 50 : 100 :: 500 : 1000 :: 5000 : 10000. But,

Cyphers prefix'd before, that is plac'd on the *left* Hand of the *Decimal* Parts, do remove them farther from UNITY (*according to which Diſtance* we are to judge of their VALUE) and conſequently *decreaſe* their VALUE: Thus,

0,0005, are ten times *leſs* than 0,005, which are ten times *leſs* than 0,05, which are ten times *leſs* than 0,5.

And now, I think, I have ſufficiently *explain'd* the NATURE of DECIMALS, and the *Manner* of their NOTATION:

The leaſt Difficulty, therefore, in the *Arithmetical* OPERATIONS to be *us'd* in this *Kind* of *Fractions*, can't, I preſume, occur to the *Learner*, when he is once *Maſter* of what has been ſaid:

But

But before I meddle with thoſe *Operations*, I ſhall ſhew how

To Reduce a VULGAR *Fraction to a* DECIMAL.

SINCE 'tis,

As the DENOMINATOR of the VULGAR *Fraction*,

To its NUMERATOR;

So is the DENOMINATOR of the DECIMAL *Fraction*,

To its NUMERATOR:

And ſince the DENOMINATOR of a DECIMAL *Fraction* is always 10, 100, 1000, &c. *i. e.* 1 with *one* or *more* CYPHERS; we conſequently gain (to *reduce* a VULGAR *Fraction* to DECIMAL *Parts*) the *following*

RULE.

To the NUMERATOR *add as many* CYPHERS *as are needful (after the ſeparating Point or Comma, which may denote the Degrees deſcending below* UNITY*) and then divide it by the* DENOMINATOR; *the* QUOTIENT *(if Nought remains) will be* DECIMAL *Parts, equivalent to the ordinary Fraction given:* For,

By *theſe Means*, DIVISION, which the NOTATION of the *Fraction* ſhews, is actually *perform'd*: thus,

$\frac{4}{5}$ is exactly = to 0,8; and $\frac{3}{8}$ exactly = 0,375; as will be manifeſt by their *Diviſion* following:

5)

5(4,0(0,8 — 8)3,0000(0,375 — 24 — 60 — 56 — 40 — 40 — 0

But if (which often happens) *such* a *Fraction* occurs which either can't *at all*, or can't *accurately* be *expreſs'd*, without a more tedious *Operation* in *Decimal Parts*, we may carry on the WORK as *far* as we think *fit*, or, until *what remains* is *ſo ſmall as* to be ſafely *neglected*; thus, $\frac{1}{3}$ is nearly = 0,3333, *&c.*; $\frac{2}{3}$ is nearly = 0,6666, *&c.*; $\frac{1}{6}$ is nearly = 0,16666, *&c.*; $\frac{5}{6}$ nearly = 0,8333, *&c.*; $\frac{10}{11}$ nearly = 0,909090, *&c.*; $\frac{5}{11}$ nearly = 0,454545, *&c.*; $\frac{3}{7}$ nearly = 0,428571428, *&c.*; as by the following OPERATIONS, according to the foregoing RULE.

3)1,0000(0,3333 + ... 10 10 10 — 1

3)2,0000(0,6666 + ... 20 20 20 — 2

6)1,0000(0,1666 + ... 40 40 40 — 4

6)5,0000(0,8333 + ... 20 20 20 — 2

11)10,000000(0,909090+ 11)5,000000(0,454545+

100 60
100 50
10 60
50
60
—
5

7)3,00000000(0,42857142 8+

20
60
40
50
10
30
20
60
—
4

(In all *which Caſes*, and *Others like them*, we may obſerve *ſooner* or *later*, a RETURN of the *ſame* NUMBERS, *which* may be *continu'd* as far as we pleaſe).

And $\frac{3}{32}$ is exactly = 0,09375; and $\frac{23}{64}$ exactly = 0,359375.

(In *which*, and *Others like them*, the propos'd *Fractions* are *exactly* REDUC'D, tho' late, into DECIMAL *Parts*:) As will be manifeſt by *dividing* their NUMERATORS 3 and 23 (firſt adding to them a ſufficient Number of CYPHERS) by the DENOMINATORS 32 and 64.

32)

32)3,00000(0,09375
288
120
96
240
224
160
160
0

64)23,000000(0,359375
192
380
320
600
576
240
192
480
448
320
320
0

In *Caſes* of *that Kind*, *where* the *Decimal Parts run out* too tediouſly, if not infinitely, 'tis ſufficient to take what is very near the *Truth*, for the *Truth itſelf*, *i. e.* We may, having carried on the *Operation* to a convenient Length, neglect the *Remainder*: As thus, for $\frac{1}{3}$, $\frac{5}{6}$, $\frac{10}{11}$, $\frac{3}{32}$, *&c.* may be taken 0,3333; 0,8333; 0,90909; 0,0937: Or alſo, 0,333; 0,833; 0,909; 0,094: Or may be 0,33: Or, 0,83; 0,91; 0,09: According as there is greater or leſſer Accuracy requir'd.

ADDITION *of* DECIMAL *Fractions.*

WHETHER *pure* DECIMAL *Fractions*, or INTEGERS with DECIMAL *Fractions*, are propos'd to be *added together*, great Care muſt be taken to place them in *Rows* or *Columns*, ſo orderly one under another, that the *Figures* may ſtand reſpectively in their *proper* or *homogeneous* PLACES, *viz.*

Of the WHOLE *Numbers*, the UNITS muſt make *one Column*, the TENS *another*, the HUNDREDS *another*, and ſo on; So,

Of the *Decimal* PARTS, the TENTHS muſt make *one Row*, the HUNDREDTHS *another*, the THOUSANDTHS *another*; and ſo on, according to their reſpective NOTATION.

The *Figures* being thus truly *plac'd*, may be *added together* by *this*

RULE.

Begin at the FIRST COLUMN *on the right Hand, and* ADD *juſt as if they were all* INTEGERS *of* ONE DENOMINATION; *and from their* AGGREGATE *or* SUM, *cut off, for* DECIMAL PARTS, *all the* FIGURES *that are to the* RIGHT HAND *of the* UNITS *Place.*

The RATIONALE of this PROCESS, is the *ſame* with THAT in WHOLE *Numbers.*

Take the following *Examples:*

EX-

EXAMPLE I.

Let the *Decimals* 0,6; 0,48; 0,542; 0,075; 0,009, be to be *added* together.

Place them *thus*,	0,6	Not *thus*,	,6
	0,48		,48
	0,542		,542
	0,075		,075
	0,009		,009
	1,706		

EXAMPLE II.

Let 0,1855; 0,3489; 0,1875; 0,1642; 0,0139, be to be *added* together.

0,1855
0,3489
0,1875
0,1642
0,0139

0,9

Note, When the *Decimals* propos'd to be *added*, conſiſt not *all* of the *ſame* Number of *Places*, you may, if you pleaſe, fill up the *vacant Places* with *Cyphers*; for their VALUES will *not* be thereby *alter'd*, as has been before ſhewn.

EXAMPLE III.

Let 25,876542; 36,87878; 41,7999; 10,575; 6,58, be to be *added* together,

Thus,	25,876542	Or *thus,*	25,876542
	36,87878		36,8787800
	41,7999		41,7999000
	10,575		10,5750000
	6,58		6,5800000
	121,710222		121,710222

EXAMPLE IV.

Let 88,05; 0,789; 643,0029; 246,7081; 9,2, be to be *added* together,

Thus,	88,05	Or *thus,*	88,0500
	0,789		0,7800
	643,0029		643,0029
	246,7081		246,7081
	9,2		9,2000
	987,75		987,7500

SUB-

SUBTRACTION *of* DECIMAL *Fractions.*

WHEN DECIMALS, or INTEGERS with DECIMALS, are propos'd to be *ſubtracted* from *one another*, the *ſame* particular Care muſt be taken to ſubſcribe the *Figures* of the SUBTRAHEND, in their *homogeneous Places*, according to the DIRECTIONS given *above* in ADDITION of *Decimals*, and then they may be *ſubtracted* by this

RULE.

Begin at the RIGHT *Hand, and ſubtract as if they were all* WHOLE *Numbers; and from the* REMAINDER *or* DIFFERENCE, *cut off, for* DECIMAL PARTS, ALL *the* FIGURES *that are to the* RIGHT *Hand of the* PLACE *of* UNITS.

The RATIONALE of *this* RULE, is alſo the *ſame* with THAT of SUBTRACTION of INTEGERS.

EXAMPLE I.

Suppoſe the DECIMAL *Fraction* 0,852, be to be *ſubtracted* from the DECIMAL *Fraction* 0,972.

0,972
0,852
0,120

EX-

EXAMPLE II.

Suppoſe the *Decimal* PARTS 0,575 be to be *taken* from the INTEGER 41.

Thus,	41,	Or *thus,*	41,000
	0,575		0,575
	40,425		40,425 The *Remainder.*

EXAMPLE III.

Suppoſe 242,9 be to be *taken away* from 848,725.

Thus,	848,725	Or *thus,*	848,725
	242,9		242,900
	605,825		605,825 The *Remainder.*

EXAMPLE IV.

Suppoſe 19 to be *ſubtracted* from 99,75.

Thus,	99,75	Or *thus,*	99,75
	19,		19,00
	80,75		80,75 The *Remainder.*

EX-

EXAMPLE V.

Suppoſe 184,00085 be to be *ſubtracted* from 786,125.

Thus,		Or *thus,*	
786,125		786,12500	
184,00085		184,00085	
602,12415		602,12415	The *Remainder.*

EXAMPLE VI.

Suppoſe 32,0625 be to be *ſubtracted* from 33,0625.

33,0625	
32,0625	
1,0000	Or 1, The *Remainder.*

In ſome of the foregoing *Examples*, you ſee the *vacant Places* in the *Decimal* PARTS may be *fill'd up*, if you *pleaſe*, as is *noted* in ADDITION.

And now, having plainly ſhewn *how* DECIMALS are to be *manag'd* in ADDITION and SUBTRACTION, I ſhall proceed to ſhew how they are to be *multiply'd* and *divided.*

MULTIPLICATION *of* DECIMAL *Fractions.*

WHETHER the FACTORS, or *two Numbers* propos'd *to be* MULTIPLY'D into *one another*, are BOTH *pure* DECIMALS, or BOTH MIX'D *Numbers*; or ONE of *them* an INTEGER, and the OTHER, *either* a *pure* DECIMAL, or a MIX'D *Number*, the MULTIPLICATION may be *perform'd* (without the *Care* of *placing* them in ſuch *Order* as is requir'd in ADDITION and SUBTRACTION) by only obſerving the following *general*

RULE.

Place and multiply the FACTORS, *as if they were* ALL WHOLE *Numbers, then cut off from the* PRODUCT, *ſo many* FIGURES *for* DECIMAL PARTS, *as there are* PLACES *of* DECIMAL PARTS *in* BOTH *the* FACTORS *together: But if (after* MULTIPLICATION *is finiſh'd) it happens that there are not ſo many Figures in the* PRODUCT *as the* RULE *requires to be cut off for* DECIMAL PARTS, *ſuch* DEFECT *muſt be ſupply'd by preſixing* CYPHERS *to the* LEFT *Hand of the* PRODUCT.

EXAMPLE I.

Let 8,5 be to be *multiply'd* by 6.

8,5
6

51,0 The *Product.*

EX-

EXAMPLE II.

Suppoſe 8,5 be to be *multiply'd* by 6,5.

```
 8,5
 6,5
-----
 425
510
-----
55,25  The Product.
```

EXAMPLE III.

Suppoſe ,05 be to be *multiply'd* by ,05.

```
 ,05
 ,05
-----
,0025  The Product.
```

The TRUTH of the *foregoing* RULE may appear plain, by conſidering *whether* the PRODUCTS *obtain'd* by IT in the *preceding* familiar *Examples*, are the TRUE *Products or not*; thus,

Firſt, In *Example* I. Let us conſider *what 'tis* we *do* when we *multiply* 8,5 by 6: Why, we *firſt multiply* the 5 *Tenths* by 6, which manifeſtly produces 30 *Tenths*, which are = 3 INTEGERS; and then we *multiply* the 8 *Integers* by 6, which produces 48 *Integers* more; now 48 + 3 are = 51, *which* 51 (not 510) is the *true* PRODUCT *requir'd*; therefore the 0 is *cut off* for a DECIMAL, according to the DIRECTIONS of the RULE, there being but *one* DECIMAL *Place* in *both* FACTORS *together*. Again,

In *Example* II. The MULTIPLICAND 8,5 *multiply'd*, in the firſt Place, by ,5, produces 4,25, *viz.* the Half of 8,50, which is = 8,5; for to *multiply* any *Quantity* by ,5 (= $\frac{1}{2}$) is but to *halve* it, or *divide* it by 2; and then the *ſame* MULTIPLICAND 8,5, *multiply'd* by 6, produces 51, as has been juſt now ſhewn; the *Sum* of theſe 2 PRODUCTS, therefore, *viz.* (4,25 + 51 =) 55,25, is the *true* PRODUCT *requir'd.*

But, by the RULE, the SAME *Product* is *obtain'd*, *which* PROVES the RULE to be TRUE.

Or otherwiſe,

Let the DENOMINATORS of the *given* DECIMALS be *actually ſet down* under *their reſpective* NUMERATORS, and then let the *propos'd* FACTORS be *reduc'd* and *multiply'd* together, as VULGAR *Fractions* are; thus,

(In the *ſame Example*) If 8 $\frac{5}{10}$ be *multiply'd* by 6 $\frac{5}{10}$, *i. e.* (*Theſe* MIX'D *Numbers* being *reduc'd* to IMPROPER *Fractions*): If $\frac{85}{10}$ be *multiply'd* by $\frac{65}{10}$, we ſhall find the PRODUCT to be $\frac{5525}{100}$, *which* IMPROPER *Fraction* is (by REDUCTION) = 55 $\frac{25}{100}$, or 55,25, as before: And again, $\frac{5}{100}$ *multiply'd* by $\frac{5}{100}$, *produce* $\frac{25}{10000}$ = 0025, as in *Example* III.

Whence the *Neceſſity* of *prefixing* always juſt *ſo many* CYPHERS to the PRODUCT (in order to *determine* its *true* VALUE) *as* will *ſupply* ſuch *Deficiency* (when *any* happen) as is above mentioned, is plainly ſeen: From *all which*, I *conclude*, that the TRUTH of the RULE is ſufficiently *evinc'd.*

E X-

EXAMPLE IV.

Multiply 728,543 by 24,98.

728,543
24,98

5828344
6556887
2914172
1457086

18199,00414 The *Product.*

EXAMPLE V.

Multiply 3546 by ,0725.

3546
,0725

17730
7092
24822

257,0850 The *Product.*

EXAMPLE I.

Suppoſe I would *multiply* 58,25435 by 21,234, and would have in the *Product* (as ſufficient) but TWO *Decimal Places*, altho', if theſe FACTORS were *multiply'd* together at large, there would be EIGHT *Decimal Places* in the *Product*, by the *general* RULE.

Now, if they are *plac'd*, and the *Operation* wrought as above directed, they'll ſtand *thus*,

```
 58,25435
 432,12
-----------
116508
  5826
  1165
   174
    24
-----------
1236,97
```

The *Operation* at large.

```
 58,25435
   21,234
-------------
  23|301740
 174|76305
1165|0870
5825|435
116508|70
-------------
1236,97386790
```

EX-

EXAMPLE II.

Suppoſe I would *multiply* 234,012 by 36,08, and would have the *Product* INTEGERS, without any *Decimal* PARTS.

Here, according to the *Directions* above, the UNIT's *Place* of the *leſſer* FACTOR muſt be ſet under the UNIT's *Place* of the *greater*, becauſe *all* the *Decimal Parts* are to be left out of the *Product*; thus,

```
234,012
80,63
-------
7020
1404
  19
-------
8443
```

The *Common* Way.

```
   234,012
     36,08
   -------
    18|72096
1404 |0720
7020 |36
-------------
8443,15296
```

EXAMPLE III.

Suppoſe I would *multiply* the *Decimal Parts* 0,23456 by the ſame *Decimal Parts* 0,23456, and would have a *Product* of SIX *Decimal Places.*

The *Figures* PLAC'D, and the OPERATION *perform'd*, according to the DIRECTIONS above, will be thus,

```
 0,234560
   65432,0
 -------
   46912
    7036
     938
     117
      15
 -------
 ,055018
```

The *Operation* at large.

```
   ,23456
   ,23456
 -------
  140736
 117280
 93824
 70368
 46912
 -------------
 ,0550183936
```

If

If THREE *Decimal Places* ſhould be thought a *ſufficient* PRODUCT by the *Multiplication* of the *ſame* FACTORS, the OPERATION would be no more than *this*,

0,23456
65432,0

46
9

,055 The PRODUCT *requir'd.*

When it happens (as it does in this *Example*) that the *Number* of *Figures* produc'd by this *contracted* Method of MULTIPLYING, are not *ſo many as* are *aſſign'd* for *Decimal* PLACES in the PRODUCT, the DEFECT muſt be *ſupply'd* by *prefixing* CYPHERS in *like* Manner, *as* was *obſerv'd* under the *general* RULE.

And here, I think, it may not be improper to ſhew that this *Contraction* of MULTIPLICATION may be ſometimes uſeful or advantageous in WHOLE *Numbers*; as in the *following* ASTRONOMICAL CALCULATION, and the *like.*

Let the *Sun's* DECLINATION be requir'd when *it* is in *Taurus* 27° 34′, *its greateſt* DECLINATION being 23° 29′.

This will be found by the *Rule* of *Proportion*; thus,

As the *Radius*

To the *Sine* of the *Sun's* GREATEST *Declination*;

So is the *Sine* of *its Longitude*,

To the *Sine* of *its* PRESENT *Declination*, i. e.

100000 : 39848 :: 84402 : The *Sine* ſought.

84402

	79696
159	3920
1593	92
31878	4

100000)33632|50896(The *Sine* of *its* PRESENT *Declination* = 19° 39′ +.

Here, in *this* OPERATION at *large*, becauſe the FIVE *laſt Figures* of the PRODUCT are to be *cut off*, for the *Five* CYPHERS of the DIVISOR, the *remaining Figures*, *viz.* the FIVE *firſt*, being the QUOTIENT, and *ſufficient* for *this* USE, may be *ſecur'd* by *abridging* the WORK; *thus*,

39848
20448

31878
1593
159
2

33632 = The *Sine* of 19° 39′ +, as above.

The REASON of this *abridg'd Method* of MULTIPLICATION, muſt appear very plain by only *comparing* it with the *Operation* AT LARGE, and *from what follows*.

'Tis plain, that *at what Diſtance* or *Number* of PLACES *ſoever* from the *Unit's* PLACE of the MULTIPLICAND, you *place* the *Unit's* PLACE of the MULTIPLIER, *juſt ſo many* PLACES in the PRODUCT will be *cut off* in the INTEGERS, or ſecur'd in the *Decimal* PARTS; As for Inſtance;

If

If you would cut off 5 in the INTEGERS, you ſet it under the 5th PLACE in the INTEGERS from UNITY, and begin to *multiply* it by the *Figure* in that 5th PLACE; And,

If you would ſecure 5 PLACES in the PARTS, you ſet the *Unit's* PLACE of the MULTIPLIER, under the 5th PLACE of PARTS of the MULTIPLICAND, and do the like.

Now, by *placing* the reſt of the Figures of the MULTIPLIER in *the Inverſe Order*, and *multiplying* as above directed, 'tis plain that you *multiply* by the LAST Figure *firſt*, and the LAST, SAVE ONE, *next*, and ſo on; by which means you all along *drop* thoſe *Figures* which ſtand in *thoſe Places* you deſign to *cut off*.

DIVISION *of* DECIMAL *Fractions.*

AS MULTIPLICATION, *ſo* is DIVISION of DECIMALS to be perform'd, exactly as the *Operation* is when the DIVIDEND and DIVISOR are *both* WHOLE NUMBERS: But then let the *true* VALUE of the QUOTIENT be *determin'd* by the following *general*

RULE.

After the DIVISION *is perform'd, there muſt always be juſt as many* PLACES *of Decimal* PARTS *in the* DIVISOR *and* QUOTIENT *together, as there are in the* DIVIDEND *alone.*

The REASON of this is manifeſt, it being plainly deduc'd from *that* of MULTIPLICATION: For,

Since

Since the *Decimal* PLACES in the PRODUCT, ariſing from the *Multiplication* of any two given FACTORS, muſt be always = the *Number* of PLACES of *Decimal* PARTS in *both* the FACTORS together; and ſince the *Multiplication* of the DIVISOR and QUOTIENT always produces the DIVIDEND, the RULE above manifeſtly follows.

Now, the *particular* CASES that may happen in DIVISION of DECIMALS, are *four*, *viz.*

I. It may happen that the PLACES of *Decimal* PARTS in the DIVIDEND, do, in Number, *exceed* THOSE in the DIVISOR: In *which* CASE, by the *general* RULE, the *Exceſs* muſt be cut off for *Decimal* PARTS in the QUOTIENT.

II. It may happen that the *Number* of PLACES of *Decimal* PARTS in the DIVIDEND, is = the *Number* of THOSE in the DIVISOR: In *which* CASE, the QUOTIENT, by the *general* RULE, will be always INTEGERS.

III. It may happen that the *Number* of PLACES of DECIMALS in the DIVIDEND, is *leſs* than the *Number* of THOSE in the DIVISOR: In *which* CASE, CYPHERS muſt be added to the *Right Hand* of the DIVIDEND, 'till the NUMBER of *Decimal* PLACES in it, is either *equal to*, or *greater than* THAT in the DIVISOR, and then (the DIVISION being *perform'd*) the QUOTIENT muſt, accordingly, be *either* all INTEGERS (as in CASE II.) *or* muſt contain a determinate *Number* of *Decimal* PARTS, as in CASE I.

IV. It may happen, when the DIVISION is *perform'd*, that the QUOTIENT does *not* conſiſt of *ſo many* FIGURES as the *general* RULE requires to be aſſigned, or *cut off* for *Decimal* PARTS therein: And in *ſuch* CASE, the *Defect*

Defect muſt be *ſupply'd* with CYPHERS *prefix'd*, or *plac'd* to the *Left Hand* of the *Figures* in the QUOTIENT, which then will be the *true* PRODUCT requir'd.

All which particular CASES and DIRECTIONS ſhall be *illuſtrated*, in *Order*, by the following *Examples*.

CASE I.

EXAMPLE I.

Let 699.678 be *divided* by 56.7; *Thus*,

```
56.7)699.678(12.34 the Quotient.
     1326
      1927
       2268
     ------
        0
```

In this *Example*, there being *three* PLACES of *Decimal* PARTS in the DIVIDEND, and but *one* in the DIVISOR, the *Exceſs*, *viz.* *two* FIGURES, muſt be therefore, by the *general* RULE, *cut off* for *Decimal* PARTS in the QUOTIENT: And thus the *true* QUOTIENT is, *not* 1234; *not* 123,4; *not* 1,234; but 12,34; and the like in others.

CASE

CASE I.

EXAMPLE II.

Suppoſe 699.678 be to be *divided* by 567.

567)699,678(1,234 The *true Quotient.*
1326
1927
2268
0

CASE I.

EXAMPLE III.

Suppoſe ,699678 be to be *divided* by ,567.

0,567)0,699678(1,234 The *true Quotient.*
1326
1927
2268
0

CASE II.

EXAMPLE IV.

Suppoſe 672,75 be to be *divided* by 3,45.

3,45)672,75(195 The *true Quotient.*
3277
1725
0

CASE

CASE II.

EXAMPLE V.

Suppoſe 0,9855 be to be *divided* by 0,0135.

0,0135)0,9855(73 The *true Quotient.*
405

0

In theſe 2 laſt *Examples*, the PLACES of *Decimal* PARTS in the DIVIDENDS, are reſpectively *equal* in *Number* to THOSE in the DIVISORS; therefore the QUOTIENTS are WHOLE NUMBERS, according to the *general* RULE.

CASE III.

EXAMPLE VI.

Suppoſe 128,2 be to be *divided* by 5,128.

5,128)128,200(25 The *true Quotient* as in CASE II.
25640

0

CASE III.

EXAMPLE VII.

Suppoſe 976 be to be *divided* by 0,125.

0,125)976.000(7808 The *true Quotient per* CASE II.
1010
1000
0

CASE III.

EXAMPLE VIII.

Suppoſe 89,8 be to be *divided* by 6,25.

6,25)89,80000(14,368 The *true Quotient per* CASE I.
2730
2300
4250
5000
0

In the foregoing 6th, 7th, and 8th *Examples*, the *Learner* may obſerve, that CYPHERS are *annex'd* to the given DIVIDENDS, as CASE III. directs, in order to obtain the *true* QUOTIENTS.

CASE

CASE IV.

EXAMPLE IX.

Suppoſe 4,12992 be to be *divided* by 86,4.

86,4)4,12992(,0478 The *true Quotient.*
6739
6912

0

CASE IV.

EXAMPLE X.

Suppoſe 0,00003264 be to be *divided* by 0,136.

0,136)0,00003264(,00024 The *true Quotient.*
544

0

And here it may be alſo obſerv'd, that, in the 9th *Example*, the DIVISION affords but 3 *Figures* in the QUOTIENT; whereas, by the *general* RULE, there ſhould be 4 *Decimal* PLACES in *it*; for there are *five Decimal* PLACES in the given DIVIDEND, and but *one* in the DIVISOR; therefore *one* CYPHER is *prefix'd* as the 4th CASE directs, that the *Defect* may be *ſupply'd*, and the *true* QUOTIENT *obtain'd*.

The *like* may be obſerv'd of *Example* X.

Moreover, it may be obſerv'd, that in *All* the foregoing *Examples* the QUOTIENTS are *exact*; but when

it otherwiſe happens (as it very often does) *viz.* That when the DIVISION is *perform'd* there be a REMAINDER; the QUOTIENT then is NOT *exact*:

Now, if *ſuch* QUOTIENT be at any time thought not *ſufficient*, you may, by adding always a CYPHER to the *remaining Number*, continue the DIVISION in *Decimal* PARTS as far as you pleaſe, 'till at length, either an *exact* QUOTIENT (by having no *Remainder*) be obtain'd, or at leaſt, an *Approximation* to what Degree of *Accuracy* you think fit: And this is generally the *Caſe* when VULGAR *Fractions* are to be *reduc'd* to DECIMALS, which has been before ſhewn: For,

(The NUMERATOR of a *Fraction* being of the NATURE of a DIVIDEND; and the DENOMINATOR, of a DIVISOR) you are to add to the NUMERATOR, or DIVIDEND, *what Number* of CYPHERS you pleaſe, and then *divide* by the DENOMINATOR, to obtain a QUOTIENT in *Decimal* PARTS, exactly *equivalent* to the VULGAR *Fraction* given, or ſo near it as you ſhall think *neceſſary* to approach.

See the ſeveral *Examples* under the RULE for *reduceing* of VULGAR *Fractions* to DECIMALS.

EXAMPLE XI.

Suppoſe 16,0 be to be *divided* by 128.

128)16,0(0,125 The *exact Quotient*.
320
640
0

EX-

EXAMPLE XII.

Suppoſe 2,345 be to be *divided* by 55,3.

55,3)2,345(,04240506 + The *Quotient* nearly.

1330
2240
2800
3500

182 The *Remainder*.

In this laſt *Example* there being a *Remainder*, the QUOTIENT may, according to the DIRECTIONS above, be carried on as far as you pleaſe.

Note, In DIVISION it is univerſally true that *whatſoever* be the *Value* or *Quality* of that PLACE, or FIGURE of the DIVIDEND, under which the UNIT's *Place* of the DIVISOR, at asking the *firſt Queſtion*, falls; the *ſame Value* or *Quality* the *firſt* FIGURE of the QUOTIENT will always be of.

As (for Inſtance) in *Example* I.

56.7)699.678(12.34
567

0

In asking the *firſt Queſtion*, viz. *How many times* 56,7 are there in 699? the 6 in the UNIT's *Place* of the DIVISOR, falls under the 9, which is in the *Place* of TENS in the DIVIDEND; therefore (1) the FIRST *Figure* of the QUOTIENT, muſt ſtand in the *Place* of TENS; wherefore the NEXT *Figure* (2) will, of courſe, ſtand in

in the UNIT's *Place*; and conſequently *All* the *following* FIGURES muſt be *Decimal* PARTS. Again,

In the *laſt Example* 55,3)2,345(42, *&c.*
553

0

In asking the *firſt Queſtion*, the 5 which is in the UNIT's *Place* of the DIVISOR, falls under the 4 which is the *Place* of TENTHS, or the ſecond *Place* of the DECIMALS in the DIVIDEND; therefore the FIRST *Figure* obtained in the QUOTIENT *(viz.* 4*)* is ſo many TENTHS, and muſt therefore ſtand in the SECOND *Place* of DECIMALS; and to make it do ſo, *one* CYPHER muſt be prefix'd to it; and thus 0,042, *&c.* is the *true* QUOTIENT.

And by this Method (which is a very good one) may the *Value* or *Quality* of the QUOTIENT, in *all Caſes* in DIVISION, be truly *determin'd*.

And by theſe Means it happens that there are always as many *Places* of *Decimal* PARTS in the DIVISOR and QUOTIENT together, as there are in the DIVIDEND. And thus the TRUTH of the foregoing *general* RULE is otherwiſe *illuſtrated* and *confirmed*.

And now, as I have ſhewn how to *abridge*, occaſionally, a *long* OPERATION in MULTIPLICATION, ſo I ſhall here give the *Learner* a CONTRACTION of DIVISION, which may be very uſeful, and ſave much Labour, when the DIVISOR is an *irrational* or *indefinite* Number, either of INTEGERS, or *Decimal* PARTS; or whene'er the *laſt* FIGURE in the DIVIDEND, or DIVISOR, is *uncertain*; for in *this Caſe*, all beyond this FIGURE is alſo *uncertain*; ſo that the DIVIDEND and DIVISOR may, in *ſuch Caſes*, be both *curtail'd*: And the *latter*

Part

Part of the WORK *omitted* as *unneceſſary*, and the CONTRACTION perform'd after the following *Method*;

Take as many of the *firſt* FIGURES of the given DIVISOR, as you judge *neceſſary* or *ſufficient* for your *firſt* DIVISOR, with which begin to *divide*, and then at each following particular DIVISION or OPERATION, *drop* or *omit* ONE of the FIGURES of the DIVISOR towards the LEFT *Hand* (always having Regard to what would ariſe and be brought to the *next* FIGURE, from the FIGURE ſo *omitted*) and thus you muſt go on, 'till you have got a competent QUOTIENT: An *Example* or *two* will make all this very plain.

EXAMPLE I.

Suppoſe 1236,97286790 be to be *divided* by 21,234.

The OPERATION *contracted*.

21,234)1236,97(58,25 +

106170

17527
16987

540
424

116
106

10

The

The OPERATION at *large*.

```
21,234)1236.97|286790(58,25 +
       106170
       -------
        17527|2
        16987|2
        -------
          540|08
          424|68
          ------
          115|406
          106|170
          -------
            9'236
```

EXAMPLE II.

Suppose 864 be to be *divided* by 2,12540125, *&c.*

```
2,1254)864.00(406,51 +
       85016
       -----
        1384
        1275
        ----
         109
         106
         ---
           3
           2
           -
           1
```

Thus,

This, ſays OUGHTRED, is an excellent *Contraction* of DIVISION, and of great *Uſe* in *Aſtronomical* COMPUTATIONS, *e. g.*

If the *Sun's* DECLINATION be 19° — 39′ — 9″ when it is in ♉ 27° — 34′, find what *its* DECLINATION is when the *Sun* is at the Beginning of CANCER, *viz.* *Its greateſt* DECLINATION, by the RULE of PROPORTION, *thus*,

As the *Sine* of the *Sun's* LONGITUDE 57° — 34′.

To the *Sine* of *its preſent* DECLINATION 19° . 39′ . 9″;

So is the RADIUS,

To the *Sine* of the *Sun's greateſt* DECLINATION, *i. e.*

84402 : 33632 :: 100000 : The *Sine* ſought.

100000

84402)3363200(39847 = the *Sine* of the *Sun's greateſt* DECLIN. 23° . 29′.

253206

83114
75961

7153
6752

401
337

64
60

4

Here, in *multiplying* by the RADIUS, I ſet down but *one* CYPHER, which gives a DIVIDEND *ſufficient* to help me to get the *firſt* FIGURE in the QUOTIENT, and for the reſt of the QUOTIENT, I *diminiſh* the DIVISOR, as above *directed*. Again,

When the RADIUS is *not* ONE of the given TERMS in the PROPORTION, the *Advantage* ariſing from *theſe* ABRIDGMENTS of OPERATION is *double*; the ONE from that of MULTIPLICATION, the OTHER from that of DIVISION.

For Inſtance:

Suppoſe the *Sun's* AZIMUTH be demanded when it is 38°. 18′ high, at $\frac{1}{2}$ an Hour after 3 in the Afternoon, and having then 19° — 39′ — 9″ *North* DECLINATION.

To find which, the PROPORTION is *this*;

As the *Coſine* of the *Sun's* HEIGHT 38° — 18′,

To the *Sine* of the HOUR from *Noon* 52° — 30′;

So is the *Coſine* of the *Sun's* DECLINATION 19° — 39′ — 9″,

To the *Sine* of *its* AZIMUTH from the *Meridian*, *i. e.*

78478 : 79335 :: 94175 : The *Sine* ſought.
57149

714015
31734
793
555
40

78478)747137(95203 = the *Sine* of 72° — 11′,
706302 which is the AZIMUTH ſought.

40835
39239

1596
1569

27
24

3

And thus have I plainly and largely ſhewn how DECIMALS are to be *manag'd* in MULTIPLICATION and DIVISION, as well as in ADDITION and SUBTRACTION.

I ſhall now ſay ſomething

Of SEXAGESIMAL *FRACTIONS.*

IN SEXAGENARY *Fractions every* INTEGER is ſuppos'd to be *divided* into 60 PARTS, which are call'd MINUTES, or PRIMES; and *each* of *theſe* into 60 *Parts* call'd SECONDS; and again, *each* of *theſe* into *as many* THIRDS; and ſo on, continually in the *ſame* RATIO.

Thus, for Inſtance, an HOUR is *divided* into 60 MINUTES, and *every* MINUTE into 60 SECONDS, and *every* SECOND into *as many* THIRDS, *&c.* Alſo,

Every DEGREE of the CIRCUMFERENCE of a *Circle*, or of any ARCH of the *Circumference*, is *divided* in the *ſame* RATIO, into MINUTES, SECONDS, THIRDS, *&c.* Now,

The DECIMAL *Fractions* are *manag'd* (as has been already *ſhewn)* with as much *Eaſe* as INTEGERS are, whilſt *this* SEXAGENARY *Progreſſion* is much *perplex'd*, and requires VARIOUS REDUCTIONS: And,

'Tis in *this* the great Advantage of the DECIMALS above the SEXAGESIMALS does chiefly conſiſt, which will plainly enough appear by this *one*

EXAMPLE.

Let it be ſuppos'd, that the *Sun's Annual Courſe* thro' the *whole Circumference* of the ZODIAC, is perform'd in 365 *Days*, 5 *Hours*, 49 *Minutes*, 16 *Seconds*, 46 *Thirds*, &c. And let *its diurnal mean Motion* be ſought.

Now,

Now, according to the Laws of the *Golden* RULE, 'twill be,

Days H.

As the *Annual Time* 365 . 5 . 49′ . 16″ . 46‴,

To the *Diurnal Time*, 24 *Hours*;

So is the *mean Annual Motion* 360 *Degrees*,

To the *Diurnal Motion* ſought, or to HOW MANY *Degrees?*

Now, becauſe this FIRST *Term*, which is to be the DIVISOR, conſiſts of *many* DENOMINATIONS, that DIVISION may the better be perform'd, THIS muſt firſt be *reduc'd* to *one* DENOMINATION:

To which End, the Number of *Days*, 365, muſt be *multiply'd* by 24, and the 5 *Hours* (adjoining in the *Queſtion*) muſt be *added*, to get the Number of *Hours* in the *propos'd* TIME, *viz.* 8765: Theſe *Hours* muſt be *reduc'd* into *Minutes* by *multiplying* them by 60, and *adding* the 49′ (in the *Queſtion*) to the PRODUCT, which makes the Number of *Minutes* in the *propos'd* TIME to be 525949; but then theſe *Minutes* are *reduc'd* into *Seconds* (by *multiplying* by 60, *adding* the 16″ to the PRODUCT) which *Seconds* are in Number 31556956; And laſtly, Theſe *Seconds* being, in like Manner, *reduc'd* into *Thirds*, *adding* 46‴, we gain 1893417406 *Thirds*, which are = the *Annual Time* propos'd.

Days

Days	*H.*	′	″	‴
365	. 5 .	49 .	16 .	46

× 24

1465
730

8765 *Hours.*
× 60

525949 *Minutes.*
× 60

31556956 *Seconds.*
× 60

1893417406 *Thirds.*

But now, the *Annual* TIME being thus reduc'd to *Horary* THIRDS, the *Diurnal* TIME, or 1 *Day*, muſt alſo be reduc'd into the *ſame*, by *multiplying* it firſt by 24, and then thrice continually by 60, whereby *one* DAY will be found = 5184000 THIRDS.

1
× 24

24
60

1440
60

86400
60

5184000 *Thirds.*

This

This *Preparation* being made, the *Question* may be stated after *this* Manner;

If in a *whole Year*, *i. e.* in 1893417406 *Horary* THIRDS, the *whole Circumference* be perform'd, *i. e.* 360 *Degrees*, how many *Degrees* will be pass'd over in *one* DAY, *i. e.* in 5184000 *Horary* THIRDS?

And the *Operation* of the RULE of *Three* being rightly perform'd, 'twill be found that, indeed, not *one* whole Degree is pass'd over in one Day, but only $\frac{1866240000}{1893417406}$ of a Degree, as is manifest from the adjoin'd Process.

$$\begin{array}{rcl} ''' & ''' & ° \\ 1893417406 : & 5184000 :: & 360 : \text{to } \textit{how many?} \\ & 360 & \\ \hline & 311040000 & \\ & 155520000 & \\ \hline \end{array}$$

$1893417406)1866240000(0\ \frac{1866240000}{1893417406}$ of a *Deg.*

But lastly,

That this *mean Diurnal Motion* thus found, design'd by the ordinary *Fraction* $0\ \frac{1866240000}{1893417406}$ of a *Degree*, may be exhibited in SEXAGESIMAL *Parts*, by the following Method, *viz.*

Multiply the NUMERATOR by 60, and divide the PRODUCT by the DENOMINATOR; and the QUOTIENT shews the MINUTES; and the REMAINDER, after *Division*, if there be *any*, being likewise multiply'd by 60, and the PRODUCT divided by the DENOMINATOR of the *given Fraction*, the QUOTIENT will exhibit the SECONDS; and thus, also, are the THIRDS, FOURTHS, &c. produc'd, by continuing this Practice as long as is needful. And thus, at length, the Sun's *Diurnal mean Motion*

Motion is found in SEXAGESIMAL *Parts* to be 0 *Deg.* — 59′ — 8″ — 19‴ — 37⁗ very nearly.

1866240000
60
1893417406)111974400000(59′
9467087030
17303529700
17040756654
262773046
60
1893417406)15766382760(8″
15147339248
619043512
60
1893417406)37142610720(19‴
1893417406
18208436660
17040756654
1167680006
60
1893417406)70060800360(37⁗
5680252218
1325827818o
1325392184 2
4356338

But

But now the whole Buſineſs is preſently perform'd, and with the leaſt Pains, by DECIMAL *Fractions*; for,

Suppoſing that the Sun's *annual Courſe* is perform'd through the whole *Circumference* of the ZODIAC in *Days* 365,242555, the *diurnal Motion* will be found, by *one only* Operation in DIVISION, to be 0,9856463735 very nearly, as is manifeſt by the *Calculation.*

Days *Degr.* *Day* *Deg.*
365,242555 : 360 × 1 :: 1 : 0.985646374

365,242555)360,0000000, &c.(0,985646374
3128170050
2062296100
2360833250
1693779200
2328089800
1366344700
2706170350
1494724650
3754430

And now, that both this *Propoſition* in DECIMAL *Parts*, and its Reſolution in *ſuch* Parts, are of the *ſame Value* with that laborious one in SEXAGESIMAL *Parts*, will, in the next Place, appear, where I ſhall ſhew the Method of *reducing* SEXAGESIMAL *Parts* to DECIMAL, and THESE to THEM.

Firſt then, I ſhall ſhew how to reduce SEXAGESIMAL *Parts* to DECIMAL *ones.*

SEXAGESIMALS are turned into DECIMALS by *dividing* by 60; thus the SEXAGESIMAL $0^h,75'$, *divided* by 60, *quotes* the *equivalent* DECIMAL ,75 of an *Hour*; for, 60 : 45 :: 1 : 0,75.

But if there are ſeveral *Species* or *Degrees* of *Sexageſimals* annex'd to the INTEGERS (as in the propos'd annual Quantity of 365 Days, 5 Hours, 49′. 16″. 46‴) ſuch *ſexageſimal* Parts (and, indeed, any other Quantities deſign'd by many Denominations, *mutatis mutandis*) are *reduc'd* to *Decimal* Parts by the following Method; thus,

Becauſe in this Inſtance every *Third* is $\frac{1}{60}$ of one *Second*, and every *Second* $\frac{1}{60}$ of one *Minute*, and every *Minute* $\frac{1}{60}$ of an *Hour*, and every *Hour* $\frac{1}{24}$ of one *Day*; therefore 46 *Thirds* are $= \frac{46}{60}$ of one *Second*, *i. e.* (by *dividing* 46 by 60) 'tis 0,7666, *&c.* of a *Second*; and ſo 16″. 46‴ are = 16,7666 of a *Second*; and ſince theſe *Seconds* are ſo many *Sixtieths* of one *Minute*, by *dividing* 16,7666 by 60, we get 0,2794 of a *Minute*; and thence 49′ : 16″ : 46 (= 49′. 16″,7666) are = 49′,2794 of a *Minute*:

And, in like manner, ſince *Minutes* are ſo many *Sixtieths* of an *Hour*, by *dividing* again by 60. 49′.2794 is = 0^{h},8213 of an *Hour*; and ſo 5 Hours 49′. 16″. 46‴ are = 5^{h},8213 of an *Hour*:

And laſtly, Becauſe an *Hour* is $\frac{1}{24}$ of a *Day*, 5^{h},8213 *divided* by 24, will *quote* 0^{d},2425 of a *Day*; and ſo 365 *Days*, 5 *Hours*, 49′. 16″. 46‴, are very nearly = 365^{d},2425.

60)46.0000(0″,7666 60)16.7666(0′,2794

H. *D.*

60)49.2794(0,8213 24)5,8213(0,2425, *&c.*

Thus we ſee the *Propoſition* of the *Sun*'s annual *Courſe* being 365^{d},2425, *&c.* is agreeable, or of the ſame *Value* with it when propos'd in *Sexageſimal* Parts.

And

And now I ſhall ſhew how to *reduce* DECIMAL *Parts* to SEXAGESIMAL *ones.*

As SEXAGESIMALS are *turn'd* into DECIMALS by *dividing* by 60, ſo, on the contrary, DECIMALS are *turn'd* into SEXAGESIMALS by *multiplying* by 60; thus,

The DECIMAL 0,75 of an *Hour*, multiply'd by 60, produces the *equivalent* SEXAGESIMAL 45′: For,

H. H.
1 : 0,75 :: 60′ : 45′

And when there are any RESIDUAL *Decimals*, they muſt be *multiply'd* continually by 60; thus, for Inſtance,

The *diurnal Motion* of the *Sun*, *Gr.* 0,985646374 is *reduc'd* to the *Sexageſimal Parts*, 59′ . 8″ . 19‴ . 37⁗ very nearly, as is manifeſt from the *following* OPERATION.

0,985646374
× 60
59′)138782440
× 60
8″)326946400
× 60
19‴)616784000
× 60
37⁗)007040

And thus we alſo ſee that the *Reſolution* of the ſame PROPOSITION, in DECIMAL *Parts*, is of the *ſame* VALUE with THAT more *laborious* one in SEXAGESIMAL *Parts*.

Mr. OUGHTRED's *Method* of *reducing* SEXAGESIMALS to DECIMALS, and the *contrary*, is *this*, *viz.*

To *reduce* SEXAGESIMALS to DECIMALS, he *ſets* the SEXAGESIMALS under the INTEGERS in an oblique Deſcent, *each* one Place forwarder towards the right Hand, and then divides *each* by 6, beginning with the *loweſt*, and ſetting *each Quotient* in the *Rank* next above it, 'till he comes to the INTEGERS; thus,

Gr.
0,985646373 are the DECIMALS found = the
′59.13878240 SEXAGESIMALS *Gr.* 0 . 59′ . 8″
″8.3269444 19‴ . 37⁗.
‴19.616666
6)⁗37

And on the contrary, to *reduce* DECIMALS (ſuppoſe *Gr.* 0,985646374) to SEXAGESIMALS, he multiplies them continually by 6, and writes the *Products* under one another, cutting off the SEXAGESIMALS in an oblique Deſcent, *each* one Place farther towards the right Hand; thus,

0.985646374 × 6
′59,13878244
″8,3269464
‴19,616784
⁗37,007

Thus the *given* DECIMALS *Gr.* 0.985646374, are found = the SEXAGESIMALS *Gr.* 0 . 59′ . 8″ . 19‴ . 37⁗, very nearly.

And

And thus, I think, ASTRONOMICAL, or SEXAGESIMAL *Fractions*, have been ſufficiently handl'd.

In the next Place I ſhall treat of EVOLUTION, or the EXTRACTING of ROOTS.

The Extraction *of the* SQUARE *Root.*

WHEN *any Number* (whether *whole*, *broken*, or *mix'd*) is multiplied by itſelf, the *Product* obtain'd thereby is call'd the SQUARE of *that Number*; and contrarily, *That Number* from *whence* ſuch SQUARE ariſes, is call'd the *Square* ROOT thereof; thus,

9 is the SQUARE of 3; for 3 multiply'd by itſelf, produces 9; and thus is 625 the SQUARE of 25; for $25 \times 25 = 625$:

And, on the contrary, 3 is the *Square* ROOT of 9; and 25 the *Square* ROOT of 625.

And thus the SQUARE *Numbers*, whoſe ROOTS are the 9 *Digits*, are as expreſs'd in the following

TABLE.

Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

To extract the SQUARE *Root* then of any propos'd Number, is to find out, by ſome Method or other, a Number,

Number, which being multiply'd into itſelf, will produce the Number propos'd: Not that the Root can be exactly extracted out of every propos'd Number whatſoever; for there are an infinite Number of Numbers that may be propos'd, whoſe *Square Roots* can't poſſibly be preciſely expreſs'd in Numbers: As for Inſtance,

The *Square Root* of the *Number* 2 can't, there being no effable *Number (whole* or *broken)* which being multiply'd into itſelf can produce 2: And *ſuch Numbers* are commonly call'd SURD, or IRRATIONAL *ones.*

But tho' we can't always *extract* the EXACT *Root*, yet we may approach it very nearly, as ſhall be ſhewn by-and-by.

Whate'er then the *Number* be that is propos'd, we muſt always imagine it to be a *perfect ſquare Number* (be it ſo or not) and proceed to the *extracting* of its ROOT after the following *Manner.*

When the *Square* ROOT of any *Number* is to be *extracted*, it is convenient to diſtinguiſh it into ſeveral *Portions*, by POINTS ſet over the *proper Figures*, *viz.* by putting a POINT over the *Place* of UNITS, and thence proceeding towards the left Hand, and putting a POINT over every *other Figure*; and *ſo many* POINTS *as* are in that Manner *plac'd*, of *ſo many Figures* the *Square* ROOT ſought will conſiſt: The *Reaſon* of which is manifeſt enough from the very *Conſtitution* of a SQUARE: For,

Suppoſing the ROOT 500 to be multiply'd by itſelf 500, the SQUARE *Number* produc'd will be 250000, *i. e.* 5 with 2 *Cyphers*, multiply'd into 5 with 2 *Cyphers*, will make juſt 25 with 4 *Cyphers*; and ſo elſewhere, every *Cypher* or *Place* following a *Figure* in the ROOT, makes *two* in the SQUARE: The *Pointings* therefore are *order'd* accordingly.

E X-

EXAMPLE I.

Let us extract the *Square* ROOT out of the *Number* 78783376.

Having firſt *pointed* it as above directed; thus,

78783376

Seek a Number for the firſt Figure of the *Quotient*, or *Root*, whoſe *Square* ſhall be equal to, or the next *leſs Square* to 78, the Figures to the firſt *Point*, and you'll find it to be 8: For 9 × 9, or 81, is *greater* than 78; and 8 × 8, or 64, *leſs*; wherefore 8 will be the firſt Figure of the *Root*; the *Value* of which is indeed 8 thouſand; for by the Number of *Points* you ſee the *Root* will conſiſt of four Figures.

This 8 therefore being placed in the *Quotient*, as the firſt Figure of the *Root*, from 78 ſubtract 64, the *Square* of 8, and to the Remainder 14, bring down the next two Figures, *viz.* 78, the Figures to the next *Point*, and you'll have 1478; and then, to obtain the next Figure of the *Root*, you muſt divide this 1478 by 16, the Double of the *Quotient* 8, *viz.* neglecting the laſt Figure 8, ſay, How many times is 16 contain'd in 147? and (upon Trial) you'll find it to be 8 times; wherefore place this 8 in the *Quotient* (being the ſecond Figure of the *Root*) and alſo annex it to the *Diviſor* 16, which, by that means, becomes 168; then multiply this 168 by 8, the Figure laſt obtain'd in the *Root*, and ſubtract the *Product* 1344 from the aforeſaid Number 1478 (ſometimes call'd the *Reſolvend*) and there will remain 134; to which bring down 33, the Figures to the next *Point*, and you'll have 13433 for the next *Reſolvend*:

Now,

Now, to obtain the third Figure of the *Root*, this *Reſolvend*, neglecting 3, the laſt Figure, as before, muſt be divided by 176, the Double of the *Quotient* 88; by which Diviſion you'll find the third Figure of the *Root* to be 7, which place in the *Quotient*, and annex to the *Diviſor* 176, making it 1767; then, as before, multiply this 1767 by 7, the Figure laſt obtain'd in the *Root*, and ſubtract the *Product* 12369 from the laſt *Reſolvend* 13433, and to the Remainder 1064, bring down the two laſt Figures 76, and you'll have 106476 for the laſt *Reſolvend*.

And laſtly, to obtain the laſt Figure of the *Root*, divide this laſt *Reſolvend* (neglecting its laſt Figure 6) exactly after the ſame Manner, *viz.* by ſaying, How many times is 1774 (the Double of the *Quotient* 887) contain'd in 10647, and you'll find it to be 6, which place in the *Quotient*, and annex to the *Diviſor* 1774, making it 17746; and then, by multiplying and ſubtracting, as you did before, you'll find 0 remain; wherefore the Buſineſs is done, and the *Square* ROOT extracted, appears to be exactly 8876; which is eaſily prov'd to be the TRUE *Root* ſought by multiplying it into itſelf.

The *Arithmetical* OPERATION, purſuant to the foregoing Directions, follows at large.

. . . .

78783376

```
  78783376 (  8876 = the Root ſought.
  64          8876
  --          ----
168)1478      53256
    1344      62132
    ----      71008
1767)13433    71008
     12369    ------
     -----    78783376
17746)106476
      106476
      ------
           0 Remains.
```

EXAMPLE II.

Suppoſe the *Square* ROOT of 368863 be to be *extracted.*

The WORK, by the foregoing Method, will ſtand thus,

```
  368863(607
  36
  --
1207) 8863
      8449
      ----
       414 Remains.
```

Let the Learner here obſerve, that tho' the firſt *Example* propos'd, 78783376, was a *perfect* SQUARE Number, its *exact* ROOT being *extracted* without any thing remaining, yet this ſecond propos'd Number, 368863, is a SURD, as appears by the *Remainder* 414,

whereby he may know that the Number 607 is not the *Square* ROOT of that propos'd Number preciſely, but that it is ſomething (tho' not an *Unit*) too *little*, 608 being too *much*; yet *how much* the *true* ROOT is *greater* than 607, can't poſſibly be preciſely diſcovered by any known RULES.

Wherefore, if you would approach nearer the *true* ROOT of this, or of any other propos'd SURD Number, you muſt carry on the Approximation in *Decimal* Parts, by adding to the *Remainder* two *Cyphers* at each OPERATION.

Thus, in this Caſe, the *Remainder* 414 having two *Cyphers* annex'd to it, becomes 41400; which being divided by 1214, the Double of the *Quotient* 607, exhibits 3 for the *firſt* DECIMAL *Place* in the ROOT; then having accordingly plac'd 3 in the QUOTIENT, *multiply* and *ſubtract* ſtill according to the foregoing DIRECTIONS, and the next *Remainder* will be found to be 4971:

And ſo again, by adding two more *Cyphers* to this *Remainder*, the WORK may be repeated and carried on at Pleaſure, 'till at length comes forth the ROOT 607,34092, *&c.* as appears by the *whole* OPERATION following.

```
        368863(607.34092, &c.
        ------
1207)8863
     8449
     ----
12143)41400
      36429
      -----
121464)497100
       485856
       ------
12146809)112440000
         109321281
         ---------
121468182)311871900
          242936364
          ---------
           68935536
```

After the same Manner the *Square* ROOTS are to be *extracted* out of DECIMAL *Numbers*.

Thus the ROOT of 6225,21 is 78,9; and the ROOT of 6556,999 is 80,97, &c. and the ROOT of 0.06556999 is ,25606, &c. as is manifest from the *following* OPERATIONS.

EXAMPLE III.

Let the *Square* ROOT be *extracted* out of 6225,21.

```
        6225,21(78,9
        49
      ---
  148)1325
      1184
      ----
 1569)14121
      14121
      -----
         0
```

EXAMPLE IV.

```
        6556,9990(80,97, &c.
        64
      --
 1609)15699
      14481
      -----
16187)121890
      113309
      ------
        8581
```

EX.

EXAMPLE V.

```
  . . . .
0,06556999(,25606, &c.
   4
   —
45)255
   225
   ———
506)3069
    3036
    ————
51206)339900
      307236
      ———————
       32664
```

Hence 'tis plain, that by this very Method the *Square* ROOT of any VULGAR *Fraction* may be *extracted*, the propos'd VULGAR *Fraction* being first *reduc'd* into DECIMALS; thus,

EXAMPLE VI.

Let us *extract* the *Square* ROOT of $\frac{4}{64}$.

In the first Place the DECIMAL *Fraction* equivalent to this propos'd VULGAR *one* $\frac{4}{64}$, will, by *Reduction*, be found to be 0,0625, whose ROOT, *extracted* by the RULES above taught, is equivalent to the ROOT of the given VULGAR *Fraction*.

```
  . . .
0,0625(0,25
   4
   —
45)225
   225
   ———
     0
```

Thus,

Thus, 0,25 is found equivalent to the ROOT requir'd; for,

$$0{,}25 \times 0{,}25 = 0{,}0625 = \tfrac{4}{64}.$$

But when the NUMERATOR and DENOMINATOR of the propos'd VULGAR *Fraction*, are both of them *perfect* SQUARE Numbers, as they are in this *Example*, you have nothing to do but to *extract* the ROOTS of them severally, and the ROOT of the NUMERATOR of the *propos'd* FRACTION, is the NUMERATOR of its ROOT; and the ROOT of the DENOMINATOR of the *propos'd* FRACTION, is the DENOMINATOR of its ROOT; as here,

The ROOT of the NUMERATOR 4 is 2, and the ROOT of the DENOMINATOR 64 is 8; thus,

$\frac{2}{8}$ ($= 0{,}25$) is found to be the ROOT of the *propos'd* FRACTION $\frac{4}{64}$: And this is evidently the *true* ROOT from the NATURE of MULTIPLICATION of *Fractions*; for,

$\frac{2}{8} \times \frac{2}{8}$, produces its SQUARE $\frac{4}{64}$: And the like in others.

But when either the NUMERATOR or DENOMINATOR of a propos'd FRACTION hath *not* a *perfect Square* ROOT, then *reduce* it to DECIMALS, and proceed as before.

EXAMPLE VII.

Extract the *Square* ROOT of $\frac{5}{8}$.

This propos'd VULGAR *Fraction*, turn'd into a DECIMAL *one*, becomes 0,625; the *Square* ROOT whereof may be found to be 0,790569, &c. thus,

```
  0,6250(0,790569 = the ROOT sought
    49                very nearly.
  ---
149)1350
    1341
    ----
 15805)90000
       79025
       -----
158106)1097500
        948636
        ------
1581129)14886400
        14230161
        --------
          656239 The Remainder.
```

Note, That the DECIMAL out of which the ROOT is to be *extracted*, must always consist of an *even* Number of PLACES, as *Two*, *Four*, *Six*, *Eight*, *Ten*, &c. PLACES; wherefore, to the propos'd DECIMAL, if it consist of an *odd* Number of *Places* (as in the last *Example*) I annex one or more *Cyphers* at Pleasure, and then proceed to the EXTRACTION of its ROOT.

Note also, That, in order to prove the ROOT to be TRULY *extracted*, so far as you have thought fit to approach, you must, after *multiplying* such ROOT into itself, *to the Product add the Remainder, and* (if it be truly *extracted*) *the Sum will be the* SURD *Number propos'd.*

I shall

I ſhall now give a DEMONSTRATION, or a *rational Account* of the *foregoing* METHOD of *extracting* the SQUARE ROOT.

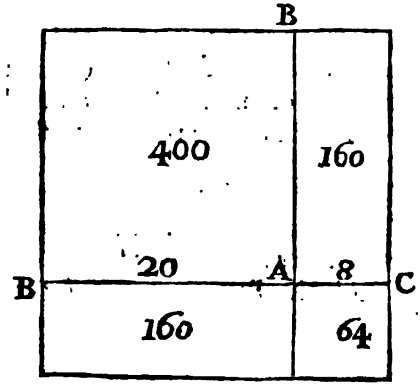
The foregoing Method of *extracting* the SQUARE *Root* of any Number, is deduc'd from *Prop.* IV. *Book* II. of EUCLID, which aſſerts, and 'tis there *demonſtrated*, That,

If any right Line be divided into two unequal Parts, the SQUARE *of the* WHOLE *Line is* = *both the* SQUARES *of the* PARTS *added to twice the Rectangle contained under the* PARTS. Or in other Words,

The SQUARE of the SUM of any two Numbers, is = the SUM of the SQUARES of thoſe two Numbers, together with twice the *Product* under thoſe two Numbers: Thus, for Inſtance,

Let the *Line* A B conſiſt of 20 *equal Parts*, and the *Line* A C of 8 of the *like Parts*.

Now the SQUARE of the *Line* A B, *i. e.* 20 × 20 is = 400; and the SQUARE of the *Line* A C, *i. e.* 8 × 8 is = 64; and the *Product* or *Rectangle* under thoſe two *Lines*



Lines is = 160, *i. e.* = 20 × 8; the Double of which *Product* is = 320: Now 400 + 64 + 320 is = 784 = 28 × 28, *i. e.* = the SQUARE of the SUM of the two propos'd Numbers, 20 and 8, or of the WHOLE *Line* B C.

Hence therefore ariſes the REASON of the *foregoing* METHOD of *extracting* the SQUARE *Root*: For,

If B C, or the ROOT of this SQUARE Number 784, was to be *extracted*, the firſt Figure we ſhould (by the *precedent* DIRECTIONS) obtain in the ROOT, would be 2, whoſe *true* VALUE is 20, ſince it ſtands in the Place of *Tens*, the ROOT ſought conſiſting but of *two Figures*, which we know by the Number of POINTS:

Now 20 being known to be a Part of the ROOT ſought, it's manifeſt, from the forementioned *Propoſition* of EUCLID, that if the *Square* of *this Part* 20 (*viz.* 400) be *ſubtracted* from the *whole* given *Square* 784, the *Remainder* 384 is *equal to* the *Square* of the *remaining Part* of the ROOT ſought, together with *twice* the PRODUCT under the *known Part* 20, and the (as yet) *unknown Part* of the ROOT ſought; but it's already well known from common *Multiplication* and *Diviſion*, that if once this PRODUCT be *divided* by the *known Part* (or FACTOR) 20, the QUOTIENT will be the *other Part* requir'd; or if *twice* this PRODUCT be *divided* by *twice* the *known Part* of the ROOT, the QUOTIENT ſhall be the *other Part*, which is evidently the REASON why the *firſt Figure* of the QUOTIENT or ROOT ſought, is *doubl'd* for a *Diviſor*:

Thus the *Remainder* 384 *divided* by 4 (the *Double* of 2) that is, indeed, by 40 (the *Double* of 20) *quotes* 8 for the *other Part* of the ROOT ſought; which laſt obtain'd Number 8, annex'd to the *Diviſor* 4 (as above directed) makes it 48; which 48, *multiply'd* by 8, muſt neceſſarily produce a Number equal to the *Square* of *that*

Part 8, together with the *Double* of the PRODUCT under the *Parts* 20 and 8, and conſequently *nothing* can remain; for,

Twice 20×8, that is $\overline{40 \times 8} + \overline{8 \times 8}$ is $48 \times 8 = 384$: Whence the REASON for annexing the QUOTIENT *Figure* to the DIVISOR, in the Manner before taught, and of the *reſt* of the PROCESS, is plainly ſeen.

Thus BC, or the ROOT of the propos'd SQUARE 784, is found to conſiſt of the *two Parts* 20 and 8, *i. e.* the ROOT ſought is 28.

But now, if the ROOT conſiſts of more than *two Figures*, it may nevertheleſs be imagin'd to be compos'd but of *two* Numbers, or *Parts*.

As the ROOT of the firſt *Example* 78783376, conſiſts of the *four Figures* 8876, yet may be conceiv'd to be *compos'd* of the *two Parts* 8870 and 6; but ſince the *former* of theſe *Parts* can't be conveniently diſcover'd at once, it is neceſſary that we ſeek it *out* by piecemeal (as we do the QUOTIENT in common DIVISION) by repeating the OPERATIONS as often as there is Occaſion.

Thus having deſcry'd 8 (*i. e.* 8000) to be the firſt Figure of the ROOT, by *doubling* it for a DIVISOR, I proceed (as before taught) to diſcover the *next Figure*, and finding it to be 8, I have got 88 (*i. e.* 8800) in the QUOTIENT; all which I then conſider as the *firſt Part* of the ROOT, which I *double* for a DIVISOR, and begin (as it were, *de Novo*) to find the *next Figure*, which is 7; and then again, having got 887 (*i. e.* 8870) in the QUOTIENT, I uſe *it* as *one Part* of the ROOT ſought, by *doubling* it for a DIVISOR, in order to find the *other Part*, *viz.* 6.

And,

And, in like Manner may a ROOT conſiſting of any *Number* of *Places* whatſoever, be conſider'd as one compos'd of *two Parts* (call'd a BINOMIAL *Root)* and be thus gradually extracted.

And now, having given ample *Directions* for *extracting* the SQUARE ROOT, and plainly prov'd the TRUTH thereof, I ſhall ſhew the Learner how to *extract* the CUBE *Root*.

The EXTRACTION *of the* CUBE ROOT.

ALL POWERS ariſe from the *Multiplication* of the SIDE or ROOT, *once* or *more times* continually into *itſelf*; thus,

As any *Number* (as a ROOT) *multiply'd* into itſelf, produces a SQUARE, or *ſecond* POWER, ſo again, that PRODUCT or SQUARE *multiply'd* into the *ſame* ROOT, produces a CUBE, or *third* POWER; and *that Number* from whence *ſuch* CUBE ariſes, is call'd the CUBE ROOT thereof.

Thus 343 is a CUBE *Number*, whoſe ROOT is 7; for 7 times 7 is 49, the SQUARE of 7; and again, 7 times 49 is 343, the CUBE of 7.

And, on the contrary, 7 is the CUBE *Root* of 343; and thus the CUBES, as well as the SQUARES of the 9 *Digits*, are expreſs'd in the following

TABLE.

Cubes	1	8	27	64	125	216	343	512	729
Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

To *extract* the CUBE *Root* then of any propos'd *Number*, is to find out a *Number* which, being *multiply'd* into *its* SQUARE, will produce the CUBE *Number* propos'd: And *this* may be done by the *following* METHOD.

Firſt, Let the propos'd *Number* be properly diſtinguiſh'd into *Portions* by POINTS, *viz.* by putting a POINT over the *Place* of UNITS, and from thence over every *third Figure* towards the *left* Hand, as in the *following* EXAMPLE; and *ſo many* POINTS *as* are in that Manner *plac'd*, of *ſo many Figures* the CUBE ROOT requir'd will conſiſt:

For 5 with two *Cyphers*, multiply'd by 5 with two *Cyphers*, makes 25 with four *Cyphers*; and this again into 5 with two *Cyphers*, makes 125 with ſix *Cyphers* after it; *every Cypher* or *Place* following a *Figure* in the ROOT, making *two* in the SQUARE and *three* in the CUBE.

EXAMPLE I.

Let it be propos'd to *extract* the CUBE *Root* out of the *Number* 146363183.

Having duly *pointed* the propos'd *Number*, as the *Conſtitution* of this POWER requires; thus,

146363183

I ſee

I ſee by the *Number* of POINTS, that the ROOT is to conſiſt of *three* FIGURES.

Then I ſeek (by myſelf, or) by Help of the foregoing TABLE, a *Number* for the *firſt Figure* of the ROOT, whoſe CUBE is the *greateſt* in the TABLE that's *below* 146, the *Figures* to the *firſt* POINT, which I find to be 5, and place it in the *firſt Place* of the QUOTIENT; and having *ſubtracted* its CUBE 125, from 146, there will remain 21; to which *Remainder* I bring down, or annex 363 (*i. e.* the *Figures* to the *next* POINT) by which Means I get the RESOLVEND 21363.

Now, to get a new DIVISOR (which muſt be found at *each* particular OPERATION) I *ſquare* the QUOTIENT, and *triple* that SQUARE; thus,

5 times 5 is 25, and 3 times 25 is 75, which I ſet for a *new* DIVISOR: Then, to obtain the *next Figure* of the ROOT, I ask, How many times 75 I can have in 213 (*i. e.* in *all* the *Figures* of the RESOLVEND 21363, ſave *two*, as in *extracting* the SQUARE *Root*, it was in *all* the *Figures* ſave *one*) and finding it *twice*, I place 2 in the QUOTIENT (as the *ſecond Figure* in the ROOT) by which I *multiply* the DIVISOR 75, and *place* the PRODUCT 150 *two Places* ſhort of the *laſt Figure* of the RESOLVEND; as may be ſeen in the OPERATION following:

Then I *ſquare* 2, the *laſt Figure* of the QUOTIENT, and it makes 4; which SQUARE being *multiply'd* by 3 times the *former Figure* of the QUOTIENT (*i. e.* by 3 times 5, or by 15) makes 60, which I ſet *one Place farther* on the *right Hand*; that is, *one Place ſhort* of the *laſt Figure* of the RESOLVEND.

And laſtly, I *Cube* 2, the *laſt Figure* of the QUOTIENT, which is 8, and *ſet* it ſtill *one Place farther on*

to

to the *right Hand*, under the RESOLVEND, *viz.* under the *Figure* 3, then I *add* theſe *three* PRODUCTS together, and *ſubtract* their *Sum* 15608 (which muſt never be *greater* than the RESOLVEND) from the RESOLVEND 21363, and find 5755 remaining; to which *Remainder* I bring down, or annex the *next three Figures* 183 (*i. e.* the *Figures* to the *next* POINT) whereby I get 5755183 for the *next* RESOLVEND.

In like Manner, by repeating the foregoing WORK, I obtain every *following Figure* of the ROOT; thus,

To get the *third* and *laſt Figure* thereof, I *ſquare* ALL the QUOTIENT 52, and *triple that* SQUARE, which produces 8112 for a *new* DIVISOR:

Then (as before) I ask, *How many* times is 8112 contain'd in 57551 (*i. e.* in all the *Figures* of the RESOLVEND 5755183, ſave *two*)? Or, *How many* times 8 in 57? And (upon Trial) I find it to be 7 times, which 7 I *place* in the QUOTIENT (being the *laſt Figure* of the ROOT):

Then I *multiply* the DIVISOR 8112 by this *laſt Figure* 7, and *ſet* the PRODUCT 56784, *two Figures ſhort* of the *laſt Figure* of the RESOLVEND, as before:

Then I *multiply* 49, the SQUARE of the *laſt* obtain'd *Figure* 7, by 156, the *Triple* of (52) *all* the *former Figures* of the QUOTIENT, and *ſet* the PRODUCT 7644 *one place farther on* to the *right Hand*:

And, laſtly, *under this* PRODUCT, but ſtill *one Place farther on* to the *right Hand*, I ſet 343, the CUBE of the *laſt* obtain'd *Figure* 7, and then I *add* theſe *three* PRODUCTS together, as before, and *their* SUM 5755183, being *ſubtracted* from the *laſt* RESOLVEND, leaves 0 remaining.

Whence it is evident, that the CUBE ROOT of 146363183 is *exactly* 527.

The Learner muſt obſerve, that whene'er the SUM of the *three forementioned* PRODUCTS is *greater* than the RESOLVEND, 'tis a Sign that the *Quotient*-FIGURE was taken *too great*, which muſt be corrected by placing a *leſſer Figure* in the QUOTIENT.

The *whole* OPERATION, according to the foregoing DIRECTIONS, will ſtand thus,

```
. . .
146363183(527 = the CUBE Root ſought.
125
---
 21363
 ----
 150
   60
    8
 -----
 15608
 -----
  5755183
  -------
  56784
    7644
     343
  -------
  5755183
  -------
     0 Remains.
```

Which

Which may be thus PROV'D;

527 = the ROOT.
527

3689
1054
2635

277729 = the SQUARE.
527

1944103
555458
1388645

146363183 = the CUBE.

When o remains, after the OPERATION is finiſh'd, we may be certain that the propos'd *Number* (as *that* above) is a *perfect* CUBE, *whoſe* ROOT is *preciſely* EXTRACTED; but *here*, as well as in the *Extraction* of the SQUARE *Root*, it very often happens, that the *Number* propos'd is a SURD one, *whoſe* ROOT can never be *truly* EXTRACTED, but *ſomething* will always *remain*; but yet, in *ſuch Caſe*, the ROOT may be *continu'd on* in DECIMAL *Parts* to what *Exactneſs* we pleaſe, by continually annexing to the *Remainders* THREE *Cyphers* (as the NATURE of *this third* POWER requires) and *repeating* the *foregoing* PROCESS; as in the following EXAMPLE.

EXAMPLE II.

Let the CUBE *Root* of 2 be propos'd to be *extracted*.

The OPERATION will ſtand thus,

2(1,2599,

2(1,2599, &c. The CUBE *Root* nearly.
1

2)1000

6
12
8

728

432)272000

2160
900
125

225125

46875000

421875
30375
729

42491979

4383021000

42797187
305937
729

4282778799

100242201 *Remainder.*

And, to PROVE *ſuch* ROOT to be thus far *truly* EXTRACTED, to the CUBE *thereof* you muſt *add* the *Remainder*, and the SUM will be the SURD *Number* propos'd; thus,

The CUBE of 1.2599 is 1,9998997577 99; to which, if the REMAINDER 10024220 1, be *added*, the SUM will be 2, the very SURD *Number* propos'd.

After the ſame Manner is the CUBE ROOT of any DECIMAL FRACTION to be *extracted*, as muſt neceſſarily appear very plain, when the *preceding* PRACTICE is rightly underſtood; and therefore I think it needleſs to give any *more* EXAMPLES.

Hence 'tis plain, that the CUBE ROOT of *any* VULGAR *Fraction* may be found in DECIMALS by the *ſame Method*; the given VULGAR *Fraction* being firſt *reduc'd* to a DECIMAL *one*, conſiſting of a ſufficient *Number* of PLACES; the CUBE ROOT of which ſhall, in DECIMALS, be very nearly equal to the CUBE ROOT of the VULGAR *Fraction* propos'd; thus,

If the CUBE ROOT of $\frac{3}{7}$ be requir'd, *its* equivalent DECIMAL may be found to be

. . . . .

0,428571428571, *&c.*

The CUBE ROOT of which will, by the *foregoing* DIRECTIONS, be found to be 0,7539, *&c.* which is the CUBE ROOT of $\frac{3}{7}$ very nearly.

But any VULGAR *Fraction* is *cub'd* by *cubing* its NUMERATOR and DENOMINATOR ſeverally, and forming a *Fraction* with thoſe two CUBES; which *Fraction* will be the CUBE requir'd.

The

2

The CUBE of $\frac{3}{4}$, for Inſtance, is $\frac{27}{64}$; for the CUBE of the NUMERATOR 3 is 27, and *that* of the DENOMINATOR 4 is 64: Wherefore, on the contrary, when the *Cube* ROOT of any VULGAR *Fraction* is propos'd to be *extracted*, if it be not a SURD, it may be readily done, by *extracting* the CUBE ROOT of the NUMERATOR, reſerving it for the NUMERATOR of the CUBE ROOT requir'd; and, by *extracting* the CUBE ROOT of the DENOMINATOR, for the DENOMINATOR of the CUBE ROOT requir'd; thus,

If the CUBE ROOT of $\frac{27}{64}$ was demanded, it would be found to be $\frac{3}{4}$; for the *Cube* ROOT of the NUMERATOR 27 is 3, and that of the DENOMINATOR 64 is 4:

Thus the *Cube* ROOT of $\frac{1}{8}$ is $\frac{1}{2}$:

And likewiſe thus the *Cube* ROOT of $\frac{3375}{4096}$ may be found to be $\frac{15}{16}$.

And now, having (I think) given ſufficient practical DIRECTIONS for *extracting* the CUBE ROOT, nothing remains, but that I give the *Learner* a DEMONSTRATION, or a *reaſonable* and *ſatisfactory* ACCOUNT of the *ſame*; which I ſhall endeavour to do in as eaſy and plain a Manner as poſſible.

When the ANALYSIS, or the RESOLUTION of any thing into the PARTS of *which* it is *compos'd*, is requir'd, it muſt certainly be very neceſſary that we firſt know *what thoſe conſtituent* PARTS are; ſo from conſidering the *Nature* of the GENESIS, or COMPOSITION of *Powers* (commonly call'd INVOLUTION) we may beſt learn the ANALYSIS, or the EXTRACTION of *their* ROOTS.

How the SECOND and THIRD *Powers*, or the SQUARE and CUBE are *generated* from a *ſingle* or *entire* ROOT, has been already ſhewn:

And it has alſo been ſhewn, of *what* PARTS *that* SQUARE conſiſts *which* ariſes from a BINOMIAL ROOT, or a ROOT that is divided into *two* PARTS, *viz.* That it conſiſts of the SQUARE of the *greater* PART, of the SQUARE of the *leſſer* PART, and of *two* RECTANGLES or PRODUCTS made of *thoſe* PARTS.

And I ſhall now ſhew, by Way of LEMMA, that the CUBE *generated* from a BINOMIAL ROOT, conſiſts of *eight* PIECES or PARTS, *viz.* Of the CUBE of the *leſſer* PART, of *thrice* the SOLID, made by the *Multiplication* of the *greater* PART, into the SQUARE of the *leſſer*:

As for *Example*;

Suppoſe a CUBE be to be conſtituted from the ROOT 28, conſider'd as a BINOMIAL *Root*, conſiſting of the *two* PARTS, 20 and 8 (the *former* PART 20, being repreſented by A, the *other* 8, by E).

In the firſt Place, this BINOMIAL *Root* A + E, *multiply'd* by A + E, produces the SQUARE thereof, conſiſting of A A (the SQUARE of the *greater* PART) + 2 A E (*twice* the PRODUCTS of the PARTS) + E E (the SQUARE of the *leſſer* PART): And then *this* SQUARE (*viz.* A A + 2 A E + E E) *multiply'd* by the *ſame* ROOT, A + E, will *produce* the CUBE thereof, conſiſting of A A A (the CUBE of the *greater* PART) + 3 A A E (*thrice* the SOLID under the *leſſer* PART, and the SQUARE of the *greater*) + 3 A E E (*thrice* the SOLID under the *greater* PART, and the SQUARE of the *leſs*) + E E E (the CUBE of the *leſſer* PART): Thus, A A A, or A × A × A is = 20 × 20 × 20 = 8000; and 3 A A E, or 3 times the SQUARE of A *multiply'd* into E, is = 400 × 8 × 3 = 9600; and 3 A E E, or 3 times the SQUARE of E *multiply'd* into A is = 64 × 20 × 3 = 3840; and E E E, or E × E × E is = 8 × 8 × 8 = 512:

But

But now the SUM of *theſe* PARTS, *i. e.* 8000 + 9600 + 3840 + 512, is = 21952, *which* is the CUBE of the *entire* ROOT 28: Whence it appears, that the PIECES or PARTS above mention'd, are the *conſtituent* PARTS of a CUBE; *which was to be ſhewn.*

Now from *this* GENESIS of the CUBE, are naturally *deduc'd* the DIRECTIONS before given, for its ANALYSIS or the EXTRACTION of *its* ROOT; for by duly *comparing* the *one* with the *other*, it muſt plainly appear, that *thereby* we UNDO or RESOLVE the CUBE into the *ſame* PARTS *which* it is *compos'd* of: As in the preſent Caſe;

Suppoſe the CUBE ROOT of 21952 was requir'd to be *extracted:*

Firſt we *point* it thus, 21952, whereby we know the ROOT ſought will conſiſt of two *Figures.*

Then, by ſeeking in the TABLE, we find that the CUBE of 21 (the *Figures* to the *firſt* POINT) is 2 (the CUBE of 3 being 27) the VALUE of *which* we know is 20, and may be call'd A, or the *greater* PART of the ROOT.

And then we *ſubtract* 8 (the CUBE of 2) from 21, and 13 *remains*; to *which* REMAINDER we bring down and annex the *Figures* to the *next* POINT, whereby we get 13952 for a *new* RESOLVEND; that is, in *effect*, from the *whole given* CUBE 21952, we *ſubtract* 8000, being *one* of its *conſtituent* PARTS, *viz.* the CUBE of the *greater* PART of the ROOT ſought, and find 13952 to *remain.*

Now, by the foregoing LEMMA 'tis evident, that this REMAINDER muſt therefore conſiſt of ALL *the reſt* of the *conſtituent* PARTS of the given CUBE; wherefore, by *dividing* this REMAINDER or RESOLVEND 13952, by 3 A A, that is, by 1200, or 3 times the SQUARE of the

the *greater* PART of the ROOT, we may obtain the *lesser* PART of the ROOT, E = 8 (for *so* we really *do* when we *divide* ALL the *Figures* of the RESOLVEND, save *two*, by 12, the *Triple* of the SQUARE of the QUOTIENT *Figure* 2, as above directed; and *this* is the REASON why the *two last Figures* of the RESOLVEND are thus *omitted*).

And then, lastly, when 8, the *lesser* PART of the ROOT, is thus obtain'd, if we *multiply* it into the DIVISOR 3 A A, or 1200, we get 9600 = 3 A A E, or *thrice* the SOLID under the *lesser* PART of the ROOT, and the SQUARE of the *greater*; and if we *multiply* 64, the SQUARE of the *lesser* PART of the ROOT, into 3 times 20 (for *so* we, in effect, *do*, when, according to the DIRECTIONS, we *square* the LAST *Figure* of the QUOTIENT, and *multiply* it by 3 times ALL the *former*, and *place* the PRODUCT *one Figure* short of the LAST *Figure* of the RESOLVEND) we get 3840 = 3 A E E, or *thrice* the SOLID under the *greater* PART of the ROOT, and the SQUARE of the *lesser*; and if we also *cube* this *lesser* PART of the ROOT, the SUM of *these three* RESULTS *subtracted* from the *last* RESOLVEND, will leave 0 *remaining*.

The OPERATION, according to the *practical* DIRECTIONS, will stand thus,

```
      21952(18.
       8
      —
  12)13952
     ———
      96..
      384.
       512
     ———
     13952
     ———
        0
```

Which,

Which, from the *foregoing* RATIONAL ACCOUNT, in *effect*, is *this*,

```
      .   .  A + E
     21952(20 + 8 = 28 the CUBE ROOT requir'd.
      8000 = AAA
     ------
1200)13952
     ------
     9600 = 3AAE
     3840 = 3AEE
      512 = EEE
     ------
     13952
     ------
        0
```

From *all which*, the TRUTH and REASON of the PRAXIS is sufficiently *manifest*.

The E N D.

APPEN-

THE

APPENDIX.

TABLE I.

Of Engliſh MONEYS, *and their* PROPORTIONS *to each other.*

Farthings					
4	*A Penny*				
48	12	*A Shilling*			
320	80	6 $\frac{2}{3}$	*A Noble*		
640	160	13 $\frac{1}{3}$	2	*A Mark*	
960	240	20	3	1 $\frac{1}{2}$	*A Pound*

TABLE II.

Contains the different MEASURES *of* TIME, *of which the moſt* uſeful, *at leaſt to* Us, *is the* YEAR (*which, from its Author* JULIUS CÆSAR) *is called* the Julian Year.

Minutes						
60	*An Hour*					
1440	24	*A Day*				
10080	168	7	*A Week*			
40320	672	28	4	*An equal Month*		
			W. *d.* *h.*	*M.* *d.* *h.*		
525960	8766	365 $\frac{1}{4}$	52—1—6	13—1—6	*A Julian Year*	

N. B. This TABLE may be extended farther, *viz.* to SECONDS, THIRDS, *&c. viz.* by continually *multiplying* the next foregoing *Number* by 60.

Ex. Gr.

$$\left.\begin{matrix} 1' \\ 60 \\ 1440 \\ \&c. \end{matrix}\right\} \times 60 \text{ gives } \left\{\begin{matrix} 60'' \\ 3600 \\ 86400 \end{matrix}\right\} = \left\{\begin{matrix} 1 \textit{ Minute.} \\ 1 \textit{ Hour.} \\ 1 \textit{ Day.} \end{matrix}\right.$$

Alſo,

$$\left.\begin{matrix} 1'' \\ 60 \\ 3600 \\ \&c. \end{matrix}\right\} \times 60 \text{ gives } \left\{\begin{matrix} '''60 \\ 3600 \\ 216000 \\ \&c. \end{matrix}\right\} = \left\{\begin{matrix} 1 \textit{ Second.} \\ 1 \textit{ Minute.} \\ 1 \textit{ Hour.} \end{matrix}\right.$$

N. B.

N. B. An *equal Month* (mention'd in the foregoing TABLE) is ſo *call'd*, becauſe it always conſiſts of 28 *Days.*

On the contrary, a *Kalendar Month* is *unequal*, becauſe it conſiſts of *unequal* DAYS, according to the following *memorial Verſes*,

Thirty Days hath SEPTEMBER,
APRIL, JUNE, and NOVEMBER;
FEBRUARY hath 28 alone,
And ALL the REST have 31.

TABLE III.

Containing the various PARTS *into which the* CIRCUMFERENCE *of a* CIRCLE *is ſuppos'd to be divided, and again ſubdivided; and by which* ASTRONOMERS *meaſure* DISTANCES *in the Heavens; by which* GEOGRAPHERS *meaſure this terraqueous* GLOBE; *and laſtly, by which* MATHEMATICIANS *meaſure any propos'd* ANGLE.

Thirds	*A Second*	*A Minute*	*A Degree*	
60	60			
3600	60	60		
216000	3600	60	*A Degree*	
77760000	1296000	21600	360	*A Circle*

And ſo on to FOURTHS, FIFTHS, *&c.* by continually *multiplying* the *Number* immediately foregoing by 60, as in TABLE II.

TABLE IV.

Containing the various Engliſh MEASURES *of* LENGTHS.

N. B. It's more than probable, that the various MEASURES of LENGTHS were at firſt deduc'd, by the *Dictates* of NATURE, from the different *Dimenſions* of the *Parts* of the human *Body* (whoſe *Names* they ſtill retain) thus,

From *thence* were taken the DIGIT, or *Finger's Breadth*; a HAND; a SPAN; a FOOT; a PACE; and ſeveral others: But becauſe the *Dimenſion* of the human *Body* is *various* and *uncertain*, and the *Proportion* in MEASURES muſt neceſſarily be *fix'd* and *certain*; hence, I ſay, *every* NATION has pitch'd upon ſome *ſettl'd* and *determin'd* EXTENSION to uſe in their MEASURES of *Lengths*.

Upon this Conſideration our *Engliſh* LAWS have *determin'd* the *Extent* of the MEASURE, which is kept (for a STANDARD *one*) in GUILD-HALL, in *London*, call'd the *Yard-Standard*: And to *this*, *all* the *other* ENGLISH *Meaſures* of *Lengths* ought to obſerve their *preſcrib'd* PROPORTIONS; thus,

An *Engliſh* FOOT is a *third Part* of the *Meaſure* juſt mention'd: Thus alſo,

An INCH is the 36th *Part* of the *ſame*:

Therefore, thoſe who deduce the *Dimenſion* of the *Engliſh* INCH from *three Grains* of BARLEY, are not altogether *right*, ſince *our* INCH has not any *Reſpect*, at leaſt any *immediate one*, to *thoſe* GRAINS (or any *other*) but to the foremention'd *Standard-Meaſure*.

Theſe

These Things being *premis'd*, we may now insert the TABLE of LONG MEASURES accordingly.

A Digit, or an Inch	*A Palm*	*A Span*	*A Foot*	*A Cubit*	*A Yard*	*An Ell*	*A Pace*	*A Fathom*	*A Pole*	*A Furlong*	*A Mile*
3											
9	3										
12	4	1 $\frac{1}{3}$									
18	6	2	1 $\frac{1}{2}$								
36	12	4	3	2							
45	15	5	3 $\frac{3}{4}$	2 $\frac{1}{2}$	1 $\frac{1}{4}$						
60	20	6 $\frac{2}{3}$	5	3 $\frac{1}{3}$	1 $\frac{2}{3}$	1 $\frac{1}{4}$					
72	24	8	6	4	2	1 $\frac{3}{4}$	1 $\frac{1}{5}$				
198	66	22	16 $\frac{1}{2}$	11	5 $\frac{1}{2}$	4 $\frac{2}{5}$	3 $\frac{3}{10}$	2 $\frac{3}{4}$			
7920	2640	880	660	440	220	176	132	110	40		
63360	21120	7040	5280	3520	1760	1408	1056	880	320	8	

Again, The easier to understand several *French*, *Italian*, and *Dutch* Mathematical and Philosophical Authors, it may not be useless to shew the *Proportion* of the *English* FOOT, to the *Italian*, or *Roman* FOOT, the *Royal Paris* FOOT, &c.

TABLE

TABLE V.

Some of the more useful *Feet* compar'd with the *Engliſh* Foot.	The *Engliſh* Foot divided into 1000 equal Parts.	The *Engliſh* Foot divided into Inches & *Decimal Parts* of an Inch.
		Feet Inches
The *London*, or *Engliſh* Foot	1,000	0 12,0
The *Roman* Foot or the *Italian one*, from the *Monument*		
Of *Coſſutius*	,967	0 11,6
Of *Statilius*	,972	0 11,7
The *Grecian* Foot	1,007	1 00,1
The *Amſterdam* Foot -	,942	0 11,3
The *Leyden* Foot which is vulgarly call'd the *Rhinland one*	1,033	1 00,4
Strasburg Foot	,920	0 11,0
The *Royal Paris* Foot -	1,068	1 00,8

Again,

One *Paris Royal* Foot	is divided into	12 *Pouces* or *Inches.*
One *Pouce*, or *Inch*		12 *Lines.*
One *Line*		12 *Points.*

TABLE VI.

Of SQUARE MEASURES.

A ſquare Inch							
144	*A ſq. Foot*						
1296	9	*A ſq. Yard*					
3600	25	2,77	*A ſq. Pace*				
39204	272,25	30,25	10,89	*A ſq. Pole*			
1568160	10890	1210	435,6	40	*A ſq. Rood*		
6272640	43560	4840	1742,4	160	4	*A ſq. Acre*	
		3097600	1115136	102400	2560	640	*A ſq. Mile*

TABLE VII.

Containing the various CUBIC MEASURES.

A Cubic Inch		
1728	*A Cubic Foot*	
46656	27	*A Cubic Yard*

TABLE

TABLE VIII.

Containing the POUND TROY *and its* PARTS; *by which* Gold, Silver, Jewels, Amber, Bread, Corn, Liquors, *&c. are weigh'd; and by which the* Proportion *of* GRAVITY, *in* Philoſophical EXPERIMENTS, *which any two* BODIES *have to each other, is uſually try'd; as ſuppoſe of* Water *to* Quickſilver, *&c.* Gold *to* Silver, *&c.*

A Grain			
24	*A Penny Wt.*		
480	20	*An Ounce*	
5760	240	12	*A Pound*

TABLE IX.

Containing ſuch Parts *of a* Pound TROY, *as* APOTHECARIES *now uſe to compound their* MEDICINES *by, commonly call'd* APOTHECARIES WEIGHT.

Grains			
20	*A Scruple*		
60	3	*A Dram*	
480	24	8	*An Ounce*
5760	288	96	12 *A Pound*

TABLE

TABLE X.

Shewing the Parts *of a* Pound, *commonly called a* Pound Averdupois; *by* which *are* weigh'd *all* Phyſical Drugs, Butter, Cheese, Fleſh, Grocery, Rosin, Wax, Pitch, Tar, Tallow, Soap, Hemp, *and* all Things *that have* Waste, all baſe Metals *and* Minerals, *as* Iron, Steel, Lead, Tin, Copper, Alom, Copperas, *&c.*

Drams					
16	*An Ounce.*				
256	16	*A Pound*			
7168	448	28	*A Quarter*		
28672	1792	112	4	*An Hund. Wt.*	
573440	35840	2240	80	20	*A Tun*

The *Pound* Troy, is to the *Pound* Averdupois; as 14 to 17.

An *Ounce* Troy, is to the *Ounce* Averdupois; as 56 to 51.

TABLE XI.

Exhibiting the Engliſh MEASURES *of* CAPACITY, *by which* Things DRY *are meaſured, as* GRAIN, SALT, COALS, &c.

Cubic Inches					
$33\frac{3}{7}$	*Pints*				
$268\frac{4}{7}$	8	*Gallons*			
$537\frac{3}{7}$	16	2	*Pecks*		
$2150\frac{2}{7}$	64	8	4	*Buſhels*	
$17203\frac{1}{7}$	512	64	32	8	*Quarter*

Note, *The* MEASURES *are here exhibited, as the legal* Wincheſter BUSHEL *was ſettled, by an* Act *of* Parliament *in* 1697, *according to the* STANDARD *in his* Majeſty's EXCHEQUER: *Altho' the* Corn GALLON *commonly receiv'd, conſiſts of* 272 $\frac{1}{4}$ ſolid INCHES.

TABLE

TABLE XII.

Shewing the Engliſh MEASURES *of* CAPACITY, *by* which All WINES, BRANDIES, SPIRITS, STRONG WATERS, MEAD, PERRY, CYDER, VINEGAR, OYL, HONEY, &c. *are* meaſured.

Solid Inches								
$28\frac{7}{8}$	*Pints*							
231	8	*Gallons*						
4158	144	18	*Rundlets*					
$7276\frac{1}{2}$	252	$31\frac{1}{2}$	$1\frac{3}{4}$	*Barrels*				
9702	336	42	$2\frac{1}{3}$	$1\frac{1}{2}$	*Terces*			
14553	504	63	$3\frac{1}{2}$	2	$1\frac{1}{2}$	*Hogſheads*		
19404	672	84	$4\frac{2}{3}$	$2\frac{2}{3}$	2	$1\frac{1}{3}$	*Puncheons*	
29106	1008	126	7	4	3	2	$1\frac{1}{2}$	*Butts or Pipes*
58212	2016	252	14	8	6	4	3	2 *Tun*

N. B. *A* BARREL *of* Wine *contains but* $31\frac{1}{2}$ Gallons; *but a* BARREL *of* Ale *contains* 32 Gallons; *and of* Beer 36.

Now this *Difference* between the *Ale* and the *Beer Barrel*, is only us'd within *London*, and the *Weekly Bills* of *Mortality*; for, in *all other Places* of *England*, 34 *Gallons* ſhall be accounted a *Barrel* of *Beer* or *Ale*, whether Strong or Small, by *Stat.* 1. of *Will.* and *Mary*.

Moreover, the Capacity of a *Gallon* of *Wine* ought to be 231 *Cubic Inches*, and to hold 8 *Pounds*, 1 *Ounce*, 11 *Drams* AVERDUPOIS, of pure, or Rain *Water*; or 9 *Pounds*, 10 *Ounces*, $1\frac{1}{4}$ *Penny Weight* TROY: But the Capacity of a *Gallon*, by which is meaſured both *Ale*

and *Beer*, ought to be 282 *Cubic Inches*, and to hold of pure or Rain *Water*, 10 *Pounds* 3,426 *Ounces* AVERDUPOIS.

Laſtly, A *ſolid* FOOT contains 1728 *ſolid* INCHES; that is, 7,48 *Wine* GALLONS; and a HOGSHEAD (*Wine-Meaſure*) contains 8,421 *ſolid* FEET; or if you allow 8 *Feet* and an half to be in an HOGSHEAD, then the PIPE will contain 17, and the TUN 34 *ſolid* FEET.

The TABLES exhibiting the MEASURES of *Capacity*, whereby *Beer* and *Ale* are meaſur'd, are as follow:

TABLE XIII.

Shews the MEASURES *for* Beer, *as us'd in* LONDON.

Solid Inches						
35$\frac{1}{2}$	*A Pint*					
282	8	*A Gallon*				
2538	72	9	*A Firkin*			
5076	144	18	2	*A Kilderkin*		
10152	288	36	4	2	*A Barrel*	
15228	432	54	6	3	1$\frac{1}{2}$	*A Hogſhead*

TABLE

TABLE XIV.

Shews the MEASURES *for* Ale *in* LONDON.

Solid Inches						
$35\frac{1}{4}$	*A Pint*					
282	8	*A Gallon*				
2256	64	8	*A Firkin*			
4512	128	16	2	*A Kilderkin*		
9024	256	32	4	2	*A Barrel*	
13536	384	48	6	3	$1\frac{1}{2}$	*A Hogſhead*

TABLE XV.

Shews the MEASURES *both for* Beer *and* Ale *in* all Places *of* ENGLAND, *except* LONDON.

Solid Inches						
$35\frac{1}{4}$	*A Pint*					
282	8	*A Gallon*				
2397	68	$8\frac{1}{2}$	*A Firkin*			
4794	136	17	2	*A Kilderkin*		
9588	272	34	4	2	*A Barrel*	
14382	408	51	6	3	$1\frac{1}{2}$	*A Hogſhead*

TABLE

TABLE XVI.

Exhibiting the SPECIFIC Gravity *of ſeveral* FLUIDS *and* Solid BODIES, *very nearly.*

	19,642		Pure *Gold.*
	14,001		*Mercury.*
	11,345		*Lead.*
	11,092		Fine *Silver.*
	8,875		*Copper.*
	8,350		*Braſs.*
	7,852		*Steel.*
	7,643		*Iron.*
	7,312	So is a *Bulk* of	*Tin.*
	4,875	*common Water,*	*Loadſtone.*
As 1 is to	2,709	to the ſame	*Marble.*
	1,177	*Bulk* of	*Ebony.*
	0,927		*Dry'd Oak.*
	0,237		*Cork.*
	1,028		*Sea Water.*
	0,994		*Diſtill'd Water.*
	1,003		*Vinegar.*
	0,927		*Brandy.*
	1,031		*Cow's Milk.*
	0,913		*Olive Oyl.*

QUESTIONS, for the Learner's *Exerciſe, in* ADDITION.

I. An *Houſekeeper* in one *Week* expends 1 *l.* 5 *s.* 7 *d.* $\frac{3}{4}$; in the ſecond *Week* 1 *l.* 18 *s.* 8 *d.* $\frac{1}{2}$; in the third *Week* 1 *l.* 15 *s.* 3 *d.* $\frac{3}{4}$; and the fourth *Week* 2 *l.* how much hath he expended that *Month* in *Houſekeeping?*

Anſwer. 6 *l.* 19 *s.* 8 *d.*

II. A

II. A *Merchant*, in the *Year* 1727, imported by *one Ship* 8 *Tuns* of *Claret*; by *another Ship* 12 *Tuns*, 3 *Hogſheads*, 42 *Gallons* of *Red Port*; by *another* 14 *Tuns*, 2 *Hogſheads*, 11 *Gallons* of *Sherry*; and by *another* 5 *Tuns*, 3 *Hogſheads*, 10 *Gallons* of *Canary*: What Quantity of *Wines* did he import *that Year* in all?

Anſwer. 41 *Tuns*, 1 *Hogſhead.*

¶ The *Chriſtian* ÆRA began in the 4714th *Year* of the *Julian* PERIOD. Hence, if to the given *Year* of the *Chriſtian* ÆRA be added 4713, the Sum will be the *Year* of the *Julian* PERIOD agreeing to the given *Year* of our *Lord.*

III. *What Year* therefore of the *Julian* PERIOD is it now, in this preſent *Year* of our *Lord* 1728?

Anſwer. 6441.

IV. From the *Creation* of ADAM, to the *Birth* of SETH (*Gen.* v. *v.* 3.) 130 Years; thence to the *Birth* of ENOS (*v.* 6.) 105 Years; thence to the *Birth* of CAINAN (*v.* 9.) 90 Years; thence to the *Birth* of MAHALALEEL (*v.* 12.) 70 Years; thence to the *Birth* of JARED (*v.* 15.) 65 Years; thence to the *Birth* of ENOCH (*v.* 18.) 162 Years; thence to the *Birth* of METHUSELAH (*v.* 21.) 65 Years; thence to the *Birth* of LAMECH (*v.* 25.) 187 Years; thence to the *Birth* of NOAH (*v.* 28.) 182 Years; thence to the *Univerſal* DELUGE (*Gen.* vii. *v.* 6.) 600 Years: *How many Years* therefore was it from the *Creation* of ADAM, to the DELUGE, according to the *Hebrew Computation?*

Anſwer. 1656.

V. From the *Foundation* of the WORLD, to the Beginning of the DELUGE, is accounted 1656 *Years*; and from the Beginning of the DELUGE, to the *Birth* of ARPHAXAD, *Helvicus* counts 2 *Years*; and thence to TERAH 220 *Years*; thence to the *Birth* of ABRAHAM 70 *Years*;

70 *Years* (*Gen.* xi.); thence to the *Promiſe* given (mention'd *Gen.* xii.) *He* counts 75 *Years*; and thence to the *Going out* of EGYPT, *He* counts 430 *Years* (*See Gal.* iii. *v.* 17. and *Exod.* xii. *v.* 41.); and from that *Going out*, to the TEMPLE of SOLOMON, *He*, in like manner, counts 480 *Years*; and thence to the *Birth* of CHRIST, *He* counts 1015 *Years* compleat; and he ſuppoſes that from thence to the *Beginning* of the common CHRISTIAN or DIONYSIAN ÆRA, 2 *Years* have elapſed; and thence to the *preſent Year* wherein this was wrote, we account 1727 *Years* compleat; according to this *Chronology* of HELVICUS, how long is it ſince the *World* was *made*?

Anſwer. 3677 *Years.*

VI. A *Surveyor* having meaſured 5 ſeveral Pieces of *Land*, finds *one* of them to contain 7 *Acres*, 3 *Roods*, 24 *Perches*; *another* to contain 18 *Acres*, 1 *Rood*, 16 *Perches*; *another* 3 *Acres*, 3 *Roods*, 37 *Perches*; *another* 12 *Acres*, 10 *Perches*; and the *laſt* to contain 15 *Acres*, 2 *Roods*: *How many Acres* were ſurveyed in all?

A. *R.* *P.*

Anſwer. 57 3 7.

VII. In the *Year* 1563, 20000 Perſons died of the *Plague* in LONDON; in 1593, 10635 died of the ſame Diſtemper; in 1603, 30578; in 1625, 54265 (of the *Plague* but 35417); in 1665, 97351 dy'd (and of them 68586 of the *Plague*): *How many* dy'd of the *Plague* between the *Years* 1563 and 1665, both *Years* included?

Anſwer. 165216.

QUESTIONS

QUESTIONS in SUBTRACTION.

I. GEORGE, the ſecond, *King* of GREAT BRITAIN, was *born October* 30. in the *Year* of *our* LORD 1683: How old is he *October* 30. 1728?

Anſwer. 45 *Years.*

II. *Queen* CAROLINE was born in the *Year* of our LORD 1682, *March* 1. What is her Age *Mar.* 1. 1728?

Anſwer. 46 *Years.*

III. ROME was built, according to VARRO, in the *Year* of the *Julian* PERIOD 3960; CHRIST was *born* in the *Year* of the *Julian* PERIOD 4714: How many *Years* before CHRIST's *Birth* was ROME built, according to the *common Computation?*

Anſwer. 754 *Years.*

IV. In the *Year* 1174, HENRY II. conquer'd IRELAND, and annex'd it to the *Title* of the *Crown* of ENGLAND: How long is that ago?

Anſwer. 554 *Years.*

V. ROME was built, according to the *Computation* of VARRO, in the 3960th *Year* of the *Julian* PERIOD: Our *Saviour* CHRIST was *born* in the 4719th *Year* of the ſame PERIOD: How many *Years* therefore, according to *common Computation*, was ROME built before the *Birth* of CHRIST?

Anſwer. 759.

VI. From the *Creation* of ADAM, to the *univerſal* DELUGE, according to the *Hebrew Computation*, is 1656 *Years*; and from the *Creation* of ADAM, to the *Birth* of METHUSELAH 687 *Years*: How many *Years* did the DELUGE happen after METHUSELAH was *born?*

Anſwer. 969.

VII. *Tobacco* was firſt us'd in ENGLAND in the *Year* 1585; the preſent *Year* is 1728, how long is that ſince?

Anſwer. 143.

VIII. From the *Creation* of ADAM, to this *Year*, 1728, are, by *Computation*, 5677 *Years*; and from the *Creation* of ADAM, to the Beginning of the *Chriſtian* or *Dionyſian* ÆRA (which HELVICUS ſuppoſes to begin *two Years* after the *Birth* of CHRIST) are accounted 3949 *Years*, what *Year* of the *Chriſtian* ÆRA is *this*?

Anſwer. 1728.

QUESTIONS in MULTIPLICATION.

I. A *great Circle*, whether *Cœleſtial* or *Terreſtrial*, is divided into 360 *Degrees*, and each *Degree* into 69 ½ *Miles* + 14 *Poles*; how many *Engliſh* MILES is (a *great Circle* of the EARTH, or) the *Circumference* of the EARTH?

Anſwer. 25035 *Miles* + 240 *Poles*, i. e. 25036 nearly.

¶ Each *Olympiad* conſiſts of a *Cycle* of 4 Years, and the FIRST *Olympiad* was celebrated in the Year of the *Julian* PERIOD 3938, on the *full Moon*, ſubſequent to the *Summer* SOLSTICE: Wherefore any *Olympiac Year* being given, if the compleat *Olympiads* are × by 4, and to the Product be added the given *Year* of the current *Olympiad*, and then 3937 be added, the Sum will be the Year of the *Julian* PERIOD agreeing to the *Olympiac Year* given, *viz.* from the *full* Moon ſubſequent to the *Summer* SOLSTICE.

II. *Ariſtotle*

II. *Ariſtotle* was born in the firſt Year of the 99th *Olympiad.* What Year of the *Julian* PERIOD agrees with that *Olympiac* Year?

Anſwer. 4334.

¶ The *mean* SEMIDIAMETER of the EARTH is defin'd to 4000 *Engliſh* Miles, equal to 21120000 ENGLISH *Feet.*

III. LIGHT runs thro' the Space of 1000 *Diameters* of the Earth in one Minute's Time, how many Engliſh *Feet* does it run thro' in the ſame Time?

Anſwer. 42,240,000,000.

IV. The SUN is eſteem'd diſtant from the EARTH 1000 of the Earth's *Diameters*: How many *Engliſh* FEET are they diſtant?

Anſwer. 422,400,000,000.

V. The *mean Diſtance* of the MOON from the EARTH's *Center* is accounted about 60 of the EARTH's *Semidiameters*: How many ENGLISH *Feet* is it?

Anſwer. 1,267,200,000.

¶ The *Area* of EVERY *Parallelogram* WHATSOEVER is found by *multiplying* the *Length* and true *Breadth* together. *Euclid. Schol.* 35. 1.

This being granted, the young *Arithmetician* may uſefully enough employ himſelf in the following and ſuch like *Queſtions.*

VI. An *Acre* of Land in ENGLAND, by the *Statute*, is *four* RODS *broad*, and *forty long*: How many *ſquare Rods* does an *Acre* contain?

Anſwer. 160.

VII. There are four Fields, each of them in the Form of a *Parallelogram* (no matter whether *Right* or *Oblique* Angl'd): The *Length*, as alſo the true *Breadth* of the *Firſt* is 60 *Rods*, *Poles*, or *Perches*; the *Length* of the *Second* is 90 *Rods*, and its true *Breadth* 40; the *Length* of the *Third* is 120, and *Breadth* 30; and the *Length* of the *Fourth* is 180 *Rods*, and its true *Breadth* 20 *Rods*. What is the *Area* of *Each*?

Anſwer. 3600 ſquare *Rods*, *Poles* or *Perches*.

VIII. There is a Place whoſe *Length* is 64 *Feet*, and true *Breadth* 42 *Feet*, to be paved with Stone: How many Stones, each a Foot ſquare, will this Pavement take up?

Anſwer. 2688.

¶ The Solidity of every CUBE and of every PARALLELOPIPED is had by *multiplying* the three *Dimenſions*, LENGTH, BREADTH, and DEPTH into *one another*; or, which is the ſame thing in the End, by finding the AREA of the *Baſe* (which may be *any* Side at Pleaſure) and multiplying it into the Height. *Euclid.* Cor. 40. 11.

IX. There is a *Stone* CUBE, or a STONE whoſe three *Dimenſions* are equal to each other, *viz.* *each* 12 *Inches*; what is the *Solidity* of that CUBE?

Anſwer. 1728 Cubic *Inches*.

X. There is a *Beam* 10 *Feet* long, 14 *Inches* broad, and 11 *Inches* thick: What is its ſolid *Content*?

Anſwer. 18480 ſolid *Inches*.

XI. There is a *Wall* to be built whoſe *Length* muſt be 100 *Feet*, *Breadth* 2 *Feet*, *Height* 30 *Feet*: How many *Bricks*, *one Foot* long, $\frac{1}{2}$ a *Foot* or [5 broad, and $\frac{1}{4}$ of a *Foot* or [25 deep, will do the Buſineſs?

Anſwer. 48000.

Viz.

Viz. 100 *ſuch* Bricks plac'd length-ways, will be the *Length* of the *Wall* propos'd; 4 breadth-ways will be the *Breadth* of the *Wall*, and 120 plac'd one upon another will be the *Height* of the *Wall*. But 100 × 4 × 120 equal to 48000. But here note, that in this Calculation no Account is had of any Room for Mortar for *Joints*.

¶ By the ſame Artifice, by which is found the *Solidity* of a CUBE or PARALLELOPIPED, may alſo be found the *Capacity* of any *Veſſel* of the Form of either CUBE or PARALLELOPIPED. EX. gr.

XII. There is a *Veſſel*, each of whoſe *Sides* within is 7 *Feet*; what is the *Capacity* of that *Veſſel*?

Anſwer. 343 Cubic *Feet*, or 2565 *Gallons*, equal to 40 $\frac{1}{7}$ *Hogſheads*.

XIII. There is a *Ciſtern* 8 *Feet* long, 2 broad, and 4 deep; how many Cubic *Feet* of Water will it hold?

Anſwer. 64.

¶ The *Semidiameter* × into the *Semi-periphery* gives the *Area* of the CIRCLE; or (to avoid *Fractions*) the fourth Part of the *Product* of the *Diameter* and *Periphery* is the *Area* of the CIRCLE.

XIV. Let the *Diameter* of the CIRCLE be 28 *Inches*, the *Periphery* 88. What is its *Area*?

Anſwer. 616 ſquare *Inches*, or 4 *ſquare* FEET + 40 *ſquare Inches*.

XV. Suppoſe the Earth's CIRCUMFERENCE to be 25132 *Engliſh* MILES, and its DIAMETER to be 8000 of the ſame MILES; what is the *Area* of a *Great Circle* of the EARTH?

Anſwer. 50,264,000 ſquare *Engliſh* MILES.

¶ The

¶ The *Diameter* of a SPHERE multiply'd into the *Periphery* gives the *Superficies* of the SPHERE.

XVI. The *Diameter* of a SPHERE is 28 *Inches*, its *Periphery* 88, what is its *Superficies*?

Anſwer. 2464 ſq. *Inches.*

¶ The SOLIDITY of a SPHERE is produc'd from the *Superficies* × the *Sixth* Part of the *Diameter*, or $\frac{1}{3}$ Part of the *Semidiameter*; or, to ſhun *Fractions*, the *Product* of the *Semidiameter* and *Superficies* TRISECTED, gives the SOLIDITY of the SPHERE:

XVII. What is the SOLIDITY of the SPHERE juſt propos'd?

Anſwer. 11498 $\frac{2}{3}$ *Cubic* INCHES.

XVIII. The MOON's *Diameter* is 2175 *Miles*; what is its *Surface*, and what its *Solidity*?

Anſwer. Surface 14,840,025 *ſquare Miles*; and *Solidity* 5,379,509,062 *Cubic Miles.*

XIX. The SUN in *Diameter* is 763000 *Miles*; what is its *Surface*, and what its *Solidity*?

Anſwer. Surface 1,828,936,179,000 *ſquare Miles*, and *Solidity* 232,579,717,429,500,000 *Cubic Miles.*

XX. MERCURY, the nearcſt to the SUN of the known *Planets*, is in *Diameter* 4240 *Miles*; what *Surface* has it, and what *Solidity*?

Anſwer. Surface is 56,476,800 *ſquare Miles*, and its *Solidity* is 39,910,272,000 *Cubic ones.*

XXI. The *Planet* VENUS is farther from the SUN, and is in *Diameter* 7900 *Miles*; what *Surface*, and what *Solidity* has it?

Anſwer. Its *Surface* is 196,062,200 *ſquare Miles*, and its *Solidity* is 258,148,563,333 *Cubic Ones.*

XXII. The

XXII. The EARTH is the *next* PLANET to VENUS; it is in *Diameter* 8000 *Miles*; what is its *Surface*, and what its *Solidity?*

Anſwer. Its *Surface* is 201,056,000 *ſquare Miles*, and its *Solidity* is 268,074,666,666 *Cubic Ones.*

XXIII. MARS is ſtill higher in the *Syſtem*; it is in *Diameter* 4440 *Miles*; what *Surface* has it, and what *Solidity?*

Anſwer. Its *Surface* is 61,929,120 *ſquare Miles*, and its *Solidity* 45,827,548,800 *Cubic Ones.*

XXIV. JUPITER, the largeſt of *all* the PLANETS, is much *higher* in the SYSTEM; it is in *Diameter* 81000 *Miles*; what *Surface* has it, and what *Solidity?*

Anſwer. *Surface* 20,611,908,000 *ſquare Miles*, and *Solidity* 278,260,758,000,000 *Cubic Ones.*

XXV. SATURN, the *Higheſt* and moſt *Remote* of all the *known* PLANETS, is in *Diameter* 68000 *Miles*; what *Surface* has it, and what *Solidity?*

Anſwer. Its *Surface* is 14,526,704,000 *ſquare Miles*, and its *Solidity* 164,635,978,666,666 *Cubic Miles.*

¶ The *Curv'd Superficies* of a *Right* CYLINDER is had by *Multiplying* the *Periphery* of the BASE by the *Height.*

XXVI. Let the BASE's *Periphery* be 88 *Inches*, and the HEIGHT of the CYLINDER 48: What is its *Superficies?* (*i. e.* the *Curv'd* Part) the *Superficies* of both BASES being excepted.

Anſwer. 4224 *ſquare Inches.*

¶ The *Solidity* of a CYLINDER is obtain'd by *Multiplying* the BASE (*i. e.* the *Area* of the BASE) into the HEIGHT.

XXVII.

XXVII. What then is the *Solidity* of the CYLINDER whoſe BASE is 616 *ſquare* INCHES, and HEIGHT 48 *Inches?*

Anſwer. 29568 *Cubic* INCHES.

¶ The *Curv'd Superficies* of a *Right* CONE is produc'd by *Multiplying* the *Periphery* of the BASE into *half* the SIDE of the CONE.

XXVIII. Let the PERIPHERY of the CONE's BASE be 88 *Inches*, and its SIDE 48; what is its SUPERFICIES?

Anſwer 2112 *ſquare Inches.*

¶ The *Solidity* of a CONE is produc'd from the BASE × a third Part of the *Height.*

XXIX. What then is the *Solidity* of the CONE, whoſe BASE is 616 *ſquare* INCHES, and HEIGHT 48 *Inches?*

Anſwer. 9856 *Cubic Inches.*

Thus may the Solidity of every $\left\{\begin{matrix} Priſm \\ Pyramid \end{matrix}\right\}$ whatſoever be found by multiplying its Baſe into $\left\{\begin{matrix} \text{the Perpendicular Height.} \\ \frac{1}{3} \text{ of the Perpendicular Height.} \end{matrix}\right.$

Vide Euclid Schol. of Prop. 9. 12.

QUESTIONS

QUESTIONS in DIVISION.

I. The *Mean Diſtance* of the MOON from the EARTH's *Center* is reckon'd about 1,267,200,000 *Engliſh* FEET; how many MILES then is it, ſuppoſing 5280 *Feet* to make a *Mile?*

Anſwer. 240000.

II. The SUN is reckon'd to be 422,400,000,000 *Engliſh* FEET diſtant from the EARTH; how many MILES then are they aſunder?

Anſwer. 80,000,000.

¶ If, having the Year of the *Julian* PERIOD given, you would find the *Olympiac* Year agreeing to it, from the given Year ſubtract 3937, and divide the Remainder by 4; the Quotient will exhibit the *Olympiads* paſt, and the Remainder the Year of the current *Olympiad.* *Ex. gr.*

III. What *Olympiac* Year agrees to the firſt Year of the Building of the City of ROME; or, according to the *Varronian* Computation, to the 3961ſt Year of the *Julian* PERIOD?

Anſwer. The 4th Year of the 6th *Olympiad.* That is, whenſoever nothing remains after the Diviſion is perform'd, it is from thence evident, that in the given Year of the *Julian* PERIOD ſome Cycle of *Olympiads* or other is compleated; that is, that it is the 4th Year of the current *Olympiad.*

IV. What *Olympiac* Year agrees to the Year of our *Lord* 1727; *i. e.* to the Year of the *Julian* PERIOD 6440?

Anſwer. The 3d Year of the 625th *Olympiad.*

V. A PLANK contains 40 ſquare *Feet*; but the *Breadth* of the PLANK is 2 *Feet*: What then is its *Length*?

Anſwer. 20 *Feet.*

VI. A ROOM contains 216 ſquare *Feet*, and the *Length* of the ROOM is 18 *Feet*; what is its *Breadth*?

Anſwer. 12 *Feet.*

VII. The *Breadth* of a *Rectangular* FIELD is 8 *Rods*; what ought its *Length* to be, to contain an *Acre* of *Land*?

Anſwer. 20 *Rods.*

VIII. A *Chamber* FLOOR 12 *Feet* broad and 20 long, is to be cover'd with BOARDS a *Foot* broad, and 10 *Feet* long; how many ſuch BOARDS will do the Job?

Anſwer. 24. For the FLOOR contains 240 ſquare *Feet*, *i. e.* 12 *Feet* × 20: Wherefore, ſince every BOARD is but 1 *Foot* broad, and 240 ÷ 1 is equal to 240; it is requiſite that the *Length* of *all* the BOARDS be 240 *Feet*. But *each* BOARD is 10 *Feet* long, and 240 ÷ 10 is equal to 24. Therefore 'tis manifeſt, that 24 BOARDS, *each* 10 *Feet* long, are adequate to the requiſite *Length*.

IX. There is a CHAMBER whoſe *Length* is 16 *Feet*, *Breadth* 14, and *Height* 12; how many *Yards* of *Yard-wide* CLOTH are ſufficient to hang all the Sides of that CHAMBER?

Anſwer. 80 *Yards.*

X. How many *Yards* of *ordinary* STUFF, 2 *Yards* wide, will line a Piece of TAPESTRY 24 *Feet* long, and 8 broad?

Anſwer. 10 $\frac{2}{3}$ *Yards.*

XI. The

XI. The *Capacity* of a CISTERN is 64 Cubick *Feet*, the *Length* of it is 8 *Feet*, and *Breadth* 2 *Feet*; what is its *Depth*?

Anſwer. 4 *Feet.*

XII. A CISTERN is deſir'd which will contain 10 Cubic *Feet* (*i. e.* a little more than 74 *Gallons*, *Wine Meaſure*) of WATER; but the Place where the CISTERN is to be ſet, won't allow the *Breadth* of it to be above one *Foot*, and the *Length* 4 *Feet*: How deep then ought the CISTERN to be to hold the aforeſaid *Quantity* of WATER?

Anſwer. 2 $\frac{1}{2}$ *Feet.*

XIII. The Solidity of a *Cylinder*, whoſe Baſe contains 616 *ſquare Inches*, being 29568 *Cubic Inches*; what's the Height of that Cylinder?

Anſwer. 48 *Inches.*

XIV. The Solidity of a *Cone*, whoſe Height is 48 *Inches*, being 9856 *Cubic Inches*, what's the *Area* of its Baſe?

Anſwer. 616 *ſquare Inches.*

XV. The Solidity of a SPHERE, whoſe *Surface* contains 2464 *ſquare Inches*, being 11498 $\frac{2}{3}$ *Cubic Inches*, what's the Diameter of that Sphere?

Anſwer. 28 *Inches.*

The following Queſtions in *Reduction*, are but Queſtions in *Multiplication* and *Diviſion*, and will farther Exerciſe the Learner therein.

QUESTIONS in Reduction.

I. In 8706 *Nobles* (each 6 *s.* 8 *d.*) how many *Farthings?*

Anſwer. 2,785,920.

II. In 4858 *Guineas* (each 21 *s.*) how many *Groats?*

Anſwer. 306054.

III. In 1728 *Years* (1 Year being 365 Days 6 Hours) how many *Minutes?*

Anſwer. 908,858,880.

IV. In 253 *Pounds, Troy Weight,* how many *Grains?*

Anſwer. 1,457,280.

V. In 994 *Tuns,* how many *Pounds* Avoirdupois?

Anſwer. 2,226,560.

VI. In 144 *Hogſheads* of Wine, how many *Pints?*

Anſwer. 72576.

VII. In 640 *Acres,* how many ſquare *Yards?*

Anſwer. 3,097,600.

VIII. In 488 ſquare *Yards,* how many ſquare *Inches?*

Anſwer. 632448.

IX. In 1728 ſolid *Feet,* how many Cubic *Inches?*

Anſwer. 2,985,984.

X. In 12 *Signs,* or 360 Degrees, how many *Thirds?*

Anſwer. 77,760,000.

XI. In 2,785,920 *Farthings,* how many *Nobles?*

Anſwer. 8706.

XII. In

XII. In 306054 *Groats*, how many *Guineas* (each 21 *s.*)?

Anſwer. 4858.

XIII. In 908,858,880 *Minutes*, how many *Years* (allowing one Year to contain 365 Days 6 Hours)?

Anſwer. 1728.

XIV. In 1,457,280 *Grains*, how many *Pounds* Troy?

Anſwer. 253.

XV. In 2,226,560 *Pounds* Avoirdupois, how many *Tuns*?

Anſwer. 994.

XVI. In 72576 *Pints*, how many *Hogſheads* of *Wine*?

Anſwer. 144.

XVII. In 3,097,600 ſquare *Yards*, how many *Acres*?

Anſwer. 640.

XVIII. In 632448 ſquare *Inches*, how many ſquare *Yards*?

Anſwer. 488.

XIX. In 2,985,984 ſolid *Inches*, how many ſolid *Feet*?

Anſwer. 1728.

XX. In 77,760,000 *Thirds*, how many *Degrees*?

Anſwer. 360.

XXI. In 254 *l.* 11 *s.* 2 *d.* $\frac{1}{2}$ *Sterling*, how many *French Crowns*, at 54 *d.* $\frac{1}{2}$ *per French Crown*?

Anſwer. 1121.

XXII. In

XXII. In 4260 *Dollars* (at 4 *s.* 4 *d.*) how many *Milreas* (each 6 *s.* 6 *d.*)?

Anſwer. 2840.

XXIII. In 58940 *Ells Flemiſh* (each 3 Quarters of an *Engliſh* Yard) how many *Ells Engliſh?*

Anſwer. 35364.

XXIV. In 1121 *French Crowns*, at 54 *d.* $\frac{1}{2}$ apiece, how much Sterling?

Anſwer. 254 *l.* 11 *s.* 2 *d.* $\frac{1}{2}$.

XXV. In 2840 *Milreas* (at 6 *s.* 6 *d.* each) how many *Dollars*, at 4 *s.* 4 *d.* each?

Anſwer. 4260.

XXVI. In 35364 *Ells Engliſh*, how many *Ells Flemiſh?*

Anſwer. 58940.

QUESTIONS in the *Single Rule of Three* DIRECT.

I. If a DRAPER buys *Broad Cloth* for 16 *s.* and ſells it for 18 *s.* a *Yard*, what does he gain *per Cent.* or in laying out 100 *l.?*

Anſwer. 12 *l.* 10 *s.*

II. How many *Ounces* of PLATE, at 5 *s.* 6 *d.* an *Ounce*, will 67 *l.* 6 *s.* 1 *d.* $\frac{1}{2}$ pay for?

Anſwer. 244 *Ounces*, 15 *Penny-weights.*

III. If an *Ingot* of GOLD, weighing 9 *lb.* 9 *oz.* 12 *pwts.* be worth 411 *l.* 12 *s.* what is a *Grain* of that GOLD worth?

Anſwer. 7 *Farthings.*

IV. If

IV. If a Dozen *Ells* of HOLLAND are valued at 3 *l.* 6 *s.* how much will 8 *Pieces* (each *Piece* containing 54 *Ells*) amount to, at the ſame *Rate*?

Anſwer. 118 *l.* 16 *s.*

V. If I *buy* 2 *C.* 1 *qr.* 7 *lb.* of COFFEE for 64 *l.* 15 *s.* how muſt I *ſell* it by *Retale*, *viz.* how much *per Ounce*, to gain 21 *l.* 11 *s.* 8 *d.* by the *Whole*?

Anſwer. 5 *d.*

VI. If $\frac{3}{4}$ of a *Yard* of VELVET coſt 7 *s.* 3 *d.* how many *Yards* will 13 *l.* 15 *s.* 6 *d.* buy, at that *Rate*?

Anſwer. 28 $\frac{1}{2}$.

VII. If $\frac{1}{4}$ *Part* of a SHIP be worth 387 *l.* 15 *s.* what is $\frac{1}{16}$ Part of that SHIP worth?

Anſwer. 96 *l.* 18 *s.* 9 *d.*

VIII. What does the Whole *Pay* of a *Man of War's Crew* of 640 *Sailors* amount unto, for 32 *Months* Service, each Man's Pay being 22 *s.* 6 *d.* a *Month*?

Anſwer. 23040 *l.*

IX. If the CLOATHING of a *Regiment* of *Soldiers*, conſiſting of 680 *Men*, coſt 2574 *l.* 1 *s.* 8 *d.* how much is that a *Man*?

Anſwer. 3 *l.* 15 *s.* 8 *d.* $\frac{1}{2}$.

X. If the *Expences* in *Houſe-keeping*, Six Weeks, amount to 9 *l.* 3 *s.* 6 *d.* how long will 100 *l.* laſt, at that *Rate*?

Anſwer. $65\frac{141}{367}$ *Weeks*; which is about 1 $\frac{1}{4}$ *Year.*

XI. One *Bricklayer*, A, can build a certain *Wall* in 56 *Days*; another, B, can build the *ſame* in 42 *Days*; in how many *Days* then can they finiſh the *ſame*, if they work together?

Anſwer. 24 *Days.*

XII. A

XII. A DEBTOR owing ſeveral Perſons in *all* 1490 *l.* 5 *s.* 10 *d.* *compounds* with, and *pays* them as far as his *Effects*, which amount to no more than 931 *l.* 8 *s.* 7 *d.* $\frac{2}{4}$, will go; how much do the CREDITORS, by this *Compoſition*, receive *per Pound?*

Anſwer. 12 *s.* 6 *d.*

XIII. Suppoſe one *buys* 245 *C.* $\frac{1}{2}$ of HOPS, at 2 *l.* 16 *s.* *per C.* and *ſells* them for 7 *d.* a *Pound*; what does he gain by the *Whole*, and what *per Cent.?*

Anſwer. He gains in *all* 114 *l.* 11 *s.* 4 *d.* and 16 *l.* 13 *s.* 4 *d.* *per Cent.*

XIV. How many *Pieces* of MARBLE, each 1 $\frac{1}{2}$ *Foot ſquare*, will pave a *Hall* containing (in *Area*) 70 *ſquare Yards?*

Anſwer. 280.

XV. If, at ROTTERDAM, I buy 850 *Flemiſh Ells* of Linen for 106 *l.* 5 *s.* *Sterling*, what may I ſell an *Ell* in *London* for, to loſe nothing thereby (5 *Ells Flemiſh* being equal to 3 *Engliſh* ones)?

Anſwer. 4 *s.* 2 *d.*

XVI. A DRAPER bought 4 *Bales* of CLOTH, each *Bale* containing 6 *Pieces*, and each *Piece* 27 *Yards*, at 16 *l.* 4 *s.* *per Piece*; I demand the Price of the *Whole*, and the Rate *per Yard?*

Anſwer. 1 *Yard* coſt 12 *s.* and the *Whole* 388 *l.* 16 *s.*

XVII. A VINTNER lays out 142 *l.* 10 *s.* 9 *d.* in 4 ſeveral ſorts of WINE, and of each a like Quantity, *viz.* *Red* PORT at 5 *s.* SHERRY at 6 *s.* *French* CLARET at 8 *s.* 8 *d.* and BURGUNDY at 10 *s.* 6 *d.* a *Gallon*; what Quantity of each did he buy?

Anſwer. 94 *Gallons* $\frac{1}{2}$ equal to 1 *Hogſhead* $\frac{1}{2}$ of each.

XVIII. A

XVIII. A DRUGGIST, in ſelling a certain Quantity of RHUBARB for 86 *l.* 13 *s.* 4 *d.* gain'd after the Rate of 30 *per Cent.*; what then did that Quantity of RHUBARB coſt him?

Anſwer. 66 *l.* 13 *s.* 4 *d.*

XIX. A MERCHANT receives 1000 *Pieces* of *Eight (Mexico)* at 4 *s.* 6 *d.* each, and *ſells* them for 4 *s.* 8 *d.* $\frac{1}{4}$ apiece, what does he gain, as well by the *Whole*, as *per Cent.*?

Anſwer. His *whole Gain* is 9 *l.* 7 *s.* 6 *d.* and 4 $\frac{1}{6}$ *per Cent.*

XX. *Bought* 20 *Chaldrons* of COALS for 25 *l.* and *ſold* them for 30 *l.*; but if they had *coſt* me 30 *l.* what ſhould I have *ſold* them for, to have gain'd after the ſame *Rate*?

Anſwer. 36 *l.*

XXI. Suppoſe a FOOTMAN walks on continually 24 *Miles* a *Day*, and, after he has been gone 6 *Days*, is purſued by a HORSEMAN, who rides 36 *Miles* a *Day*; how many *Days*, and how many *Miles* muſt he ride to overtake the FOOTMAN?

Anſwer. 432 *Miles*, in 12 *Days.*

XXII. If an *Ingot* of SILVER *weighing* 7 *lb.* 6 *oz.* and worth 4 *s.* 10 *d.* an *Ounce*, be melted down together with another, weighing 9 *lb.* 2 *oz.* and worth 5 *s.* 8 *d.* an *Ounce*, what will an *Ounce* then be worth?

Anſwer. 5 *s.* 3 *d.* $\frac{1}{21}$

XXIII. If a *School-Boy* ſpends *half* a GUINEA in a *Fortnight*, how long will 10 *l.* laſt him, at that *Rate*?

Anſwer. 38 $\frac{2}{21}$ *Weeks.*

XXIV. A *Gentleman* expends 6 *l.* 6 *s.* 8 *d.* a *Day*, one Day with another, his *yearly* ESTATE being 2000 *l.*

clear: Query, Whether he lays up any thing out of his ESTATE, or out-runs it, at the *Year's End*; and how much?

Anſwer. He expends *yearly*, above his *Income*, 311 *l.* 13 *s.* 4 *d.*

XXV. A *Gentleman's* daily *Expences* (one Day with another) are 1 *l.* 8 *s.* yet at the *Year's End* he finds he hath laid up 388 *Guineas*; what's his *yearly Income?*

Anſwer. 918 *l.* 8 *s.*

XXVI. A *Gentleman*, out of an ESTATE of 1000 *l.* a *Year*, clear, wou'd *ſave* 500 MARKS *yearly*; what may he afford to *ſpend daily?*

Anſwer. 1 *l.* 16 *s.* 6 *d.* $\frac{1}{4}$ +.

XXVII. What muſt that *Gentleman*, who hath 1800 *l.* a *Year* in LANDS, pay towards the LAND-TAX, that *Year* the TAX is laid at 3 *s.* 6 *d.* in the *Pound?*

Anſwer. 315 *l.*

XXVIII. If 5 *l.* *Principal Money*, gain 6 *s.* in a *Year*, what will 100 *l.* gain in *that Time*, at that *Rate?*

Anſwer. 6 *l.*

XXIX. What is the INTEREST of 100 *l.* from the 25th Day of *December*, excluſive, to the 1ſt Day of *Auguſt* following, inclusive, at the *Rate* of 5 *l.* *per Cent.* *per Annum?*

Anſwer. 3 *l.*

XXX. Suppoſe I buy 60 *Hogſheads* of WINE for 850 *l.* 10 *s.* out of which, by chance, are leak'd 378 *Gallons*; how muſt I ſell it a *Gallon* to be no Loſer by the Bargain?

Anſwer. 5 *s.*

XXXI. Suppoſe I buy 40 *Hogſheads* of WINE for 367 *l.* how muſt I ſell it *per Gallon*, to gain 20 *per Cent?*

Anſwer. 5 *s.* 4 *d.* 3 *far.* $\frac{1}{3}$.

XXXII. If

XXXII. If I buy SATTIN at 6 *s.* 8 *d.* a *Yard*, which, being damag'd, I wou'd ſell at a *Rate*, whereby I loſe 10 *per Cent.*; what is that *Rate?*

Anſwer. 6 *s.* a *Yard.*

XXXIII. If A gives B 51 *Reams* of PAPER in *Barter*, for 3 *C.* 3 *qrs.* 5 *lb.* of TOBACCO, valued at 1 *s.* 6 *d.* a *Pound*; how's the PAPER valued *per Ream?*

Anſwer. 12 *s.* 6 *d.*

XXXIV. How much SUGAR, at 9 *d.* $\frac{1}{4}$ a *Pound*, muſt A give B for 2 *C. weight* of TEA, worth 10 *d.* an *Ounce?*

Anſwer. 32 *C.* 3 *qrs.* 7 *lb.* $\frac{35}{39}$.

XXXV. A *Merchant* has 648 *Yards* of *Broad* CLOTH, worth 14 *s.* a *Yard*, but in *Barter* values it at 16 *s.* a *Yard*; and this he wou'd *exchange* for WINE, worth, in Ready Money, 42 *l.* a *Tun*: Therefore, to proportion the *Barter* equally, what muſt a *Tun* of WINE be valued at, and how many *Tuns* muſt be given for the *Broad* CLOTH?

Anſwer. 10 *Tuns*, 3 *Hogſheads*, 12 *Gallons*, + $\frac{3}{7}$ of a *Gallon*; and valued at 48 *l.* a *Tun.*

XXXVI. Two *Corn-Factors* A and B *barter*: A hath 576 *Buſhels* of BARLEY, worth 3 *s.* 4 *d.* but, in *Barter*, reckon'd worth 3 *s.* 7 *d.* the *Buſhel*: B has WHEAT worth 5 *s.* a *Buſhel*, which, in *Barter*, he values at 5 *s.* 4 *d.* The *Queſtion* is then, how many *Buſhels* of WHEAT B muſt give A for his 576 *Buſhels* of BARLEY; and whether A or B has the *Better* of the *Bargain?*

Anſwer. B muſt give A 387 *Buſhels* of WHEAT; which are 3 *Buſhels* more than he wou'd, if the *Barter* had been equally proportion'd; and therefore B loſes ſo much by this *Barter.* For,

To proportion the *Barter* equally, B ſhould have *valued* his WHEAT at 5 *s.* 4 *d.* $\frac{1}{2}$ a *Buſhel.*

XXXVII. A *Factor* remits from *Oporto* to *London* 654 MIL-REAS and 750 REAS, at 6 *s.* 10 *d. per* MIL-REA; how much *Sterling* do they amount to?

Anſwer. 223 *l.* 14 *s.* 1 *d.* $\frac{1}{2}$.

XXXVIII. A *Merchant* at *London* pays in *Exchange* for *Roan* 647 *l.* 19 *s.* 7 *d.* $\frac{3}{4}$ STERLING, at 54 *d.* $\frac{1}{2}$ *per French* CROWN; how much TOURNOIS muſt he receive at *Roan?*

Anſwer. 8560 *Livres*, 10 *Sols*, *Tournois*, equal to 2853 $\frac{1}{2}$ *French Crowns.*

XXXIX. A *Merchant* at *London* remitted to *Amſterdam*, by *Bill* of *Exchange*, 788 *l.* STERLING at 33 *s.* 4 *d. Flemiſh per Pound* STERLING; how much *Flemiſh* Money was paid at *Amſterdam?*

Anſwer. 1313 *l.* 6 *s.* 8 *d. Flemiſh.*

XL. A *Remitter* at *London* draws upon *Dublin* for 855 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$ *Iriſh*, at 10 *d.* $\frac{1}{2}$ *per Cent.* Value receiv'd in *London*; how much STERLING was receiv'd there?

Anſwer. 774 *l.* 8 *s.* 3 *d.* $\frac{1}{4}$ + *Sterling.*

I ſhall next inſert *QUESTIONS* which relate to the *Meaſuring* the CIRCLE and SPHERE, and then give *ſome* of other Kinds, wherewith the young *Arithmetician* may be further *exerciſed* in this moſt uſeful RULE of PROPORTION, and be at the ſame Time inſtructed in ſeveral *Analogies* or *Proportions*, purpoſely ſtated for the SOLUTION of *thoſe* and ſuch like QUESTIONS, leſt he be *deceiv'd*, and take thoſe Things *to be* PROPORTIONAL which are *not ſo*; of which he hath been Caution'd in the 10th Chapter.

¶ The DIAMETERS of *Circles* and their CIRCUMFERENCES are directly *Proportional*; which being granted, the following *Queſtions* may be eaſily ſolv'd.

XLI. Sup-

XLI. Suppoſe the DIAMETER of one *Circle* to be 7 *Inches*, and the CIRCUMFERENCE 21,991 *Inches*; and the DIAMETER of another *Circle* 15 *Inches*; what is the CIRCUMFERENCE of this other *Circle*?

Anſwer. 47,123.

XLII. The CIRCUMFERENCE of one *Circle* being 21,991 (as before) and the DIAMETER 7 *Inches*; and the CIRCUMFERENCE of another *Circle* 47,123 *Inches*; what is the DIAMETER of this other *Circle*?

Anſwer. 15.

¶ The *Diameter* of every *Circle* is to its *Circumference* as 7 to 22, or as 113 to 355; or, more accurately, as 1 to 3,14159... This being granted, the following QUESTIONS may be ſolv'd.

XLIII. The DIAMETER of a *Circle* being 15 *Inches* (as in the former Propoſition), what is the CIRCUMFERENCE?

Anſwer. 47,12386.

XLIV. The CIRCUMFERENCE of a *Circle* being 47,12386 *Inches*, what is its DIAMETER?

Anſwer. 15.

¶ The *Area* of a *Circle* has the ſame *Proportion* to the *Square* of its DIAMETER, as the 4th Part of the CIRCUMFERENCE has to the DIAMETER; which being granted;

XLV. Suppoſe the DIAMETER of a *Circle* be 113 *Feet*, and its CIRCUMFERENCE 355, what is its *Area*?

Anſwer. 10028,75 *ſquare* FEET.

¶ The *Area* of a CIRCLE is to the *Square* of its DIAMETER as 11 to 14; or, as 0,785399 to 1; which being granted;

XLVI.

XLVI. What is the *Area* of that CIRCLE, whoſe DIAMETER is 113 *Feet?*

Anſwer. 10028,759 *ſquare Feet.*

¶ Again; The *Area* of a CIRCLE is to the *Square* of its PERIPHERY, as 7 to 88; or, as 0,079577 to 1. Therefore,

XLVII. If the PERIPHERY of a *Circle* be 355 *Feet*, what is its *Area?*

Anſwer. 10028,69 *ſquare Feet.*

¶ The ROOT or *Side* of the *Square* inſcrib'd in a CIRCLE is in *proportion* to the DIAMETER, as 0,707107 to 1. Which granted,

XLVIII. What's the *Side* of the SQUARE inſcrib'd in a CIRCLE, whoſe DIAMETER is 113 *Feet?*

Anſwer. 79,90309.

¶ The *Root* or *Side* of a *Square* equal to the AREA of a CIRCLE, is to the DIAMETER, as 0,886227 to 1. And this is call'd the QUADRATURE of the *Circle.*

XLIX. What then is the *Side* of the *Square* that's equal to the CIRCLE, whoſe DIAMETER is 113 *Feet?*

Anſwer. 100,1436 *Feet.*

¶ The *Superficies* of a SPHERE is to the *Square* of its *Diameter*, as 22 to 7; or, as 3,14159 to 1. Which being granted,

L. What's the *Superficies* of a SPHERE whoſe *Diameter* is 3 *Feet?*

Anſwer. 28,2743 *ſquare Feet.*

¶ The

¶ The *Side* of a *Square* equal to the *Superficies* of a SPHERE, is to the *Diameter* thereof, as 1,772454 to 1. Whence,

LI. If a SPHERE be 3 *Feet* in DIAMETER, what's the *Side* of the *Square* that's equal to the *Superficies* of that SPHERE?

Anſwer. 5,31736 *Feet.*

¶ Again; The *Side* of a *Square* equal to the *Superficies* of a SPHERE, is to the *Periphery* thereof, as 0,564189 to 1. Whence,

LII. If the *Periphery* of a SPHERE be 9,42477 *Feet*, what's the *Side* of the *Square* that's equal to the *Superficies* of that SPHERE?

Anſwer. 5,31735 *Feet.*

¶ The ROOT or SIDE of a CUBE equal to the *Solidity* of a SPHERE, is to its *Diameter*, as 0,80604 to 1;

LIII. Suppoſe then the *Diameter* of a SPHERE to be 7 *Feet*, what's the *Side* of the CUBE that's equal to the *Solidity* of that SPHERE?

Anſwer. 5,64228 *Feet.*

¶ Again; The *Side* of a CUBE equal to the *Solidity* of a SPHERE, is to the *Periphery*, as 0,256556 to 1. Which being granted,

LIV. What's the *Side* of the CUBE that's equal to the *Solidity* of a SPHERE, whoſe *Periphery* is 22 *Feet*?

Anſwer. 5,64423 *Feet.*

¶ The *Solidity* of a SPHERE, is to the *Cube* of its *Diameter*, as 0,5236 to 1. And contrarywiſe.

LV. If

LV. If therefore the *Diameter* of a SPHERE be 7 *Feet*, what's the *Solidity?*

Anſwer. 179,6 *Cubic Feet.*

¶ Again; The *Solidity* of a SPHERE, is to the *Cube* of its *Periphery*, as 0,016887 to 1. And contrary-wiſe. Whence,

LVI. If the *Periphery* of a SPHERE be 22 *Feet*, what's its *Solidity?*

Anſwer. 179,7 *Cubic Feet.*

¶ The *Area* of an ELLIPSIS, is to a *Circle* deſcrib'd on its *Tranſverſe* or *Longeſt Diameter*, as the *Conjugate* or *Shorteſt Diameter* is to the *Tranſverſe.* Wherefore,

LVII. Suppoſe the *Conjugate Diameter* of an ELLIPSIS to be 4 *Feet*, and its *Tranſverſe Axis* 6 *Feet*, and the *Content* of the CIRCLE, deſcrib'd upon the *Tranſverſe Axis*, to be 28,274 *ſquare Feet*, what is the *Content* of the ELLIPSIS?

Anſwer 18,849 *ſquare Feet.*

¶ Again; The *Area* of an ELLIPSIS, is to a *Circle* deſcrib'd on its *Conjugate Diameter*, as the *Tranſverſe Diameter* is to the *Conjugate.* Whence,

LVIII. If the *Conjugate Diameter* of an ELLIPSIS be 4 *Feet*, and its *Tranſverſe* 6 *Feet*, and the *Content* of the CIRCLE deſcrib'd upon the *Conjugate Diameter* be 12,566 *ſquare Feet*, what is the *Content* of the ELLIPSIS?

Anſwer. 18,849 *ſquare Feet.*

COROL. Hence it may be plainly inferr'd, that the *Area* of every ELLIPSIS is a *Mean Proportional* between the CIRCLES deſcrib'd on its *Longeſt* and *Shorteſt Diameters.*

¶ Our Country-man, Mr. NORWOOD, in the *Year* 1635, by making proper Obſervations, and actually *Meaſuring* from *North* to *South*, found, that for each *Degree* of *Alteration* of *Latitude*, the *Diſtance meaſured* was 69,5 *Miles.*

LIX. If then there are 69,5 MILES in 1 *Degree* of the *Circumference* of the *Earth*; how many *Miles* are there *round the Earth?*

Anſwer. 25020.

LX. The SUN is ſaid to perform one entire Revolution (or 360 *Degrees*) in the Space of 365 *Days*, 5 *Ho.* 48 *Min.* 57 *Sec.* call'd a *Tropical* or *Solar Year.* How much does it move in one *Day?*

Anſwer. 59′. 8″. 19‴. &c.

¶ By the 4th Prop. of the 6th Book of EUCLID, the following *Proportion* is juſt;

As the *Length* of the SHADOW of any *Upright Pole*, or other *Perpendicular Object*, is to the *Height* of that *Object*; ſo is the *Length* of the SHADOW, any *Building* caſts at the ſame Time, to the *Height* of that *Building.*

This being granted, we may put the following

QUESTIONS.

LXI. If a *Pole* 12 *Feet* high, ſet upright, caſt a SHADOW 18 *Feet* long; and, at the ſame Inſtant, a

Tower caſts a SHADOW 168 *Feet* long: What is the *Height* of the *Tower?*

Anſwer. 112 *Feet.*

LXII. If any *Building*, 30 *Feet* high, caſts a SHADOW 24 *Feet* long; and, at the ſame Inſtant, a STEEPLE caſts a SHADOW 216 *Feet* long; what's the *Steeple's* Height?

Anſwer. 270 *Feet.*

¶ The AREAS of all *Similar Figures* (are to each other, *not* as their *like Sides*, *but*) are in a *Duplicate Ratio* of their *Homologous* or *like Sides*; or, as the *Squares* of their *Homologous Sides.* (*Euclid* 20. 6.) And contrarywiſe.

Thus alſo CIRCLES are to each other (not as their *Diameters*, but) as the *Squares* of their *Diameters.* (*Euclid* 2. 12.) And contrarywiſe. Wherefore,

LXIII. Suppoſe the *Side* of a *Regular* or *Irregular Figure*, of what Number of *Sides* ſoever; for Inſtance, 5 *Sides*, be 4 *Feet*, and the *Area* or *Content* of this *Figure* be 68 *ſquare Feet*; and the *Homologous Side* of another *Similar* 5 *ſided Figure* be 7 *Feet*; what is the AREA of this *Figure?*

Anſwer. 208 $\frac{1}{4}$ *ſquare Feet.* Again;

LXIV. Suppoſe the *Content* of a CIRCLE, whoſe *Diameter* is 12 *Inches*, be 113,097 *ſquare Inches*, what is the *Content* of another CIRCLE, whoſe *Diameter* is 7 *Inches?*

Anſwer. 38,4844 *ſquare Inches.*

¶ The *Solidities* of *all* Similar *Priſms* and *Pyramids*, (are, to each other, *not* as their *Homologous Sides*, *but*) are in a *Triplicate Ratio* of their *Homologous* or *like Sides*; or, as the *Cubes* of their *Homologous Sides.* (*Vide Euclid* Prop. 8. 12. and Prop. 9. 12.) Thus

Thus then *Similar* CYLINDERS and CONES (are to one another, *not* as the *Diameters* of their *Baſes*, *but*) are in a *Triplicate Ratio* of that of the *Diameters* of their *Baſes*. (*Euclid* 12. 12.) And contrarywiſe. Wherefore,

LXV. Suppoſe the *Solidity* of a PARALLELOPIPED be 18 *Cubic Feet*, and any *Side* of it be 4 *Feet*; and ſuppoſe the *Homologous Side* of another *Similar* PARALLELOPIPED be 5 *Feet*, what is its *Solidity*?

Anſwer. 35,15625 *Cubic Feet.*

LXVI. Suppoſe the *Diameter* of the *Baſe* of a CYLINDER be 4 *Feet*, and the *Content* of that CYLINDER be 48 *Cubic Feet*, what ſhall the *Solidity* of another *Similar Cylinder* be, whoſe *Diameter* is 6 *Feet*?

Anſwer. 162 *Cubic Feet.*

LXVII. Suppoſe the *Diameter* of the *Baſe* of a CONE be 5 *Feet*, and the *Content* of that CONE be 56 *Cubic Feet*; what ſhall the *Solidity* of another *Similar* CONE be, whoſe *Diameter* is 8 *Feet*?

Anſwer. 229,376 *Cubic Feet.*

¶ *Triangles* and *Parallelograms*; and alſo *all* Kinds of *Pyramids* and *Priſms*; each kind compar'd among themſelves, if they have the ſame ALTITUDE, are in the ſame *Proportion* to one another, as their *Baſes*: And if their *Baſes* are equal, they are as their *Heights*. *Vide Euclid* Prop. 1. 6. and its *Scholium*; Prop. 5. 12. Prop. 6. 12. Cor. of Prop. 9. 12. Prop. 32. 11. Prop. 11. 12. and Prop. 14. 12.

LXVIII. Suppoſe two TRIANGLES to be of the ſame Height, and the *Baſe* of *one* of them to be 6 *Feet*, and that of the *other* 16 *Feet*, and the *Content* of the *former*

†

mer to be 54 *ſquare Feet*, what is the *Content* of the *latter*?

Anſwer. 144 *ſquare Feet.*

LXIX. Suppoſe the *Baſes* of two TRIANGLES to be equal, and the *Height* of the *firſt* 6 *Feet*, and that of the *other* to be 16 *Feet*; likewiſe the AREA of the *former* to be 54 *ſquare Feet*, what is the AREA of the *latter*?

Anſwer. 144 *ſquare Feet.*

LXX. Suppoſe the *Heights* of two PRISMS of any kind, (*e. g.* PARALLELOPIPEDS) to be *equal*, and the *Baſe* of *one* of them to be 8 *ſquare Feet*, and that of the *other* to be 12 *ſquare Feet*, and the *Content* of the *former* to be 48 *ſolid Feet*, what *Solidity* has the *latter*?

Anſwer. 72 *ſolid Feet.*

LXXI. Suppoſe the *Height* of a PRISM, of any kind, (a CYLINDER, for Inſtance) be 5 *Feet*, and its *Solidity* 45 *Feet*; and ſuppoſe the *Height* of another CYLINDER, of the ſame *Baſe* with the *former*, be 9 *Feet*, what is its *Solidity*?

Anſwer. 81 *ſolid Feet.*

LXXII. Suppoſe the *Baſe* of a PYRAMID, of any kind, (a CONE, for Inſtance,) be 8 *ſquare Feet*, and its *Solidity* be 72 *Feet*; and ſuppoſe the *Baſe* of another CONE of the ſame *Height* with the *former*, be 5 *Feet*, what is its *Solidity*?

Anſwer. 45 *Cubic Feet.*

LXXIII. Suppoſe the *Baſes* of any *two* PYRAMIDS, of the ſame kind, to be equal; the *Height* of *one* of them to be 10 *Feet*, and that of the *other* to be 16, and the *Content* of the *latter* to be 192 *ſolid Feet*, what is the *Solidity* of the *former*?

Anſwer. 120 *Feet.*

¶ *Similar*

¶ *Similar* CONES, whether *Right* or *Oblique*, (are not as *their* AXES, but) are in a *Triplicate Ratio* of that of their *Axes*; or, as the *Cubes* of their *Axes*. Wherefore,

LXXIV. Suppoſe the *Axis* of a CONE to be 9 *Feet*, and its *Solidity* to be 45; and ſuppoſe the *Axis* of *another* CONE, *ſimilar* to the *former*, be 12 *Feet*, what is its *Solidity*?

Anſwer. 106,666 *Feet.*

¶ Again; *Similar* CONES are in a *Triplicate Ratio* of that of their *Sides* inclin'd to their *Baſes*. Wherefore,

LXXV. Suppoſe the *ſlant Side* of a CONE be 12 *Feet*, and its *Solidity* be 42 *Feet*; and ſuppoſe the *ſlant Height* of another *Similar* CONE be 18 *Feet*, what is its *Solidity*?

Anſwer. 141,75 *Feet.*

¶ SPHERES are to each other, (*not* as their *Diameters*, but) as the *Cubes* of their *Diameters*. (*Euclid* 18. 12.) And contrarywiſe. Wherefore,

LXXVI. Suppoſe the *Diameter* of a SPHERE be 6 *Inches*, and its *Solidity* be 113,097; and ſuppoſe the *Diameter* of another SPHERE be 12 *Inches*, what is its *Solidity*?

Anſwer. 904,776 *Inches.*

¶ A SPHERE is in *Proportion* to its *Circumſcribing Cylinder*, (*i. e.* a *Cylinder* whoſe *Height* and *Baſe's Diameter*, are each equal to the *Axis* of the SPHERE) as 2 to 3: But, to a *Cone*, of ſuch *Height* and *Diameter* as 2 to 1.

LXXVII. Sup-

LXXVII. Suppoſe the *Solidity* of a SPHERE to be 18 *Feet*, what is the *Solidity* of the *Circumſcrib'd* CYLINDER?

Anſwer. 27 *Feet.*

LXXVIII. If the *Solidity* of a CYLINDER, whoſe *Height* and *Baſe's Diameter* are equal, be 36 *Feet*, what is the *Solidity* of the *Inſcrib'd* SPHERE?

Anſwer. 24 *Feet.*

LXXIX. If the *Solidity* of a CONE, whoſe *Height* and *Baſe's Diameter* are equal, be 12 *Feet*; what is the *Solidity* of a SPHERE, whoſe *Diameter* is equal to the ſame *Cone's Height?*

Anſwer. 24 *Feet.*

¶ The *Weights* of SHOT, of the ſame *Metal*, are as the *Cubes* of their *Diameters:* And on the contrary, the *Cubes* of their *Diameters* are as their *Weights.*

LXXX. It is found, that an *Iron* BULLET, whoſe *Diameter* is 4 *Inches*, weighs 9 *Pounds*; what then is the *Weight* of an *Iron* BULLET, whoſe *Diameter* is 6 *Inches?*

Anſwer. 30 *lb.* $\frac{3}{8}$, or 30,375 *lb.*

¶ The SPACES run thro' by *Deſcending Heavy Bodies*, are in a *Duplicate Ratio* (or as the *Squares*) of the Times in falling, from the Beginning of the *Fall:* And on the contrary, the Times are in a *Subduplicate Ratio* (or as the *Roots*) of the SPACES.

LXXXI. It is found, that *Heavy Bodies*, not far from the *Surface* of the EARTH, fall 16 *Feet* in a *Second of Time*; how many *Feet* will they fall in 4 *Seconds?*

Anſwer. 256.

LXXXII. SOUND

LXXXII. SOUND is found to move 1142 *Engliſh Feet* in one *Second of Time*, how far then will it paſs in a *Minute*?

Anſwer. 68520 *Feet*, equal to 13 *Miles*, very nearly.

N. B. That in Working many of the *foregoing*, as well as the *following QUESTIONS*, the *Learner* may (by way of *Exerciſe*, if he pleaſes,) very commodiouſly uſe either *Vulgar* or *Decimal* FRACTIONS: But more of this hereafter in its proper Place.

QUESTIONS in the Single Rule *of* THREE INVERSE.

I. If 16 *Men* encloſe a GARDEN with a BRICK WALL in 28 *Days*, how many *Men* will do the ſame in 64 *Days*?

Anſwer. 7 *Men.*

II. If a *Man* performs a JOURNEY in 9 *Days*, when the *Day* is 11 *Hours* long, in how many *Days* will he perform the *ſame*, when the Day is 15 *Hours* long; the *Length* of the *Day* being accounted from *Sun-Riſing* to *Sun-Set*?

Anſwer. 6 *Days*, 9 *Hours.*

III. If the *Penny White Loaf* ought to weigh 9 *Ounces*, *Troy*, when WHEAT is at 4 *s.* 6 *d.* a *Buſhel*; what ought it to weigh, when WHEAT is at 6 *s.* 9 *d.* a *Buſhel*?

Anſwer. 6 *Ounces.*

IV. At

IV. At what Price *per Buſhel* is WHEAT, when the *Penny White Loaf* weighs 5 *Ounces*, 8 *Penny-weights*, if it weighs 9 *Ounces*, when WHEAT is at 4 *s.* 6 *d.* a *Buſhel*?

Anſwer. 7 *s.* 6 *d.*

V. If 1 *l.* 2 *s.* worth of WINE will ſuffice a CLUB of 12 *Men*, when the WINE is *ſold* after the Rate of 25 *l.* 4 *s.* a *Hogſhead*; how many *Men* will 1 *l.* 2 *s.* worth ſerve, when the WINE is *ſold* after the Rate of 18 *l.* 18 *s.* a *Hogſhead*?

Anſwer. 16 *Men.*

VI. A *Governor* of a *Fort* has PROVISIONS ſufficient for 1850 *Soldiers* for 6 *Months*; but how many of them muſt he diſmiſs immediately from the *Garriſon*, that the PROVISIONS may laſt 2 *Months* longer?

Anſwer. 450.

VII. If I lend a *Man* 650 *l.* for 22 MONTHS, how long Time ought he to lend me 953 *l.* 6 *s.* 8 *d.* to be even with me?

Anſwer. 15 *Months.*

VIII. If 736 *Pieces of Money*, (*viz. Pieces of* EIGHT) at 4 *s.* 6 *d.* each, were equivalent to, and given for 144 *Pieces* of another Value *per Piece*; what was that *Value*?

Anſwer. 23 *s.* a *Piece.*

¶ Equal *Triangles*, equal *Parallelograms*, equal *Pyramids*, and equal *Priſms*, have their *Baſes* and *Heights* reciprocally Proportional.

Vide Euclid Prop. 14. 6. Prop. 15. 6. Prop. 34. 11. Prop. 9. 12. and its Cor. and Prop. 15. 12.

Theſe

Theſe being premiſed and granted, we may put the following *QUESTIONS.*

IX. Suppoſe there be 2 *equal* TRIANGLES; the *Baſe* of the *former* is 6 *Feet*, and that of the *latter* 10, and the *Height* of the *former* is 12 *Feet*; what is the *Height* of the *latter*?

Anſwer. 7,2 *Feet.*

X. Suppoſe there be 2 *equal* PARALLELOGRAMS; the *Baſe* of the *former* is 49, and its *Height* 14 *Feet*; and the *Height* of the *latter* is 21 *Feet*; what is the *Baſe* of the *latter*?

Anſwer. 32 $\frac{2}{3}$ *Feet.*

XI. Suppoſe there be 2 *equal* PYRAMIDS, *viz.* CONES; the *Baſe* of the *former* is 20 *ſquare Feet*, and that of the *latter* 12 *ſquare Feet*; and the *Height* of the *latter* is 7 $\frac{1}{2}$ *Feet*; what is the *Height* of the *former*?

Anſwer. 4 $\frac{1}{2}$ *Feet.*

XII. Suppoſe there be 2 *equal* PRISMS, *viz.* PARALLELOPIPEDS; the *Baſe* of the *former* contains 18 *ſquare Feet*, and its *Height* is 12 *Feet*; and the *Height* of the *latter* is 8 *Feet*; what is its *Baſe*?

Anſwer. 27 *ſquare Feet.*

XIII. Suppoſe there be 2 *equal Triangular* PRISMS; the *Height* of the *former* is 28 *Inches*, and that of the *latter* 34 *Inches*; and the *Baſe* of the *latter* 112 *ſquare Inches*; what is the *Baſe* of the *former*?

Anſwer. 136 *ſquare Inches.*

XIV. If a BOARD (a PARALLELOGRAM) 16 *Inches* in *Length*, and 9 in *Breadth*, be equal to a *ſquare Foot*; how many *Inches* in *Length* will make a *ſquare Foot*, when the *Breadth* is 5 *Inches*?

Anſwer. 28,8 *Inches.*

XV. If a *Strait squared Piece of* TIMBER, (a PRISM) which is terminated by *equal Areas* at the *Ends*, each containing 144 *square Inches*, must be 12 *Inches* in *Length* to make a *solid Foot*; how *long* must it be, to make a *solid Foot*, if the *Base*, or *Area* at each *End*, contains but 84 *square Inches*.

Answer. 20 $\frac{4}{7}$ *Inches.*

XVI. If 48 *Yards* of STUFF $\frac{3}{4}$ of a *Yard* wide, will *hang* the *Side* of a ROOM, how many *Yards* of *Tapestry*, 3 *Yards* wide, will *hang* the *same*?

Answer. 12 *Yards.*

XVII. If 48 *Yards* of HANGING are sufficient to *hang* the *Side* of a ROOM that's 27 *Feet long*, and 12 *Feet high*; what's the *Width* of the HANGING?

Answer. $\frac{3}{4}$ of a *Yard.*

XVIII. If 600 *Pensioners* are to be new CLOATH'D, each Man's *Garment* to contain 2 $\frac{1}{4}$ *Yards* of CLOTH, which is 1 $\frac{1}{2}$ *Yard* wide; how many *Yards* of SHALLOON, $\frac{3}{4}$ of a *Yard* wide will *line* them?

Answer. 2700 *Yards.*

Note, That *Hangings* and *Sides* of *Rooms* may be consider'd as PARALLELOGRAMS.

¶ The *Lengths* of PENDULUMS are to one another, as the *Squares* of their *Vibrations* reciprocally. And contrarywise.

XIX. It is *found* that a PENDULUM must be 39,2 *Inches* in *Length*, to *swing Seconds*, *i. e.* to make 60 *Vibrations* in a *Minute*; how *long* then must a PENDULUM be, to make 120 *Vibrations* in a *Minute*?

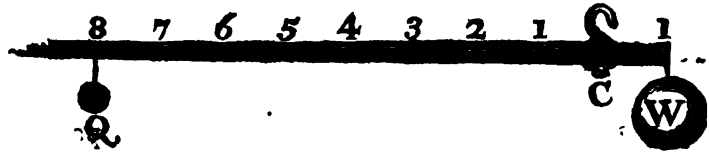
Answer. 9,8 *Inches.*

¶ If

¶ If two WEIGHTS, applied to the *Ends* of any *Lever* (as of the *Common Balance*, or the *Steel-yard*) hang in *Equilibrio*, they will be *reciprocally* proportional to their *Diſtances* from the *Center* of MOTION, or from that *Point*, by which the LEVER is *ſupported*.

XX. Suppoſing Q C W to be a *Steel-yard*, where the *Diviſions*, 8,7; 7,6; 6,5; 5,4; 4,3; 3,2; 2,1; 1,C; and C,1; are *equal*, ſuppoſe the *Weight* W to be one *Pound*; what WEIGHT *hung* at the *Point* 8, will keep the BEAM in *Equilibrio*?

Anſwer. 2 *Ounces.*



XXI. Suppoſe a LEVER put *Horizontally* upon a *Prop*, to have a *Weight* of 1000 *lb.* ſuſpended at one *End*, at the *Diſtance* of 1 *Foot* from the *Prop* or *Point* of Support; at what *Diſtance* from the ſame *Point*, muſt 100 *lb. Weight* (or ſome other *Force* equivalent to it) be applied at the other *End*, or *Arm* of the LEVER, to *counterbalance* or ſuſtain the former WEIGHT of 1000 *lb.*

Anſwer. 10 *Feet.*

¶ By the ſame RULE, the *Length* of the *Lever*, wherewith ARCHIMEDES would have lifted the *Earth*, may be calculated: Thus; Suppoſe the *Weight* of the *Earth* to be (in a *round Number*) equal to 21,000,000,000,000,000,000,000 *hundred Weight*, and that it be ſuſpended at the *Di-*

ſtance of 1 *Foot* from the *Fulcrum* or *Point* by which the *Lever* is ſupported; then, ſuppoſing the *Force* of ARCHIMEDES's *Arm* to be equivalent to 1 *hundred Weight*, I ſay, the *Lever* requir'd to move the *Earth*, muſt have been 21,000,000,000,000,000,000,000 *Feet* in *Length* from that *Point*.

But, by the by, tho' this *Lever* be, in *Theory*, truly ſufficient to move the *Earth* out of its *Place*, yet ſeveral concurring *Impoſſibilities* render it impoſſible to be done; which that Celebrated *Mathematician*, without doubt, was not ignorant of; and therefore meant no more by his *ſtupendous Propoſition*, than to denote the immenſe *Force* and prodigious *Effects* of the *Mechanical Powers*.

QUESTIONS in the Double *or* Compound Golden RULE.

I. If 100 *l. gain* 5 *l. Intereſt* in a *Year*, how much will 850 *l. gain* in 3 *Years* and 8 *Months*?

Anſwer. 155 *l.* 16 *s.* 8 *d.*

II. What *Principal* SUM will gain 155 *l.* 16 *s.* 8 *d. Intereſt*, in 3 *Years* 8 *Months*, at the *Rate* of 5 *per Cent. per Annum*?

Anſwer. 850 *l.*

III. In what *Time* will 850 *l. Principal Money* gain 155 *l.* 16 *s.* 8 *d.* at the *Rate* of 5 *per Cent. per Annum*?

Anſwer. 44 *Months* equal to 3 *Years* 8 *Months*.

IV. A USURER put out 455 *l.* at *Intereſt*, and after it had continued 3 *Years* and 4 *Months*, he receiv'd for *Principal*

Principal and *Intereſt* together 576 *l.* 6 *s.* 8 *d.* at what *Rate per Cent. per Annum* did he receive *Intereſt?*

Anſwer. 8 *per Cent.*

V. On the the 25th Day of *March* 1722, a SCRIVENER lent on a *Mortgage* 760 *l.*; and on the 25th Day of *May* 1728, he receiv'd for *Intereſt* thereof 281 *l.* 4 *s.* I demand at what *Rate, per Cent. per Annum,* his Money was *lent?*

Anſwer. 6 *per Cent.*

VI. If the *Carriage* of 60 *Hundred Weight,* 20 *Miles,* coſt 14 *l.* 10 *s.* what *Weight* ought to be carried 30 *Miles,* for 5 *l.* 8 *s.* 9 *d.* at that *Rate* of *Carriage?*

Anſwer. 15 *Hundred Weight.*

VII. If 8 *Men,* in 14 *Days,* can *mow* 112 *Acres* of GRASS, how many *Men* will *mow* 2000 *Acres,* at that *Rate,* in 10 *Days?*

Anſwer 200 *Men.*

VIII. If 170 *Buſhels* of WHEAT will ſuffice a *Regiment* of 680 *Soldiers,* 6 *Days,* how many *Quarters* will ſerve an ARMY of 79200 *Men,* at that *Rate,* 16 *Days?*

Anſwer. 6600 *Quarters.*

IX. If 2 *Horſes* eat up 8 *Buſhels* of OATS in 16 *Days,* how many *Horſes* will eat up 3000 *Quarters* in 24 *Days,* at that *Rate?*

Anſwer. 4000 *Horſes.*

X. If 275 *Pioneers* caſt a TRENCH of 250 *Rods,* in 12 *Hours,* how many *Men,* at that *Rate,* will caſt a *Trench* of 880 *Rods* in 8 *Hours?*

Anſwer. 1452 *Men.*

XI. If 2 *Penny White Loaves* ſuffice 3 *Men,* when WHEAT is at 4 *s.* 6 *d.* a *Buſhel,* how many *Penny White Loaves*

Loaves will ſuffice 9 *Men*, when WHEAT is at 9 *s.* a *Buſhel*?

Anſwer. 12 *Penny Loaves.*

XII. If 54 *Workmen* can build a FORT in 18 *Days*, when the *Day* is 17 *Hours* long; in how many *Days* will 68 *Workmen* build the *ſame*, when the *Day* is but 9 *Hours* long?

Anſwer. 27 *Days.*

If this laſt QUESTION be allow'd to come *properly* under this Head, as I ſee no Reaſon to the contrary, it occaſions the following *Remark*, *viz.* That the two Single Compoſing *Analogies*, whereby *this* is to be ſolv'd, are BOTH *of them Inverſe*, tho' it hath been aſſerted by ſome, that it NEVER happens that BOTH *Analogies* are INVERSE.

The Terms may be ſtated after either of the *Forms* following, being ALL of them *Inverſe*.

Hours.	*Days.*		*Hours.*	*Days.*
17	18	::	9	34
Men.	*Days.*		*Men.*	*Days.*
54	34	::	68	27

Men.	*Days.*		*Men.*	*Days.*
54	18	::	68	$14\frac{5}{17}$
Hours.	*Days.*		*Hours.*	*Days.*
17	$14\frac{5}{17}$	::	9	27

Hours.	*Men.*		*Hours.*	*Men.*
17	54	::	9	102
Men.	*Days.*		*Men.*	*Days.*
102	18	::	68	27

Men.

Men.	*Hours.*		*Men.*	*Hours.*
54	17	::	68	$13\frac{1}{2}$
Hours.	*Days.*		*Hours.*	*Days.*
$13\frac{1}{2}$	18	::	9	27

But, 'tis true, this *Queſtion* may be *ſolv'd* by one Single *Inverſe Analogy*: For, 'tis plain, that if the 18 *Days* be multiply'd by 17 (the *Hours* in *one Day*) the *Product* will be the *Number of Hours*, which the 54 *Men* will build the FORT in, *viz.* 306 *Hours*, then *Inverſely*,

Men.	*Hours.*		*Men.*	*Hours.*
54	306	::	68	243

which 243 *Hours* being *divided* by 9, (the *Hours* in *one Day*) will be reduc'd to 27 *Days* for the *Anſwer*, as before.

QUESTIONS in FELLOWSHIP.

I. Three *Merchants*, A, B, and C, *join* their *Monies* and *raiſe* a STOCK of 20000 *l.* of which A put in 9000 *l.* B 7000 *l.* and C 4000 *l.* with this they *traded* a certain Time, and *gain'd* 8000 *l.*; how ought this GAIN to be *ſhar'd* amongſt them?

Anſwer. A's *Share* amounted to 3600 *l.* B's to 2800 *l.* and C's to 1600 *l.*

II. Three *Merchants*, A, B, and C, *Trade* a certain Time with a *Joint-Stock* of 20000 *l.* and *gain* 8000 *l.* of which A's *Part* was 3600 *l.* B's 2800 *l.* and C's 1600 *l.* what was each *Merchant's* STOCK?

Anſwer. A's *Stock* was 9000 *l.* B's 7000 *l.* and C's 4000 *l.*

III. A

III. A *Captain* and *Lieutenant* of a *Man of War* (being *abroad*) buy a Quantity of Goods for 680 *l.* 10 *s.* of which the *Captain* depoſited 576 *l.* 5 *s.* and the *Lieutenant* the *reſt*; now by diſpoſing of theſe Goods they *gain'd* 206 *l.* 10 *s.* of which, what was the *Captain's Share?*

Anſwer. 174 *l.* 17 *s.* 3 *d.* 2 *q.* $\frac{130}{1361}$.

IV. Three *Merchants*, A, B, and C, *join* their *Monies* to make a Stock of 12400 *l.* with which they *Traded* and *Gain'd* 2480 *l.* of which A gets 686 *l.* B 870 *l.* and C 924 *l.*; what was each *Merchant's* Stock at firſt?

Anſwer. $\left.\begin{matrix}\text{A's}\\\text{B's}\\\text{C's}\end{matrix}\right\}$ Stock was $\left\{\begin{matrix}3430\ l.\\4350\ l.\\4620\ l.\end{matrix}\right.$

V. Three *Merchants*, A, B, and C, *Trading* together, *Gain'd* 516 *l.* 16 *s.* 10 *d.* A put in 816 *l.* 12 *s.* 6 *d.* B put in 913 *l.* 10 *s.* 10 *d.* and C 407 *l.* 9 *s.* 8 *d.* I demand what was each Man's Gain?

Anſwer. $\left\{\begin{matrix}\text{A's } 197\ l.\ 6\ s.\ 7\ d.\ 0\ q. \frac{33504}{513036}.\\\text{B's } 220\ l.\ 14\ s.\ 11\ d.\ 2.\ q. \frac{126952}{513036}.\\\text{C's } 98\ l.\ 9\ s.\ 3\ d.\ 1\ q. \frac{352580}{513036}.\end{matrix}\right.$

VI. Three *Men*, A, B, and C, *trade* together a little while, and *gain* 48 *l.* 13 *s.* 4 *d.* A put in at firſt 220 *l.* and B and C together, put in 330 *l.*; now B's *Share* of the Gain was 18 *l.* 5 *s.*; what then did A and C *gain*, and B and C put in reſpectively?

Anſwer. A's *Gain* was 19 *l.* 9 *s.* 4 *d.* and C's *Gain* 10 *l.* 19 *s.* B's *Stock* 206 *l.* 5 *s.* and C's *Stock* 123 *l.* 15 *s.*

VII. Three *Merchants*, A, B, and C, *freight* a *Ship* with 456 *Pipes* of Wine, *viz.* A's *Part* thereof was 190 *Pipes*, B's 152, and C's 114; but *bad Weather* oblig'd

oblig'd the *Sailors* to throw 84 *Pipes* over board; how many *Pipes* did each Merchant *Loſe*?

Anſwer. A loſt 35 *Pipes*, B 28, and C 21.

VIII. Three Perſons, A, B, and C, *put in* Money to *trade* together, A put in 60 *l.* B 70 *l.* and C a certain Sum unknown; they *gain'd* 100 *l.* of which C's *Part* came to 69 *l.*; what then did he *put in*, and what did A and B *gain*?

Anſwer. C *put in* 195 *l.*; A *gain'd* 18 *l.* 9 *s.* 2 *d.* $\frac{10}{13}$, and B *gain'd* 21 *l.* 19 *s.* 9 *d.* $\frac{3}{13}$.

IX. Two *Men*, A and B, make a JOINT-STOCK; A *put in* 100 *l.* more than B, and they *gain'd* 500 *l.* of which B's *Share* came to 218 *l.* 15 *s.*; what was each Man's *Stock* at firſt?

Anſwer. A's *Stock* was 450 *l.* and B's 350 *l.*

X. Three *Merchants*, A, B, and C, raiſed a STOCK of 3600 *l.* they *traded*, and *gain'd* 2700 *l.* of which, as oft as A receiv'd 3 *l.* B receiv'd 5 *l.* and C 7 *l.*; what was each Man's STOCK and GAIN?

Anſwer. A's, B's, C's STOCK was 720 *l.*, 1200 *l.*, 1680 *l.* and GAIN 540 *l.*, 900 *l.*, 1260 *l.*

XI. Three *Perſons*, A, B, and C, in *Traffick*, jointly *gain* 987 *C.* 1 *qr.* 19 *lb.* of SUGAR; A *depoſiting* at firſt 197 *l.* 9 *s.* 4 *d.* B 316 *l.* 18 *s.* and C 218 *l.* 7 *s.* I demand each Man's *Part* of the SUGAR?

Anſwer. A's *Part* was 266 *C.* 12 *lb.* +, B's 427 *C.* 6 *lb.* +, and C's 294 *C.* 1 *qr.* +.

XII. Four *Sailors*, A, B, C, and D, join together, and *buy* 150 *Gallons* of BRANDY for 23 *l.* 10 *s.* of which A paid 7 *l.* 10 *s.* B 6 *l.* 15 *s.* C 5 *l.* 5 *s.* and D 3 *l.* How many *Gallons* then ought each to have for his SHARE?

Anſwer. A, B, C, D ought to have 50, 45, 35, 20 *Gallons.*

XIII. Two *Merchants*, A and B, commence PARTNERS, A having 240 *l.* in *Trade* for 12 *Months*, and B having 240 *l.* in *Trade* for 14 *Months*; and they *gain* clearly 125 *l.* 9 *s.* How muſt this GAIN be *divided* between them?

Anſwer. A's *Part* is 57 *l.* 18 *s.* and B's 67 *l.* 11 *s.*

XIV. Two *Merchants*, A and B, agree thus, *viz.* That A do put into *Trade* 1500 *l.* for 12 *Months*; and that B do, 2 *Months* afterwards, *depoſite* ſuch *Sum* as will entitle him to $\frac{2}{3}$ of their GAIN, at the *Year's End*; the *Whole* of which they then find to be 250 *l.* Therefore what did B *put in*?

Anſwer. 3600 *l.*

XV. Three *Merchants* enter'd *Partnerſhip*; A put into *Trade* 900 *l.* for 15 *Months*, B 1125 *l.* for 12 *Months*, and C 750 *l.* for 18 *Months*; and they *gain'd*, in *all*, 718 *l.* 10 *s.* 6 *d.* What was each Merchant's *Share* of this GAIN?

Anſwer. 239 *l.* 10 *s.* 2 *d.*

XVI. Three *Perſons*, A, B, and C, compos'd a *ſmall* JOINT-STOCK of 90 *l.* *viz.* A had in it 20 *l.* for 3 *Months*; B 30 *l.* for 5 *Months*; and C 40 *l.* for 7 *Months*; their GAIN, in *all*, was 57 *l.* 3 *s.* 4 *d.* what was each Man's *Share* of it?

Anſwer. A's, B's, C's *Share* was 7 *l.*, 17 *l.* 10 *s.*, 32 *l.* 13 *s.* 4 *d.*

XVII. Three *Perſons*, A, B, and C, jointly *rais'd* a STOCK of 4600 *l.*; A's *Part* thereof continued in 8 *Months*, B's 6 *Months*, and C's 4 *Months*; their *total* GAIN

GAIN was 800 *l.*; of which A's *Dividend* was 400 *l.* B's 266 *l.* 13 *s.* 4 *d.* and C's 133 *l.* 6 *s.* 8 *d.* What did each Perſon *depoſite* at firſt?

Anſwer. A *put in* 1800 *l.* B 1600. *l.* and C 1200 *l.*

XVIII. Three *Perſons*, A, B, and C, enter'd *Partnerſhip*, their *Common* STOCK being 3822 *l.*; A's Money *was in* 3 *Months*, B's 5 *Months*, and C's 7 *Months*; they *gain'd* 234 *l.* which was ſo divided, as the $\frac{1}{2}$ of A's *Gain* was *equal* to $\frac{1}{3}$ of B's *Gain*, and *equal* to $\frac{1}{4}$ of C's *Gain*: What did each *gain* and *put in*?

Anſwer.

		l.		*l.*	*s.*	*d.*
A's	*Gain*	52	and	1386 :	4 :	4 $\frac{44}{193}$
B's	was	78	*Stock*	1247 :	11 :	11 $\frac{138}{193}$
C's		104		1188 :	3 :	8 $\frac{148}{193}$

Vide Hill's Arithmetick, *p.* 285.

XIX. Three *Perſons*, A, B, and C, *company'd*, and *put in* together 3822 *l.* A's *Money* was in 2 *Months*, B's 3 *Months*, and C's 4 *Months*; they *gain'd* 234 *l.* of which A's *Share* came to 52 *l.* B's to 78 *l.* and C's to 104 *l.*

Anſwer. Each *put in* 1274 *l.*

XX. Three *Partners* in *Trade* had a *Common* STOCK of 6400 *l.*; whereof the *Firſt* put in 2880 *l.* for 10 *Months*, the *Second* put in 2240 *l.* and the *Third* put in 1280 *l.* for *different Times*; but having had *ill Succeſs*, they made up their *Accompts*, and found they had *loſt* 1700 *l.*; the *Firſt* being 900 *l. Loſer*, the *Second* 560 *l.* and the *Third* 240 *l.*; how long therefore did the *Money* of the *Second* and *Third* continue in *Company*?

Anſwer. The *Second* Man's *Money* continu'd in 8 *Months*, and the *other's* 6 *Months*.

XXI. Three *Merchants* have *company'd* together; the *Firſt* put into TRADE, on the 1ſt of *January*, 600 *Guineas*; the *Second*, on the 1ſt of *April*, put in a *Diamond*; and the *Third*, on the 1ſt of *May*, put in

900 *l.* and at the YEAR's *End* their *total* GAIN was 360 *l.* whereof the *First* Merchant's *Part* was found to be 129 *l.* and the *Second's* 111 *l.* I demand what a *Guinea* went for, and what was the *Value* of the *Diamond?*

Anſwer. 1 *Guinea* went for 1 *l.* 1 *s.* 6 *d.* and the *Value* of the *Diamond* was 740 *l.*

XXII. Three *Merchants* went *Partners* 18 *Months*; A put into STOCK, at firſt, 200 *l.* at 8 *Months* End he *put in* 100 *l.* more, and 2 *Months* after that, he *put in* 50 *l.* more; B *put in* at firſt 550 *l.* at 4 *Months* End he *took out* 140 *l.* and 3 *Months* afterwards he *took out* 110 *l.* more; C *put in* at firſt 600 *l.* at 2 *Months* End he *took out* 250 *l.* and 12 *Months* after ſo doing, he *put in* 300 *l.*; and at the End of the 18 *Months* they had clearly *gain'd* 526 *l.*: What was each Merchant's *juſt Share* of the ſaid GAIN?

		l.	*s.*	*d.*
Anſwer. A's B's C's	*Share* was	133 : 179 : 213 :	5 : 8 : 5 :	11 $\frac{1757}{1973}$ 5 $\frac{1673}{1973}$ 7 $\frac{40}{1973}$

QUESTIONS in ALLIGATION.

I. A Maltſter mingles 24 *Quarters* of *high dry'd* MALT, at 25 *s.* a *Quarter*, with 30 *Quarters* of *brown* MALT, at 28 *s.* a *Quarter*, and 46 *Quarters* of *pale* MALT, at 30 *s.* a *Quarter*; the MALT being thus mingled, what is the *Worth* of a *Quarter?*

Anſwer. 28,2 *s.* equal to 28 $\frac{1}{5}$ *s.*

II. If 32 *Buſhels* of WHEAT MEAL, at 10 *s.* a *Buſhel*, 24 *Buſhels* of RYE MEAL, at 6 *s.* 8 *d.* a *Buſhel*, and 8 *Buſhels* of BARLEY MEAL, at 5 *s.* a *Buſhel*, be mixed all together, what will a *Buſhel* of that *Mixture* be *worth?*

Anſwer. 8 *s.* 1 *d.* $\frac{1}{2}$.

III. A

III. A Druggiſt wou'd *mix* three Sorts of JESUIT's BARK in ſuch a Proportion, that a *Pound* of the *Mixture* may be worth 24 *s.*; now how many *Pounds* of each Sort muſt he uſe in compoſing ſuch a *Mixture*, if the firſt Sort be worth 40 *s.* a *Pound*, the ſecond 30 *s.* a *Pound*, and the worſt Sort 16 *s.* a *Pound*?

lb.
Anſwer. { 8 / 8 / 22 } of the { 1ſt. / 2d. / 3d. } Sort.

IV. A *Goldſmith* having GOLD BULLION of 4 different Fineneſſes, *viz.* The 1ſt. being 23 *Carats fine*, the 2d. 22, the 3d. 18, and the 4th 17 *Carats fine*; how much of each *Sort* muſt he *melt* together, that the *Mixture* may be 20 *Carats fine*?

Car. *Car.*
Anſwer. { 3 / 2 / 2 / 3 } or { 2 / 3 / 3 / 2 } of the { 1ſt. / 2d. / 3d. / 4th. } Sort.

V. A *Goldſmith* having 20 *lb. Weight* of SILVER BULLION, 6 *Penny-weights* fine, wou'd *melt* it down together with another ſort 10 *Penny-weights* fine, and another ſort 12 *Penny-weights* fine, of each ſuch a Quantity, that the *whole Mixture* may bear 9 *Penny-weights* fine; what Quantity of each of the two *laſt Sorts* muſt he mix with the 20 *lb.* of the *firſt Sort* to anſwer his Purpoſe?

Anſwer. 15 *Pounds* of each.

VI. I have a *Hogſhead* of CANARY worth 2 *s.* a *Quart*, all which I would *mix* with 3 other *ſorts* of *Wine*, *viz.* One *ſort* at 22 *d.* a *Quart*, a 2d. at 16 *d.* and a 3d. at 12 *d.* a *Quart*; how much of each muſt be *mixed* with the *Hogſhead* of CANARY, that a *Quart* may then be *ſold* for 18 *d.*?

Anſwer.

Gall.

Anſwer. 21 / 42 / 63 of the 1ſt. / 2d. / 3d. Sort.

VII. A *Goldſmith* wou'd make a MASS containing 80 *Ounces*, and bearing 18 *Carats fine*, of the following ſorts of GOLD, *viz.* That of 23, 22, and 21 *Carats fine*, and of *Alloy*, of each a ſufficient Quantity; how much of *each ſort* muſt he make Uſe of?

Anſwer. 21 *Oz.* $\frac{9}{11}$ of each ſort of GOLD, and 14 *Oz.* $\frac{6}{11}$ of ALLOY.

VIII. A *Grocer* ſold an *Hundred Weight* of *Sugar* at 6 *d.* a *Pound*, which he compos'd of *Three-penny*, *Five-penny*, *Eight-penny*, and *Ten-penny* SUGAR; how much of *each* did he uſe?

lb.		*lb.*		*d.*	
22,4		44,8		3	
44,8	or	22,4	of the *Sugar* at	5	a *Pound.*
33,6		11,2		8	
11,2		33,6		10	

Anſwer.

IX. A *Tobacconiſt*, out of 5 ſeveral ſorts of SNUFF, *viz.* One ſort at 8 *s.* another at 5 *s.* another at 4 *s.* another at 2 *s.* and another at 1 *s.* 6 *d.* a *Pound*, wou'd make a *Compoſition* of 50 *Pounds weight*, which he cou'd afford to ſell at 2 *s.* 6 *d.* a *Pound*; what Quantity of *each* muſt he take?

lb.		*s.*	*d.*	
4 $\frac{8}{23}$		8	0	
2 $\frac{4}{23}$		5	0	
2 $\frac{3}{23}$	of that at	4	0	a *Pound.*
17 $\frac{7}{23}$		2	0	
23 $\frac{1}{23}$		1	6	
50				

Anſwer.

Or

	lb.		*lb.*	
Or these,	$2\frac{4}{23}$	*Or these,*	$3\frac{7}{31}$	*&c.*
	$4\frac{8}{23}$		$4\frac{26}{31}$	
	$2\frac{4}{23}$		$3\frac{7}{31}$	
	$30\frac{10}{23}$		$8\frac{2}{31}$	
	$10\frac{20}{23}$		$30\frac{20}{31}$	
	50		50	

QUESTIONS in the Rule *of* False.

I. Three Persons A, B, and C, upon Enquiry, discocover A's Age to be *double* B's Age, B's to be *triple* C's Age, and the Sum of their Ages to be 140 *Years*: What was *each* Person's Age?

Answer. $\left\{\begin{matrix}\text{A's}\\ \text{B's}\\ \text{C's}\end{matrix}\right\}$ was $\left\{\begin{matrix}84\\ 42\\ 14\end{matrix}\right\}$ *Years.*

II. Three Persons, A, B, and C, discoursing of their Ages, find that A is 6 times *older* than C, that A's Age *exceeds* the Ages of B and C together by 36 *Years*, and that A's and B's Ages together *exceeds* the Years of C by 144; what then was *each* Person's Age?

Answer. $\left\{\begin{matrix}\text{A}\\ \text{B}\\ \text{C}\end{matrix}\right\}$ was $\left\{\begin{matrix}108\\ 54\\ 18\end{matrix}\right\}$ *Years Old.*

III. Suppose 1380 *l.* be divided between 3 *Persons*, A, B, and C, so that A has $\frac{2}{3}$ of B's *Part*, and $\frac{3}{4}$ of C's *Part*; what is *each* Man's *Part*?

Answer. $\left\{\begin{matrix}\text{A's}\\ \text{B's}\\ \text{C's}\end{matrix}\right\}$ *Part* is $\left\{\begin{matrix}360\ l.\\ 540\ l.\\ 480\ l.\end{matrix}\right.$

IV. If

IV. If a certain Sum of MONEY be *divided* amongſt 4 *Men*, in ſuch ſort, that A *receives* 100 *l.* more than B, and B 100 *l.* more than C, and C 100 *l.* more than D, and D half as much as A; what was the *Sum* that was *divided* among them, and what did each Man receive?

Anſwer. The *whole Sum* was 1800 *l.* of which A *receiv'd* 600 *l.* B 500 *l.* C 400 *l.* D 300 *l.*

V. Suppoſe a SHIP from any *Parallel of Latitude ſails* directly *North*, at the Rate of 8 *Miles* an *Hour*, and 8 *Hours* after *ſhe* had *ſet ſail*, another follows her from the ſame *Parallel*, ſailing under the ſame *Meridian*, at the Rate of 10 *Miles* an *Hour*: In how many *Hours* will the *latter* come up with the *former*?

Anſwer. 32 *Hours.*

VI. If the NUMBER 39 be to be *divided* into two ſuch *Parts*, that the *Quotient*, obtain'd by dividing the *greater Part* by 5, added to the *Product*, obtain'd by *multiplying* the *Leſſer Part* by 8, may make the *propos'd Number*, *viz.* 39; what will thoſe *two Parts* be?

Anſwer. 35 and 4.

VII. If the NUMBER 18 be to be *divided* into two ſuch *Parts*, that the *Difference* of their *Squares* may be 10 times the ſaid *Number* 18, what are the *Parts* requir'd?

Anſwer. 14 and 4.

VIII. A *Gameſter*, beginning to Play with 500 GUINEAS in his *Pocket*, was ask'd (at his leaving off *Play*) how many *Guineas* he had *loſt*; to which he anſwer'd thus, if $\frac{3}{4}$ of the *Guineas* he had *loſt*, were added to $\frac{3}{8}$ of thoſe he had *left*, the *Sum* wou'd be the *Number* of *Guineas loſt*: What was that *Number* then?

Anſwer. 300 *Guineas.*

IX. Of

IX. Of 3 *Scribes*; if the *Firſt* can write 5 *Sheets* in an *Hour*, the *Second* 4 *Sheets*, and the *Third* 3 *Sheets*; in how many *Hours* will they, all together, write 150 *Sheets*?

Anſwer. 12 *Hours* and $\frac{1}{2}$.

X. A *Governour* of a FORT ſends out $\frac{1}{3}$ of his SOLDIERS, and 25 over, and has then *left* $\frac{1}{2}$ his *Soldiers* and 100 over: How many *Soldiers* had he at *firſt*?

Anſwer. 750.

XI. Into what two *Parts* muſt the *Number* 50 be *divided*, that, if 45 be *added* to the *greater Part*, and 25 to the *leſſer*, the *former* Sum will be double the *latter*?

Anſwer. 35 and 15.

XII. Two *Traders*, A and B, having a Mind to *buy* a SHIP, valued at 1800 *l.* found their CASH to ſtand thus, *viz.* if B *lent* A $\frac{2}{3}$ of his *Money*, it wou'd enable A to buy the SHIP alone; but if A *lent* B $\frac{3}{4}$ of his *Money*, then B wou'd be able to buy the SHIP: How much *Money* then had *each* Man?

Anſwer. A 1200 *l.* and B 900 *l.*

QUESTIONS in PRACTICE.

I. What will 25000 BRICKS (*i. e.* 50 *Load*) amount to, at 15 *s.* 6 *d.* a *Thouſand*?

Anſwer. 19 *l.* 7 *s.* 6 *d.*

II. What will 12 *Load* of TYLES (*viz.* 12000) amount to, at 19 *s.* 6 *d.* a *Thouſand*?

Anſwer. 11 *l.* 14 *s.*

III. What will 6 *Hundred of* LIME (a *Hundred* of LIME being 25 *Bags* or *Buſhels*) amount to, at 5 *d.* a Bag?

Anſwer. 3 *l.* 2 *s.* 6. *d.*

IV. What will 66 $\frac{1}{2}$ *Bundles* of LATHS amount to, at 1 *s.* 4 *d.* $\frac{1}{2}$ a *Bundle?*

Anſwer. 4 *l.* 11 *s.* 5 *d.* $\frac{1}{4}$

V. What will 3840 DEAL BOARDS amount to, at 7 *l.* 7 *s.* a *Hundred*, reckoning 6 *Score* to the *Hundred?*

Anſwer. 235 *l.* 4 *s.*

VI. What is 50 *ſolid Feet* (*i. e.* a *Load*) of TIMBER worth, at 10 *d.* $\frac{3}{4}$ a *Foot?*

Anſwer. 2 *l.* 4 *s.* 9 *d.* $\frac{1}{2}$.

VII. What will 12 *Rods* and $\frac{1}{4}$ of BRICK-WORK amount to, at 5 *l.* 3 *s.* 6 *d.* a *Rod?*

Anſwer. 63 *l.* 7 *s.* 10 *d.* $\frac{1}{2}$

VIII. What will the Workmanſhip of 18 $\frac{1}{2}$ *Rods* of BRICK-WORK amount to, at 1 *l.* 6 *d.* a *Rod?*

Anſwer. 18 *l.* 19 *s.* 3 *d.*

IX. If TYLING be worth 21 *s.* 11 *d.* a *Square* (or 100 *Foot*) what is 14 $\frac{3}{4}$ *Square* worth?

Anſwer. 16 *l.* 3 *s.* 3 *d.* $\frac{1}{4}$.

X. If WHITE-WASHING be worth 1 *d.* $\frac{1}{4}$ a *Yard*, what will 3336 *Yards* come to, at that Rate?

Anſwer. 17 *l.* 7 *s.* 6 *d.*

XI. If PAINTING be worth 5 *d.* $\frac{1}{2}$ a *Yard*, what will 2118 *Yards* coſt?

Anſwer. 48 *l.* 10 *s.* 9 *d.*

XII. If GLASS be worth 4 *d.* $\frac{3}{4}$ a *Foot*, what will 848 *Feet* of GLAZING come to?

Anſwer. 16 *l.* 15 *s.* 8 *d.*

XIII. What

XIII. What will 46 *Yards* of WAINSCOT come to, at 3 *s.* 9 *d.* a *Yard?*
Anſwer. 8 *l.* 12 *s.* 6 *d.*

XIV. What will 64 *Squares* of FLOORING coſt, at 1 *l.* 9 *s.* 8 *d.* a *Square?*
Anſwer. 94 *l.* 18 *s.* 8 *d.*

XV. What will 9 *Fodder* of LEAD (each 19 *C.* $\frac{1}{2}$) amount to, at 18 *s.* 8 *d.* a *Hundred Weight.*
Anſwer. 163 *l.* 16 *s.*

XVI. At 8 *d.* $\frac{3}{4}$ a *Hundred*, what will 450 *Hundred* of NAILS coſt?
Anſwer. 16 *l.* 8 *s.* 1 *d.* $\frac{1}{2}$.

XVII. At 46 *l.* 13 *s.* 4 *d.* a *Tun*, how much will 72 *Tuns* 5 *C.* 2 *qrs.* 18 *lb.* of IRON coſt?
Anſwer. 3373 *l.* 4 *s.* 2 *d.*

XVIII. At 17 *s. each*, how much STERLING will 550 *Piſtoles* come to?
Anſwer. 467 *l.* 10 *s.*

XIX. At 1 *s.* 8 *d.* a GUILDER, how much will 1842 GUILDERS amount to?
Anſwer. 153 *l.* 10 *s.*

XX. At 4 *s.* 5 *d.* a DUCAT, how much do 888 DUCATS amount to?
Anſwer. 196 *l.* 2 *s.*

XXI. At 4 *s.* 11 *d.* a FLORIN, how much do 757 FLORINS amount to?
Anſwer. 186 *l.* 1 *s.* 11 *d.*

XXII. At 8 *s. each*, what do 454 RIX-DOLLARS amount to?

Anſwer. 181 *l.* 12 *s.*

XXIII. At 13 *s.* 4 *d. each*, what do 323 RUBLES amount to?

Anſwer. 215 *l.* 6 *s.* 8 *d.*

XXIV. What will the *Net-weight* of 544 *C.* 2 *qrs.* 16 *l. Groſs, (Tare* 14 *lb. per C.* to be *deducted)* amount to, at 3 *l.* 13 *s.* 3 *d. per C. Net?*

Anſwer. 1745 *l.* 8 *s.* 2 *d.* $\frac{1}{4}$ + $\frac{3}{4}$ of a *Farthing.*

XXV. What will the *Net-weight* of 810 *C.* 3 *qrs.* 14 *l.* GROSS, (16 *lb. per C.* being deducted for TARE) amount to, at 8 *s.* 8 *d.* $\frac{3}{4}$ *per lb.* NET?

Anſwer. 33975 *l.* 13 *s.* 3 *d.*

XXVI. What will the *Net-weight* of 999 *C.* 1 *qr.* 12 *lb.* GROSS, (17 *lb. per C.* being deducted for TARE) amount unto, at 19 *s.* 6 *d. per lb.* NET?

Anſwer. 92565 *l.* 10 *s.* 6 *d.* +.

XXVII. What will the *Net-weight* of 761 *C.* 2 *qrs.* 14 *lb.* GROSS, (18 *lb. per C.* being deducted for TARE) amount unto, at 2 *l.* 16 *s.* 8 *d. per C.* NEAT?

Anſwer. 1811 *l.* 2 *s.* 6 *d.* very nearly.

XXVIII. What will the *Net-weight* of 246 *C.* 2 *qrs.* 20 *lb.* GROSS, (20 *lb. per C.* being deducted for TARE) amount unto, at 9 *s.* 9 *d.* $\frac{1}{4}$ *per lb.* NET?

Anſwer. 11087 *l.* 3 *s.* 5 *d.* $\frac{3}{4}$.

XXIX. What will 19 *lb.* 8 *oz.* 12 *pwts.* 8 *gr.* of GOLD amount unto, at 4 *l.* 4 *s.* 6 *d.* an *Ounce?*

Anſwer. 999 *l.* 14 *s.* 1 *d.* $\frac{1}{4}$ + $\frac{1}{7}$ of a *Farthing.*

XXX. What

XXX. What will 28 *lb.* 3 *oz.* 9 *pwts* of SILVER amount unto, at 3 *l.* 8 *s.* 6 *d.* a *Pound* TROY?
Anſwer. 96 *l.* 17 *s.* 8 *d.* $\frac{1}{4} + \frac{3}{10}$ of a *Farthing.*

QUESTIONS for the Exerciſe of FRACTIONS.

I. What's the SUM of 7 *Yards* $\frac{3}{4}$ and 5 *Yards* $\frac{5}{8}$?
Anſwer. 13 $\frac{3}{8}$ *Yards.*

II. What's the SUM of $\frac{3}{7}$ of an *Ounce*, $\frac{3}{5}$ of an *Ounce*, $\frac{4}{9}$ of an *Ounce*, and $\frac{2}{3}$ of $\frac{3}{4}$ of an *Ounce*?
Anſwer. $1\frac{61}{630}$ *Ounce.*

III. What's the DIFFERENCE between $\frac{1}{15}$ of an *Hour*, and $\frac{19}{60}$ of an *Hour*?
Anſwer. $\frac{1}{4}$ of an *Hour.*

IV. What's the DIFFERENCE between $\frac{5}{6}$ of a *Flemiſh Ell* and $\frac{3}{4}$ of an *Engliſh Ell*?
Anſwer. $\frac{5}{16}$ of a *Yard.*

V. How many *ſquare Feet* of GLAZING are there in that which is 4 $\frac{2}{11}$ *Feet* in *Length*, and 2 $\frac{3}{4}$ *Feet* in *Breadth*?
Anſwer. 11 $\frac{1}{2}$ *ſquare Feet.*

VI. What do 9 *oz.* 5 *pwts.* (*i. e.* 9 *oz.* $\frac{1}{4}$) of SILVER come to, at 5 *s.* 4 *d.* (*i. e.* 5 *s.* $\frac{1}{3}$) of an *Ounce*?
Anſwer. 49 *s.* $\frac{1}{3}$. i. e. 2 *l.* 9 *s.* 4 *d.*

VII. If the *Side* of a ROOM, 12 *Feet* 4 *Inches*, (or 12 *Feet* $\frac{1}{3}$) high contains 58 $\frac{1}{9}$ *ſquare Yards*, what is the *Length* thereof?
Anſwer. 42 *Feet* $\frac{15}{37}$.

VIII. What

VIII. What DECIMAL of a *Pound Sterling* is 17 *s.* 6 *d.* ?

Anſwer. 0,875 of a *Pound Sterling.*

IX. What's the VALUE of 0,875 of a *Pound Sterling* ?

Anſwer. 17 *s.* 6 *d.*

X. What's the VALUE of 0,4 of a *Year*, ſuppoſing the *Year* to be 365 *Days*, 6 *Hours*, equal to 365 *Days* $\frac{1}{4}$?

Anſwer. 146 *Days*, 2 *Hours*, 24 *Min.*

XI. If the SUN *paſſes* thro' the ZODIAC (*viz.* 360 *Degrees*) in 365 *Days*, 5 *Hours*, 49 *Minutes*, 16 *Seconds*, and 46 *Thirds*; how many *Degrees* does he *paſs thro'* in a Month, *i. e.* in 28 *Days* ?

Anſwer. 27 *Deg.* 35′. 53″. 9‴. *&c.*

I can't help *preſſing* the Learner to obſerve, That every *Number*, as well *Whole* as *Broken*, is compos'd of *two* Parts, *viz.* a DENOMINATOR, as well as NUMERATOR.

Vide NOTE I. CHAP. III.

And this *fundamental* Obſervation, attentively weigh'd, will very much *conduce* to the facilitating the Doctrine of VULGAR FRACTIONS, which is accounted by *Youngſters* ſo difficult, for no *other* Reaſon but becauſe they do not rightly underſtand, that *Integers* as well as *Fractions* have both *Numerators* and *Denominators*; and therefore believe there's a much greater *Difference* between *Whole Numbers* and *Fractions*, than really there is.

I need not (I think) here inſert any *Queſtions* in the RULE OF THREE, ſince many of the *foregoing* ones, in the *Rules of* PROPORTION, may, very properly, be *ſolv'd* by

by the Help of VULGAR FRACTIONS, or of DECIMALS, (wherein the Learner may *exerciſe* himſelf at Pleaſure), as well as by *whole Numbers* (reduc'd, when requiſite) as hath been taught in the 10th CHAPTER: For,

The *Directions* there *laid down* for the *right Ordering* or *Stating* the *Three* given Terms in any Proportion, and the *Rules* there given for finding a *Fourth*, or the Term *ſought*, hold good, *work* which Way you pleaſe: For which Reaſon it was thought needleſs to *ſay* any thing, or to *give* any *Examples*, in the RULE OF THREE, in the 17th CHAPTER, containing the *Doctrine* of FRACTIONS.

But, for the Learner's *Service* and *Satisfaction*, let us *work* the firſt two or three *Queſtions* of the RULE OF THREE theſe *three* ſeveral Ways; which will be ſufficient for him to ſee the *Manner* of applying *Vulgar* and *Decimal* FRACTIONS to *Operations* in the RULE OF THREE.

See the firſt two or three *Queſtions* in the *Single Rule of Three*, at *Page* 374.

QUEST. I.

1*mo.* By *Whole* NUMBERS, thus,

s. *s.* *l.* *l.* *s.*
16 : 2 :: 100 : *Q* equal to 12 : 10
20

2000 *Shillings.*
2

s. *l.* *s.*
16)4000(250 equal to 12 : 10
80
0

2*do*

2*do.* By *Vulgar* FRACTIONS, thus,

$$\frac{4}{5}\,l. : \frac{1}{10}\,l. :: \frac{100}{1}\,l. : ?\ \text{equal to}\ \frac{100}{40}\,l.$$

$$\frac{4}{5})\frac{100}{10}(\frac{100}{40}\,l. \text{ equal to } \frac{25}{2}\,l. \text{ equal to } 12\tfrac{1}{2}\,l. \text{ or } 12\,l. : 10\,s.$$

3*tio.* By DECIMALS, thus,

0,8 *l.* : 0,1 *l.* :: 100 *l.* : ? equal to 12,5 *l.* equal to 12 *l.* : 10 *s.*

0,1
0,8)10,0(12,5 *l.*
20
40
0

QUEST.

QUEST. II.

1*mo.* By *Whole* NUMBERS, thus,

s. *d.* *oz.* *l.* *s.* *d.* *oz.* *pwts.*
5 : 6 : 1 :: 67 : 6 : 1 $\frac{1}{2}$: Q equal to 244 : 15

12 20
66 1346
4 12
264 16153
4
oz.
264)64614(244
1181
1254
198
20
264)3960(15 *pwts.*
1320
0

2*do.* By *Vulgar* FRACTIONS, thus,

l. *oz.* *l.* *l.* *oz.*
$\frac{11}{40}$: $\frac{1}{1}$ :: (67 $\frac{49}{160}$ =) $\frac{10762}{160}$: Q = 244 $\frac{3}{4}$

oz. oz. oz. *pwts.*
$\frac{11}{40}$) $\frac{10762}{160}$ ($\frac{430760}{1760}$ = 244 $\frac{3}{4}$ = 244 : 15

176|0)43076|0(244
1787
836
1323
176|4

3*tio*. By DECIMALS, thus,

l. *oz*. *l*. *oz*.
0,275 : 1 :: 67,30625 : Q = 244,75.

oz. *oz*. *pwts*.
0,275)67,30625(244,75 = 244 : 15
12 30 ...
1 306
2062
1375
0

QUEST. III.

1*mo*. By *Whole* NUMBERS, thus,

lb. *oz*. *pwts*. *l*. *s*. *gr*. *far*.
9 : 9 : 12 : 411 : 12 :: 1 : Q equal to 7
12 20
117 8232
20 12
2352 98784
24 4
9408 56448)395136(7 *Far*.
4704 0
56448 *Grains*.

2*do*.

2do. By *Vulgar* FRACTIONS, thus,

lb. *l.* *lb.*
$9\frac{4}{7}$: $411\frac{3}{7}$ :: $\frac{1}{5760}$: ☌

That is,

lb. *l.* *lb.*
$\frac{49}{5}$: $\frac{2058}{5}$ :: $\frac{1}{5760}$: ☌ equal to $\frac{7}{960}$ of *l.*

$\frac{49}{5}\overline{)\frac{2058}{5}}\left(\frac{10290}{1411200}\right.$ equal to $\frac{7}{960}$ of *l.* equal to 7 *Farth.*

3tio. By DECIMALS, thus,

lb. *l.* *lb.* *l.* *d. f.*
9,8 : 411,6 :: 0,0001736111 + : ☌ = 0,0072916 = 1 : 2,99999 +
411,6 = 7 *Far.* very nearly.

10041666
173611
173611
694444

l.
9,8),071458287 6(0,007291662
285 20
898 ,145833240
162 12
648 1,749998880 *Pence.*
607 4
196 2,999995520 *Farthings.*
0

And after the *like* manner the LEARNER may further *exercise* himself with such other of the *foregoing* QUESTIONS as he shall think proper.

QUESTIONS requiring the Application and Uſe of the Extraction of the SQUARE ROOT.

☞ *See amongſt the* QUESTIONS *in the* Rule of Three *for* Analogies, *and Directions proper for the Solution of ſome of the following propos'd* Queſtions.

I. If an ARMY, conſiſting of 32400 *Men*, be to be *form'd into a Square, viz.* ſo that the *Men* in *Rank* be equal to thoſe in *File*; how many *Men* muſt each RANK conſiſt of?

Anſwer. 180.

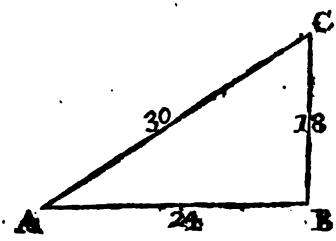
II. Suppoſe 12,544 *Soldiers* be to be *diſpos'd* into the Form of an *Oblong*, or *Long Square*, ſo that the *Number* of Men in *Rank* may be 4 times the *Number* of them in *File*; how many *Men* then muſt he *plac'd in Rank*, and how many in *File*?

Anſwer. 224 in *Rank*, and 56 in *File, viz.* The *Square* ROOT of $\frac{1}{4}$ of the *Number* of *Men* is equal to the *Breadth* of the *Oblong*. And, in like Caſes, univerſally, the ROOT of the *given Number* divided by the *Number of Times* the *Length* is to contain the *Breadth*, is equal to the *Breadth*.

III. Suppoſe the *Height* of the *Wall* of a GARRISON be 18 *Feet*, and the *Breadth* of the *Moat* ſurrounding it be 24 *Feet*; how long muſt a *Scaling-Ladder* be to *reach* from the *Out-ſide* of the *Moat* to the *Top* of the *Wall*?

Anſwer. 30 *Feet, viz.* The *Root* of the Sum of the Squares of 18 and 24. For, univerſally, in the *Triangle* A B C, the *Square* of A C (here repreſenting the *Length* of the LADDER) is equal to the *Square* of A B (the *Breadth* of the *Moat*) and

and the *Square* of B C (the *Height* of the *Wall*) taken together. (*Euclid* 47. 1.)



¶ The *Area* of any *plain Triangle* may be obtain'd thus;

From half the Sum of the *three Sides*, let each of the three Sides be *ſeverally* ſubtracted; then let the *three Differences* and the ſaid *half Sum* be multiply'd into one another continually, and the *Square Root* of the *laſt* Product ſhall be the *Content* of the *Triangle*.

Note, This RULE may oftentimes be of good Uſe in *Meaſuring* a Piece of *Land* very exactly.

IV. If one *Side* of a *Triangular* FIELD be 20 *Rods*, another 30, and the third 40; what's the *Content* of that FIELD?

Anſwer. 290,47 *ſquare Rods.*

¶ If the *Second*, of three *Numbers*, bears the ſame *Proportion* to the *Third*, as the *Firſt* does to the *Second*, then that *Second* is call'd a MIDDLE or MEAN PROPORTIONAL between the *Firſt* and *Third*; thus, 4 is a *Mean Proportional* between 2 and 8: For, 2 : 4 :: 4 : 8.

Whence,

Whence, and from what has been ſaid in CHAP. X. 'tis plain, that the *ſquare* ROOT of the *Product* of any Two *given Numbers* is the *Mean Proportional* between them.

V. What *Number* is the *Mean Proportional* between 48 and 768?

Anſwer. 192.

¶ The *Mean Proportional* between the *Longeſt* and *Shorteſt* DIAMETERS of an *Ellipſis*, will always be equal to the DIAMETER of a *Circle*, whoſe *Area* is equal to the *Area* of the *Ellipſis*. Then,

VI. What's the AREA of an *Ellipſis*, whoſe *Longeſt Diameter* is 25 *Feet*, and *Shorteſt* 16 *Feet*?

Anſwer. 314,159 *ſquare Feet.*

VII. What is the DIAMETER of that *Circle*, whoſe *Area* is 10028,76 *ſquare Feet*?

Anſwer. 113 *Feet.*

VIII. What's the PERIPHERY of that *Circle*, whoſe *Area* is 10028,7 *ſquare Feet*?

Anſwer. 355 *Feet.*

IX. What's the DIAMETER of the *Sphere*, whoſe *Superficies* contains 452,38896 *ſquare Feet*?

Anſwer. 12 *Feet.*

X. The *Content* of a *Circle*, whoſe *Diameter* is 12 *Inches*, is 113,1 *ſquare Inches*; what then is the *Diameter* of a *Circle* whoſe *Area* contains 452,4 *ſquare Inches*?

Anſwer. 24 *Inches.* Where, by the by, you may obſerve, that if the *Diameter* of *one Circle* be the *Double* of *that* of *another*, the *Area* of the *former* will be (*not* alſo juſt the *Double*, but) the *Quadruple*

†

druple of the *Area* of the *latter*: And the Reaſon is, becauſe *Circles* are not as their *Diameters*, but as the *Squares* of their *Diameters*, as before mention'd.

XI. Suppoſe the *Side* of a *Regular* or *Irregular Figure* (of what Number of *Sides* ſoever, for Inſtance, of 5 *Sides*) be 4 *Feet*, and the *Area* of this *Figure* be 68 *ſquare Feet*; what's the *Homologous Side* of another *ſimilar* 5 *ſided Figure*, whoſe *Area* is 208,25 *ſquare Feet*?

Anſwer. 7 *Feet.*

XII. A *Heavy Body*, near the *Earth's Surface*, is found to fall 16 *Feet* in a SECOND of *Time*; now, in how long Time wou'd a *Stone* fall to the *Bottom of a Well* as deep as the *Earth's Center*, ſuppoſing the *Diſtance*, from the *Surface of the Earth* to its *Center*, to be 4000 *Engliſh Miles*, or 21120000 *Engliſh Feet*?

Anſwer. In 19 *Minutes* and 9 *Seconds*, very nearly.

XIII. If a PENDULUM muſt be 39,2 *Inches* long, to *ſwing Seconds*, *i. e.* to make 60 *Vibrations* in a *Minute*; how many *Vibrations* will a PENDULUM, that's 9,8 *Inches* long, make in a *Minute*?

Anſwer. 120 *Vibrations.*

¶ The SQUARES of the *reſpective Periodic Times* of the ſeveral *Primary Planets*, *i. e.* the Times, wherein they reſpectively revolve *once* about the *Sun*, are ever as the *Cubes* of their *Diſtances* from him.

XIV. Suppoſe then that (the PLANET) the EARTH, which revolves about the *Sun* in 365 ¼ *Days*, be *diſtant* from the *Sun's Center* 20000 of the EARTH's *Semidiameters*, or 80,000,000 of *Miles*; what is the *Periodic Time* of (the PLANET) SATURN, being 770,000,000 *Miles* diſtant from the ſame *Center*? And what wou'd *that* of another

another PLANET be, which we'll ſuppoſe but 120 of the EARTH's *Semidiameters*, or 480,000 *Miles*, diſtant from the SUN's *Center*, *viz.* juſt as far again as the MOON is from the EARTH's *Center*?

Anſwer. SATURN's *Periodic Time* is nearly 30 *Years*; and the other Planet's *Periodic Time* wou'd be 4 *Hours*, 4 *Minutes*, 26 *Seconds*, 34 *Thirds*, *&c.* tho' the MOON is 27 $\frac{1}{3}$ Days nearly in revolving about the *Earth*.

SCHOL. This *Proportion* is ſo Univerſal, that *Aſtronomers* find it to hold good alſo amongſt the *Satellites* or *Secondary Planets*, revolving about their reſpective *Primaries*, without the leaſt Exception. And, if our *Earth* had more than one *Moon* revolving about it, it is not to be doubted but that there wou'd be the ſame *Law* and *Harmony* found amongſt *them*.

And thus, if the *Sun* revolv'd about the *Earth*, as well as the *Moon*, (as ſome think it does) and not the *Earth* about the *Sun*, his *Period*, *i. e.* the *Length* of the *Solar Year* wou'd by this *Univerſal Law* be found to be 167,400 *Days*, which is 458 times longer than it is now really known to be; which manifeſt Contradiction (beſides other conſequent Abſurdities) ſufficiently determines, that the Annual Revolution belongs to the *Earth*, and not to the *Sun*.

QUESTIONS

QUESTIONS for Exerciſing the Extraction *of the* Cube Root.

To aſſiſt him in the Solution of ſome of the following Propoſitions, the *Learner* may find proper *Analogies* amongſt the *Queſtions* of the Rule of Three.

I. A *Piece of* Timber exactly *Cubical*, containeth 1728 *ſolid Feet*; how many *Inches* doth *one* Side thereof contain?

Anſwer. 144 *Inches.*

II. If a Rock or Stone of a *Cubic Form* contains 2985984 *ſolid Inches*, how many Feet does *one* Side contain?

Anſwer. 12 *Feet.*

III. If the *ſolid Content* of a *Globe*, *Cylinder*, *Priſm*, *Cone*, or any *other* Body be 15625 *Inches*, what is the Side of a *Cube* of equal *Solidity*?

Anſwer. 25 *Inches.*

IV. If the *Solidity* of a Sphere be 179,6 *Cubic Feet*, what is its *Diameter*?

Anſwer. 7 *Feet.*

V. If the *Solidity* of a Sphere be 179,7 *Cubic Feet*, what's its *Periphery*?

Anſwer. 22 *Feet.*

VI. Suppoſe there be a Cube whoſe *Side* is 7 *Inches*, and I wou'd find the Side of another *Cube* that is double to it (which is commonly call'd the *Duplication* of the *Cube*); what will its Side be?

Anſwer. 8,81 *Inches.*

Here obſerve that the *Side* of this *ſecond Cube* is not (14) *double* the *Side* of the *former*; for, if ſo, then like *Solids* wou'd be in the ſame *Proportion* as their *Sides* are; but they are, as before ſaid, as the *Cubes* of their *Sides*; and *Spheres* as the *Cubes* of their *Diameters*.

VII. The *Solidity* of a SPHERE, whoſe *Diameter* is 6 *Inches*, is 113,097 *Cubic Inches*; what then is the *Diameter* of another *Sphere*, containing 904,776 *Cubic Inches*, *viz.* 8 times as big as the *former*?

Anſwer. 12 *Inches.*

Hence you may ſee, that, as by *doubling* the *Diameter* of a CIRCLE the AREA becomes 4 times as large as before; ſo by *doubling* the *Diameter* of a SPHERE, the *Solidity* becomes 8 times as *large* as before.

VIII. If one Side of a *Parallelopiped*, containing 35,15625 *Cubic Feet*, be 5 *Feet* in *Length*, what will be the *Length* of the *Homologous Side* of another *Parallelopiped*, ſimilar to the *former*, and containing 18 *Cubic Feet*?

Anſwer. 4 *Feet.*

IX. If the *Diameter* of the *Baſe* of a CYLINDER be 4 *Feet*, and the *Content* of that CYLINDER be 48 *Cubic Feet*, what will the *Diameter* of another *ſimilar Cylinder* be, whoſe *Content* is 162 *Cubic Feet*?

Anſwer. 6 *Feet.*

X. If the *Diameter* of the *Baſe* of a CONE be 5 *Feet*, and the *Content* of that CONE be 56 *Cubic Feet*, what will the *Diameter* of another *ſimilar* CONE be, whoſe *Content* is 229,376 *Cubic Feet*?

Anſwer. 8 *Feet.*

XI. If

XI. If the *Axis* of a CONE be 9 *Feet*, and its *Solidity* be 45 *Cubic Feet*, what is the *Axis* of another *ſimilar* CONE, whoſe *Solidity* is 106,667 *Feet*?

Anſwer. 12 *Feet.*

XII. If the *ſlant Side* of a CONE be 12 *Feet*, and its *Solidity* be 42 *Feet*, what is the like *ſlant Height* of another *ſimilar* CONE, the *Solidity* of which is 141,75 *Feet*?

Anſwer. 18 *Feet.*

XIII. If a SHIP of 75 *Tuns* be 40 *Feet* long at the *Keel*, how many *Feet* in *Length* will the *Keel* of a SHIP of 600 *Tuns* be, ſuppoſing the *Burdens* of SHIPS to be as the *Cubes* of their *Keels*?

Anſwer. 80 *Feet.*

XIV. If an *Iron Bullet*, which is 3 *Inches* in *Diameter*, weighs 9 *Pounds*, what is the *Diameter* of one that weighs 243 *Pounds*?

Anſwer. 12 *Inches.*

XV. If 2 *Pound* 2 *Ounces* (*i. e.* 34 *Ounces*) of GUN-POWDER be ſufficient to charge a *Gun*, whoſe *Concave Diameter* is 3 *Inches*, what is the *Concave Diameter* of that *Gun*, which 81 *Ounces* of the ſame GUN-POWDER will charge; the Capacities of *Guns* being to one another as the *Cubes* of their *Diameters*?

Anſwer. 4 *Inches.*

Vide the *Analogy* at *Queſt.* XIV. in the *Square Root*, for the *Solution* of the following:

XVI. The *Planet* VENUS revolves about the SUN in about 224 *Days*, and the *Planet* MARS in about 686 *Days*, the *Diſtance* of the *former* being 59,000,000 *Miles* from the SUN's *Center*, what's the *Diſtance* of the *latter* from the ſame?

Anſwer. 124,000,000 *Miles.*

The USE *of* TABLE XVI.

A *Cubic Inch* of *Common Water* hath by Experiments been found to *weigh* 0,527 of an *Ounce Troy*, very nearly.

From thence therefore, with the Help of the Numbers annex'd to the ſeveral kinds of *Bodies* mention'd in the ſaid TABLE, the *Weight* of any *propos'd* Quantity of any one of thoſe Bodies may be determin'd eaſily: For,

As 1 is to { 19,642 / 14,001 / &c. } So is a Bulk of common Water to the ſame Bulk of { *Pure Gold* / *Mercury* / &c. } *i. e.*

as to Gravity; as is meant by that Expreſſion in the TABLE. Thus,

Suppoſe the *Weight* of a *ſolid Foot* of *Pure Gold* was requir'd:

'Tis (by the TABLE) 1 : 19,642 :: 0,527 *oz.* (the *Weight* of a *Cubic Inch* of WATER) : 10,351334 *oz.* (the *Weight* of a *Cubic Inch* of GOLD).

Then 'tis plain that this *Weight* of 1 *Inch* of the propos'd Matter, *multiply'd* by the Number of *Inches* contain'd in the Quantity propos'd, will produce the *Weight* of that Quantity in *Ounces*.

Thus, 10,351334 × 1728 (the Number of *Inches* in a *ſolid Foot*) produces 17887,105152 *Ounces*, the *Weight* requir'd; which you may reduce into other Denominations, as *Pounds*, &c. as you pleaſe. Again,

Suppoſe

Suppoſe the *Weight* of a *Gallon* of OLIVE OYL be requir'd:

Firſt (by the TABLE) 1 : 0,913 :: 0,527 *oz.* 0,481151 *oz.* equal to the *Weight* of 1 *Cubic Inch* of OYL;

Which 0,481151 × 231 (the Number of *Cubic Inches* in a *Gallon*) is equal to 111,145881 *Ounces Troy*, the *Weight* requir'd. On the contrary,

Suppoſe the *Weight* of a Quantity of any one of thoſe *Bodies* be given, the Content of that Quantity, in *Cubic Inches*, may from thence be found: Thus,

If it were requir'd to find how many *Cubic Inches* are contain'd in 111,145881 *Ounces Troy* of OLIVE OYL;

Firſt find the *Weight* of 1 *Cubic Inch* of OYL (and the like for any of the other kinds of Matter) by the Help of the proper *Tabular Number*, as above ſhewn; which will be found to be 0,481151; then *divide* the *given Weight* (firſt reduc'd into *Ounces*, if not given in *Ounces*) by the *Weight* of 1 *Inch*, and the *Quotient* will be the Number of *Cubic Inches* contain'd in the *Bulk* or *Quantity* of Matter propos'd.

So here 111,145881 (the given Number of *Ounces* of OYL) divided by 0,481151 *Ounces* (the *Weight* of 1 *Cubic Inch* of OYL) quotes 231 *Cubic Inches*, which is the *Content* requir'd.

Laſtly, I ſhall advertiſe the ingenious Learner, that *Archimedes* hath demonſtrated, That *A* BODY *ſpecifically heavier than* WATER, *weighs leſs in* WATER, *than it does in the* AIR, *by the Weight of as much* WATER *as is equal in Bulk to the given* BODY.

Whence,

Whence, if we wou'd know what any *given Bulk* or *Quantity* of any one of the *Bodies*, mention'd in the TABLE, that are heavier than WATER, wou'd *weigh*, when immerged in WATER, we have nothing to do but to ſubtract the *Weight* of as much WATER as is equal in *Bulk* to the *Body* propos'd, from the *Weight* of that *Body* in the AIR, and the *Remainder* will be the *Weight* of the ſame in the WATER.

As for *Example*;

Suppoſe you wou'd know what a *Cubic Inch* of *Pure* GOLD *weighs* in WATER.

From the *Weight* of 1 *Inch* of GOLD in the AIR, *viz.* 10,351334 *Ounces* (as was above found) *ſubtract* 0,527 *Ounces* (the *Weight* of a *Cubic Inch* of WATER) and the *Remainder* (9,824334 *Ounces*) is the *Weight* of the *Inch* of GOLD when immerged in WATER.

And in like manner 910,656 *Ounces* (the *Weight* of a *Cubic Foot* of WATER) *ſubtracted* from 17887,105152 *oz.* (the *Weight* of a *Cubic Foot* of *Pure* GOLD in *Air*), leaves the *Weight* of the ſame GOLD in WATER, *viz.* 16976,449152 *Ounces*, equal to 1414,704096 *Pounds*, *Troy*.

And the like for the *other* Bodies in the TABLE, that are ſpecifically heavier than WATER, *mutatis mutandis*. But this is ſufficient.

FINIS.

THE

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ERRATA.

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25	13	for 19*s.*	read 11*s.*
43	12	*twice* in 400, but *two*	*once* in 400, but *one.*
	13	*four*, but *forty*	*six*, but *sixty.*
70	16	*this*	*&c.*
93	11	5	6.
120	23	$9=80$ $4:9=80$	$q=80$. $4:q=80$.
	ult	$80=9$	$80=q$.
125	13	*to* the same	*in* the same
127	4	130	$130\frac{3}{4}$
134	8	36*l.*	136*l.*
141	11	*Ingredient*	*Ingredients.*
145	20	$10+3+13\frac{1}{3}$	$10+3=13$.
146	20	$+32$	$=32$.
160	14	$=5y$.	$-5y$.
170	15	25	15.
176	12	*we* put	*were* put.
177	9	6700	6720.
	11	3900	3920.
179	12	X	Z.
187	13	*or*	*of.*
227	13	4	2.
242	17	*added*	*added to.*
247	15	$\frac{1}{4}$	$\frac{2}{4}$.
255	11	$\frac{2}{3}$	$\frac{2}{3}$.
267	2	$8\frac{2}{3}$	$18\frac{2}{3}$.
278	14	0,7800	0,7890.
311	20	*by* the fol-	*take* the fol-
313	30	$0^h, 75'$	$0^h, 45'$
335	12	21363	75)21363.
	17	5755183	8112)5755183.
337	3	2)	3)
	13	46875000	46875)46875000.
	18	4383021000	4775243)4383021000.
363	6	to 4000	*to be* 4000.
	13	1000	10000.
380	17	10*d.* $\frac{1}{2}$	10 and $\frac{1}{2}$,
400	17	16*s.*	10*s.*
403	19	Insert, *what did each put in?*	
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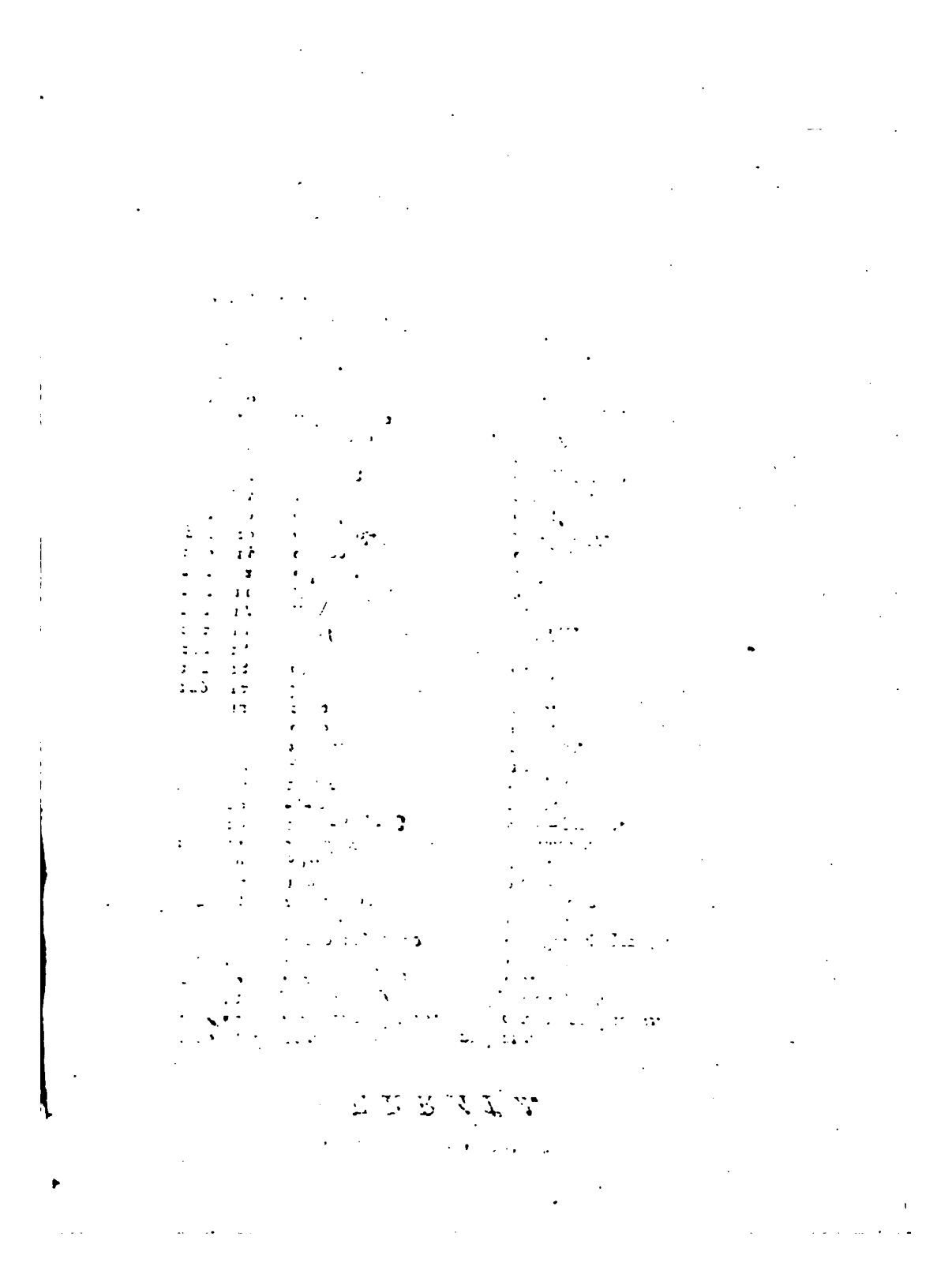

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